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ABSTRACT

The model competency-based education assessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics (OCTM) task force for implementing National Council of Teachers of Mathematics (NCTM) Standards for Grades 1 through 8. Three types of assessment for each grade are presented. The first type of model competency test has a largely traditional design. The second type of model competency based assessment is based on a checklist format to go beyond multiple choice assessment. Finally, the third type of model competency test includes authentic-type situations and is based on portfolio assessment. The assessment is done as an ongoing part of instruction. The instructional activity details how students apply mathematical skills, understanding, and thinking. Directions for the test, necessary materials, and detailed rubrics are provided. (ASK)

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Pythagorean
Theorem

Competency-Based Education Assessment Series

Mathematics

Mathematics Consultants

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Steve Meiring
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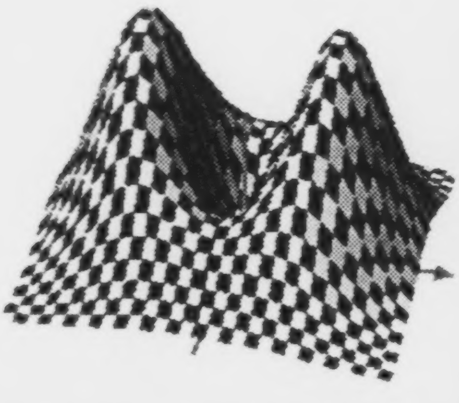

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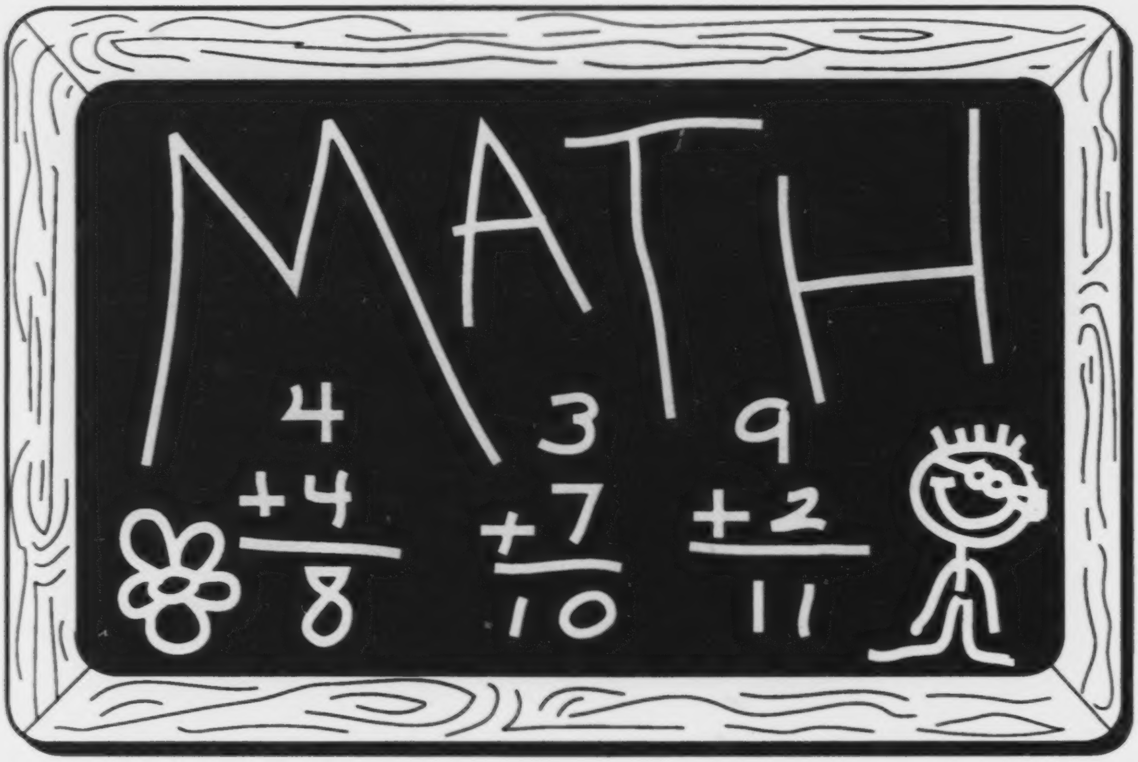
Competency-Based Education Assessment Series

First Grade Mathematics

Type 1 Assessment


MATH
4
+4
8
3
+7
10
9
+2
11

Developed by

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The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency test has a largely traditional design. The following information is intended to be a guide, not a rigid set of directions for use of this instrument. Decisions about the administration and use of this assessment are best made at the local level. District and teacher discretion should be used in administering the assessment in a manner best suited to the needs of your students.

- **How much time is needed?** This assessment is not intended to be a timed test. Students should be allowed as much time as needed to complete the test. Due to the length of the instrument, the test may best be given in two or three parts.

- **What are the directions?** A scripted set of directions is not provided. It is intended that the teacher read each question aloud and allow time for all students to respond completely before going on to the next question.

- **What materials are needed?** This assessment is designed as a paper and pencil, traditional assessment. If students routinely use counters, manipulatives, measuring tools, and calculating devices as an instrumental part of instructional activities, they should be available for use during assessment.

- **How should the assessment be scored?** The scoring guide provided is intended to be, as its name implies, a guide. Teacher discretion should be used in making judgments about the correctness of a response if it appears a student has interpreted a question differently than was intended.

Scoring Guide and Teacher Notes: There are 16 items on this assessment. Most items require students to select or provide an answer. In general these "single response" items are intended to be worth one point. There are some items, (1, 6, 9, 10, 11, 15 and 16) that contain several parts or require lengthier answers. These items would be worth more than one point. Below are specific scoring suggestions and discussion of the items.

Item 1 A and B: One point each. [Correct answers: black square for Part A and white square for Part B]

Objective: 1-1-2. The student will be able to sort objectives on multiple attributes.

This assessment focuses on students' abilities to identify objects by several criteria at the same time. Familiar shapes are used, but this assessment does not require children to know the names of those shapes.

Throughout the year, children should have frequent opportunities to sort a variety of objects (buttons, keys, cards, pictures, toys, etc.) on one or more attributes.

One of the earliest concepts in understanding probability is the understanding of certainty and chance. Some activities that can be done to build this concept include putting two colors of marbles (or bingo chips, tiles, counters, etc.) into a bag while children are watching then asking, "What will happen if you reach in without looking and choose one marble? Will it be red? Do you know for sure? Will it be blue? Will it be green? What can you tell for sure? etc." A variation is to put something in a bag or box without the children seeing it and asking questions about it. Answers could be put in three areas on a chalkboard or bulletin board:

I am sure the answer is yes	*I can't tell*	*I am sure the answer is no*

Ask questions like:
- Is the object blue?
- Is it a car?
- Is it smaller than my desk?
- Is it alive?

etc.

Item 10: One point for an X on the circle with 2 quarters and a penny. [The important concept is estimation of coin values and not explicit following of directions. The response should be marked correct if the right group of coins is indicated by any mark].

Objective 1-7-3 The student will be able to estimate the value of a given collection of coins.

Children should have practice in telling that a collection of coins is "more than" or "less than" some amount even when they cannot count the entire collection of coins. In this example, many children know that two quarters make fifty cents, but they cannot "count on" the one penny to make 51¢. Too often, children attempt to "estimate" by counting an entire collection then rounding the result. They should be encouraged to estimate (frequently using the "more than"/ "less than" framework) amounts of money, lengths, numbers of objects, etc. that they are not yet able to count exactly.

Although, for scoring purposes, each part is scored as correct (1 point) or incorrect (0 points), looking at the type of errors made can be informative. If a child chooses the hexagon in part B, for example, that student may not understand the effect of the word "not" in identifying the objects. Other patterns of choices can indicate that the child recalls only the first (or last) attribute in making a selection, or doesn't understand the vocabulary "less than" or "more than."

Item 2: One point for either correct number sentence or for circling both correct number sentences. [Correct answers: 3 + 2 = 5 and/or 5 - 3 = 2]

Objective 1-2-1	The student will be able to select apropriate notation and methods for symbolizing the problem statement and the solution process.
Objective 1-5-3	The student will be able to model a problem situation using a number phrase or sentence.

How many "more than" is a very important concept, but one that is often overlooked in teaching beginning number operations. This is the "compare" feature of objective 1-3-5 A strategy commonly used by children to solve "how many more" problems is to use a one-to-one pairing of objects. Another common strategy used by children is to start with the smaller number and "count on" until reaching the larger number. Therefore, either of the two number sentences above can represent equally valid representations of the problem solution. The more common "adult" representation of this problem is as a subtraction sentence (5 - 3 = 2), while it is quite common for children to formulate this problem in an additive way (3 + how many more = 5).

Children should be encouraged to model problems of joining, separating, and comparing in order to solve them rather than looking for "clue" words (more than, less than, difference, etc.). Also, note that one of the number sentences is given in "non-traditional" order (3 = 8 - 5). The ability to use different representations of the same number relationship is important.

Item 3: One point for choosing the strawberries "lined up" and paired for comparison.

Objective 1-3-5	The student will be able to develop the concept of addition and subtraction from situations in the environment, including joining, separating, and comparing sets of objects.

The one-to-one pairing of objects in order to determine "more than" should be modeled as a strategy for solving these problems. Students should be encouraged use many different solution strategies and representations for the problems they solve. Even if this is not their preferred strategy, they should recognize it as a correct representation of the problem.

The groups of strawberries would seem to indicate a joining of 5 strawberries and 3 strawberries into a set of 8 strawberries. It might be good to ask a child who

This problem may require the teacher to encourage children to use some of the problem-solving strategies they have learned. When given this problem, one child responded, "We haven't learned how to subtract big numbers yet." The test administrator then asked, "But can you solve the problem?" and the child said, "Yes," and proceeded to solve the problem correctly.

First graders can solve large number addition and subtraction problems and even some multiplication and division problems. Solving these using modeling with manipulatives and acting out strategies help develop the concepts needed to understand the algorithms they will learn later.

Item 6: 6 points possible. One point for each item correctly identified as having a circle or as not having a circle. Answers will vary depending on objects in the classroom.

Objective 1-4-3	The student will be able to identify two-dimensional shapes on three-dimensional objects.

To avoid confusion, the teacher should select one of each object to be used by the class in answering this question. The teacher should tell students that they should make their choices based on the objects at the front of the room. Children should have the opportunity to go to the front of the room and examine the objects if desired.

Item 7. One point for choosing the correct number sentence (5 + 2 = 7).

Objectives 1-2-1	The student will be able to select appropriate notation and methods for symbolizing the problem statement and solution process.
Objective 1-5-3	The student will be able to model a problem situation using a number phrase or sentence.

In this item, children need to demonstrate the ability to model the joining of two numbers from a real-life situation and to identify a correct number sentence to correspond to the joining.

The child who chooses the sentence "5 - 2 = 3" probably does not understand what is required in the problem, but can choose a number sentence which correctly uses the numbers in the problem. On the other hand, the child who chooses the number sentence "5 - 2 = 7" probably knows that the correct solution to the problem is 7 (possibly by counting all of the objects), but does not yet know how to symbolize the operation (s)he intuitively used.

Neither symbolic representation **only** nor concrete representation **only** are sufficient. As students develop an understand the concepts of combining, separating, and comparing, they should gradually be encouraged to record symbolic representations of their solutions.

Item 8: One point for a correct answer (beach ball).

Objective 1-6-1 The student will be able to recognize and count a collection of coins including pennies, nickels, dimes, and a quarter and determine its value.

If possible, children should be given real coins or plastic accurately colored and sized coins to use in answering this question.

Various strategies can be used in solving this problem. One strategy, of course, is to count all of the money (42¢) and compare that amount to the prices of the toys. Another strategy is to find enough money to pay for the objects--for example, the quarter and nickel make 30¢ so he could buy the car; the quarter and dime make 35¢ so he could buy the beach ball; using the quarter and dime for 35¢, he would need more than 10¢ more to buy the bear, but a nickel and two pennies is not enough to make 10¢ so there isn't enough to buy the bear.

Children can be encouraged to solve problems such as this (choosing from items in a school store, cafeteria, etc.) and to discuss their methods of solution with the class. In this way, children will learn a variety of strategies to apply.

Item 9A, B, and C: One point for each correct answer.

9A _X_ It will get dark tonight.
9B _X_ I will grow 9 inches taller by Friday.
9C _X_ I will find a quarter on the sidewalk tomorrow.

Objective 1-8-5. The student wil be able to identify events that are sure to happen, events that are sure not to happen, and those we cannot be sure about.

The answers given are the expected responses. However, an individual child's experiences may influence his/her answer. For example, a child who recently vacationed in Alaska may respond that "get dark tonight" is sure not to happen or something he/she cannot be sure about. Teachers should use professional judgment in asking students to explain questionable answers. If, in light of unusual personal circumstances, the child's answer would be correct, a score for a correct response should be given.

One of the earliest concepts in understanding probability is the understanding of certainty and chance. Some activities that can be done to build this concept include putting two colors of marbles (or bingo chips, tiles, counters, etc.) into a bag while children are watching then asking, "What will happen if you reach in without looking and choose one marble? Will it be red? Do you know for sure? Will it be blue? Will it be green? What can you tell for sure? etc." A variation is to put something in a bag or box without the children seeing it and asking questions about it. Answers could be put in three areas on a chalkboard or bulletin board:

I am sure the answer is yes *I can't tell* *I am sure the answer is no*

Ask questions like:
- Is the object blue?
- Is it a car?
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etc.

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Children should have practice in telling that a collection of coins is "more than" or "less than" some amount even when they cannot count the entire collection of coins. In this example, many children know that two quarters make fifty cents, but they cannot "count on" the one penny to make 51¢. Too often, children attempt to "estimate" by counting an entire collection then rounding the result. They should be encouraged to estimate (frequently using the "more than"/ "less than" framework) amounts of money, lengths, numbers of objects, etc. that they are not yet able to count exactly.

Item 11: One point for each correct representation of "5" (it is not necessary that the "one more" addition appear in the left position, etc.):

4 + 1
6 - 1
2 + 2 + 1 -OR- 3 + 3 - 1

Objective 1-3-6 The student will be able to learn strategies for addition of whole numbers such as

a. counting all
b. counting on
c. one more, one less
d. two more, two less
e. doubles
f. doubles plus or minus one and doubles plus or minus two
g. make ten
h. use ten frames

When you read the directions aloud to the children, you may want to hold up a copy of the example and point to "6 + 1" when you say " a 'one more' addition," etc. to help reinforce the patterns.

Throughout the year, children should be encouraged to develop their own strategies for writing and computing with numbers and to share those strategies with the class. When a child makes a statement like, "I know that 3 is 4 minus 1" is an ideal time to say, "Manuel's way is to think of 3 as a 'one less' subtraction" to reinforce terminology rather than teaching these strategies as algorithmic procedures to be memorized.

Item 12: One point for circling the correct statement (The fish is about 2 inches long because the distance from 1 to 3 is 2 inches.

Objective 1-6-2 The student will be able to measure lengths using non-standard units, centimeters, and inches.

Item 13: One point for writing 7 either on the answer line or in the circle on the number line.

Objective 1-3-2 The student will be able to decompose, combine, order, and compare numbers.

The objective is to have children order the numbers. If the directions are too difficult, an alternative would be to give the children number tiles (4, 7, and 12) to place in the shapes in the correct order.

Item 14: One point for circling statement C. " The shoe is longer than 2 keys." One additional point for **not** circling each of statements A and B.

Objective 1-6-2	The student will be able to measure lengths using nonstandard units, centimeters, and inches.

In administering this question, it is important to read each statement separately and allow children time to decide if the statement is true. After reading statement A, you might want to say something like, "Look at the picture. Then draw a circle around statement A if you think it is true. Do not draw a circle around it if you do not think it is true." Repeat similar directions for statements B and C.

This item measures children's abilities to compare an object to non-standard measures as well as their knowledge of the terms "shorter" and "longer."

Item 15: One point for each correct representation (up to a maximum of 3 for each number).

Objective 1-3-2	The student will be able to decompose, combine, order, and compare numbers.

Although many chldren in first grade have not learned addition facts up to 19, they should still be able to write 19 in more than one way. Many can use counting strategies to write representations such as

18 + 1
20 - 1
17 + 2
17 + 1 + 1

Although none of the examples used three or more addends to represent "5," if a child chooses to represent 8 as 3 + 3 + 2 or 19 as 17 + 1 + 1, his/her answer is correct.

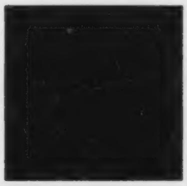
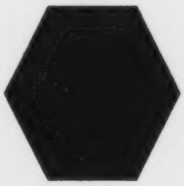
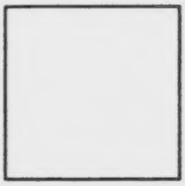
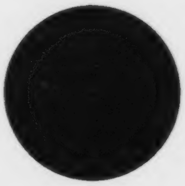
Item 16 Four points for any reasonable explanation indicating an understanding of addition as joining or combining of sets. A vague response with some elements of understanding might be worth 1, 2 or 3 points.

Information Sheet

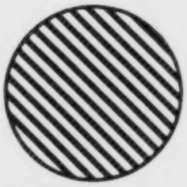
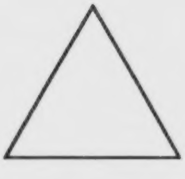
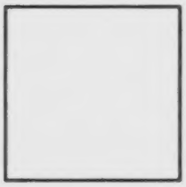
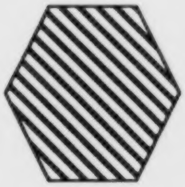
Item	Answer	Objective	Outcome Level	Critical Objective
1.	Black Square White Square	1-1-2	PS/A	Yes
2.	5 - 3 = 2.	1-2-1 1-5-3	K/S	Yes
3.	"lined-up" strawberries	1-3-5	C	Yes
4.	7	1-3-5	PS/A	Yes
5.	16	1-3-5	PS/A	Yes
6.	Answers will vary	1-4-3	K/S	Yes
7.	5 + 2 = 7	1-2-1 1-5-3	C	Yes
8.	Beach Ball	1-6-1	PS/A	Yes
9.	X ⌢ O	1-8-5	C	Yes
10.	See "About this Instrument"	1-7-3	C	Yes
11.	Answers will vary	1-3-6	K/S	Yes
12.	The fish is about 2 inches...	1-6-2	C	Yes
13.	Third Number Line	1-3-2	K/S	Yes
14.	C.	1-6-2	C	Yes
15.	Answers will vary	1-3-2	K/S	Yes
16.	Answers will vary	1-3-7	C	Yes

1. Circle the shapes described.

A. This shape
- has less than five sides
- has equal length sides
- is black

B. This shape
- has more than 3 sides
- has equal length sides
- is not striped

2. **Circle a number sentence that goes with the story problem.**

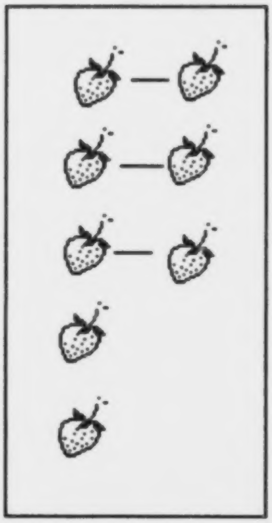
Dan had 5 pencils. Tim had 3 pencils. How many more pencils did Dan have than Tim?

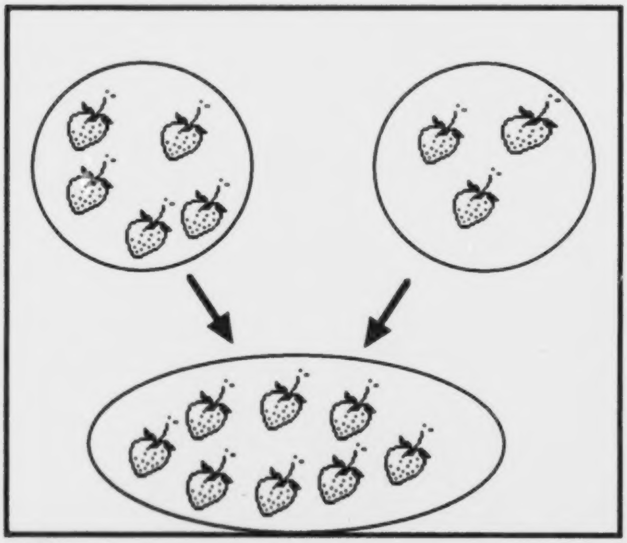
5 + 3 = 8   *5 - 3 = 2*

3 = 8 - 5   3 + 2 = 5

3. Put an X on the picture that helps to solve the story problem.

Mary has 5 strawberries. Sam has 3 strawberries. How many more strawberries does Mary have than Sam?





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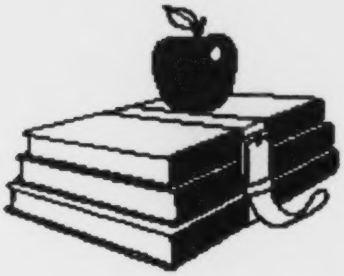
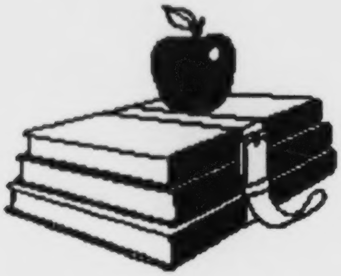
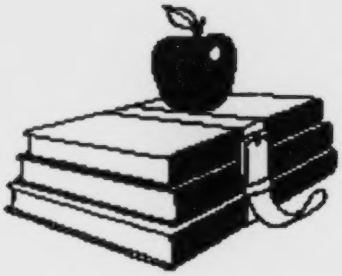


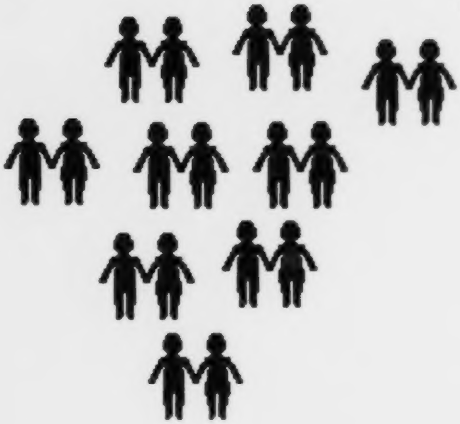


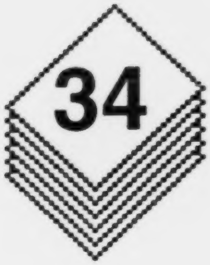


4. Paul read 3 pages in the morning. After lunch he read 4 more pages. How many pages did Paul read in all during the day?

Paul read_____pages in all.

5.





We have 34 sheets of red paper. How many sheets will be left after we give one to each of the 18 children here today?

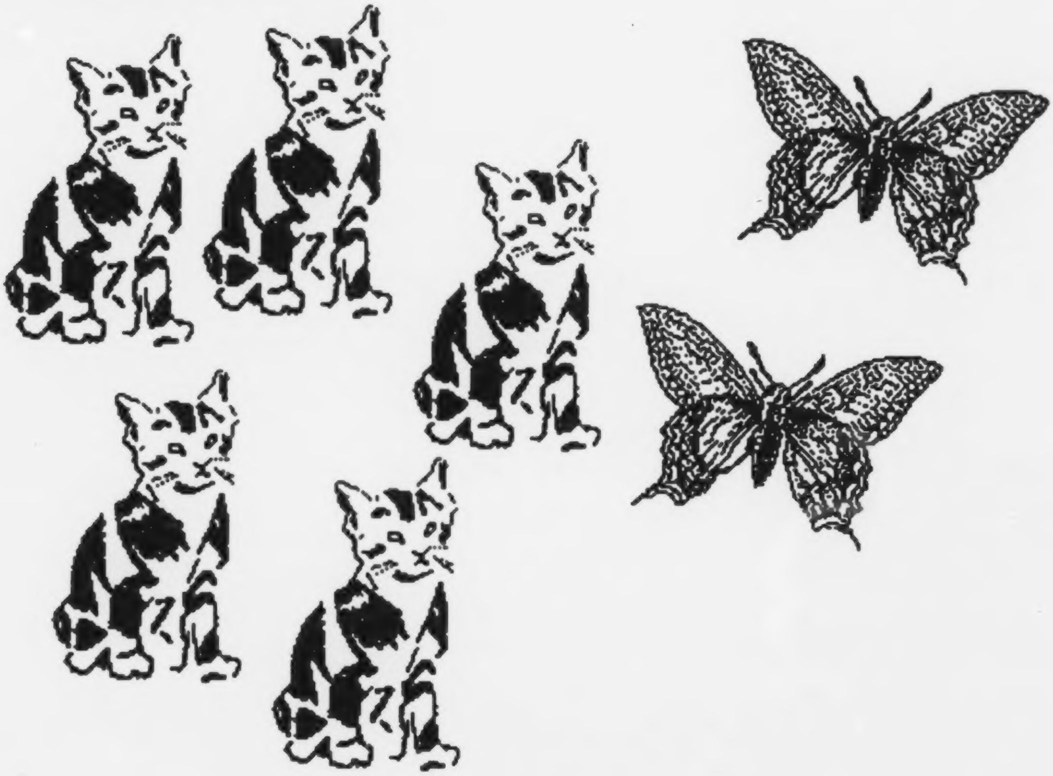
There will be ______ sheets left.

6. Some objects that we see every day are made up of many shapes.

Draw a line from the word CIRCLE to each object that has a circle as part of its shape.

Wastebasket
Window
Desk
Clock
Eraser
Pencil

7. **Circle the number sentence that tells how many butterflies and cats there are all together?**

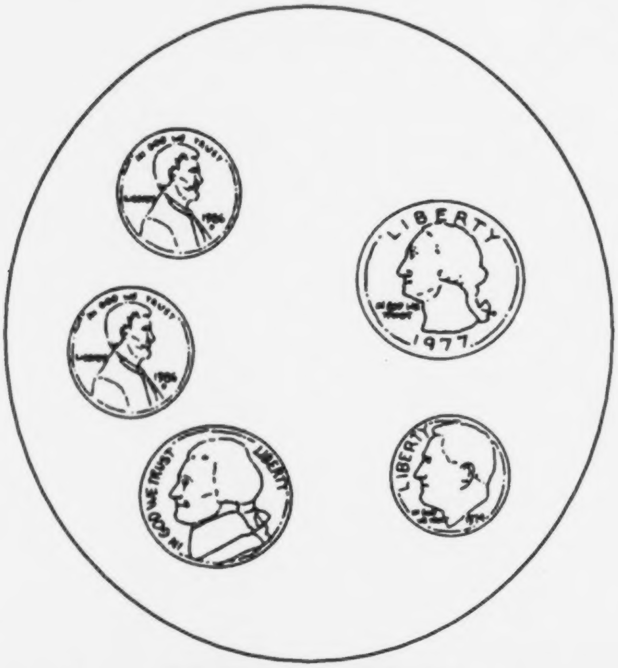


5 - 2 = 3

5 - 2 = 7

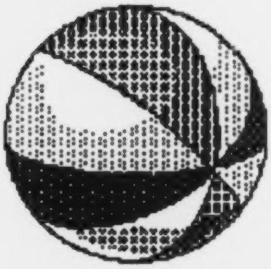
5 + 2 = 7

8.

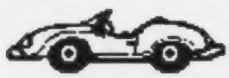


Ramon's Money

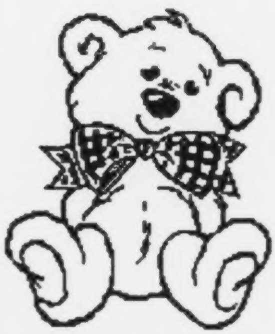
Look at the picture of Ramon's money . Look at the prices of the toys below.
Draw a circle around the most expensive toy that Ramon can buy.



35¢



29¢



46¢

9A. Think about the future.

Put an X by the thing you are sure will happen.

___It will get dark tonight.

___I will find a quarter on the sidewalk tomorrow.

___I will grow 9 inches taller by Friday.

9B. Think about the future.

Put an X by the thing you are sure will not happen.

__It will get dark tonight.

__I will find a quarter on the sidewalk tomorrow.

__I will grow 9 inches taller by Friday.

9C. Think about the future.

Put an X by the thing that may or may not happen.

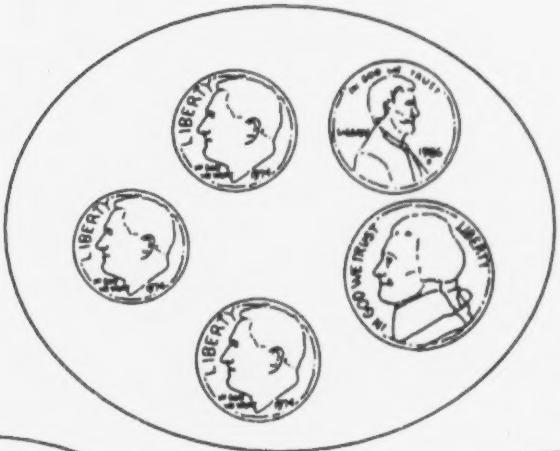
__It will get dark tonight.

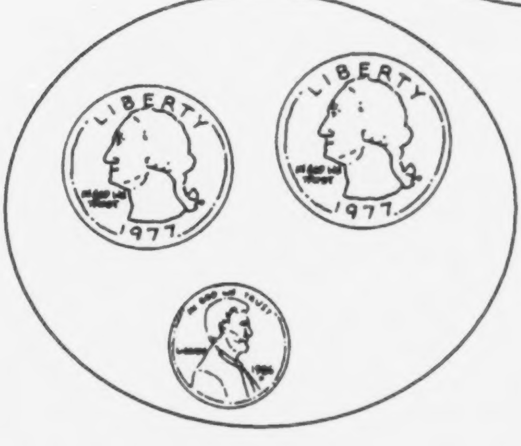
__I will find a quarter on the sidewalk tomorrow.

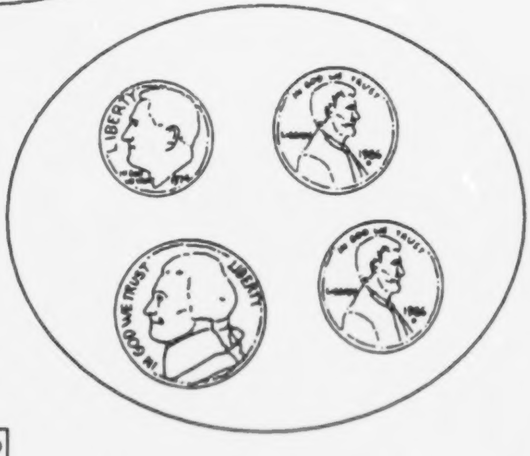
__I will grow 9 inches taller by Friday.

10. Estimate the values.

Put an X on the set of coins worth more than 50¢



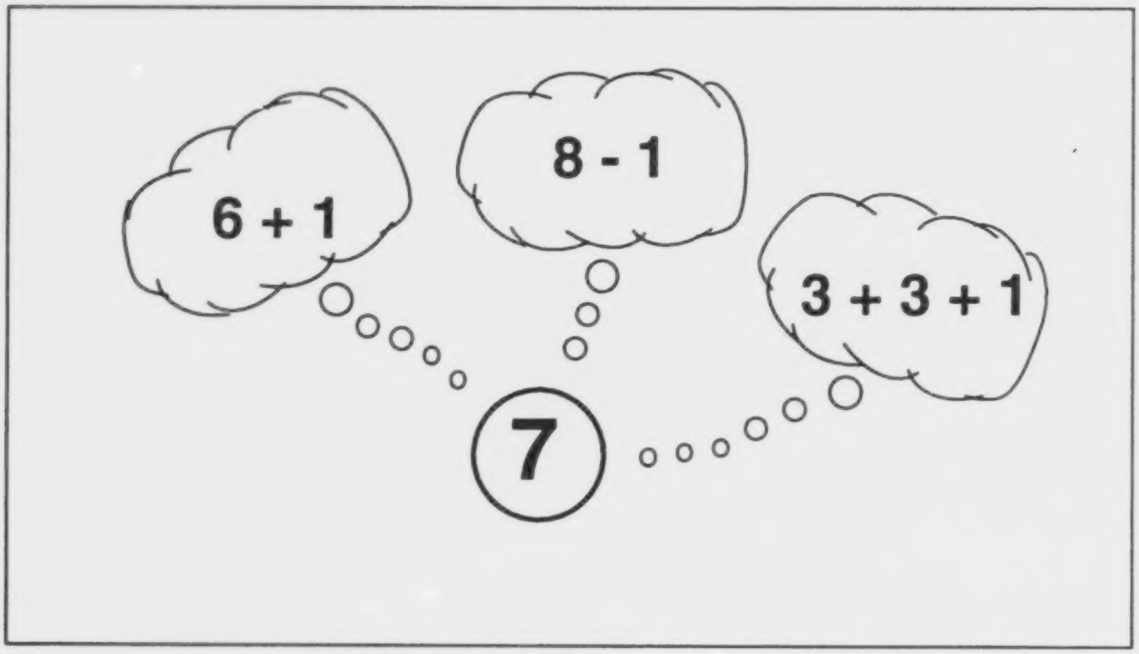


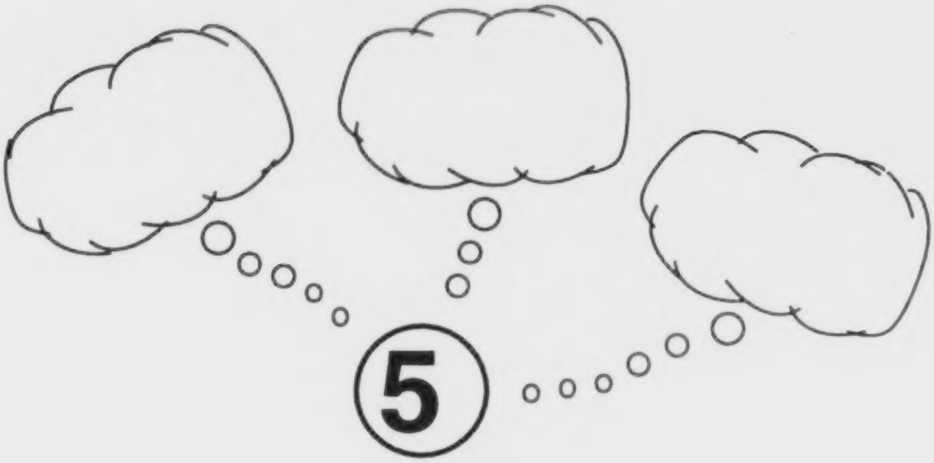


11. Look at the example:
The numeral 7 in the circle
is written as

- a "one more" addition
- a "one less" subtraction
- a double plus one

Example :

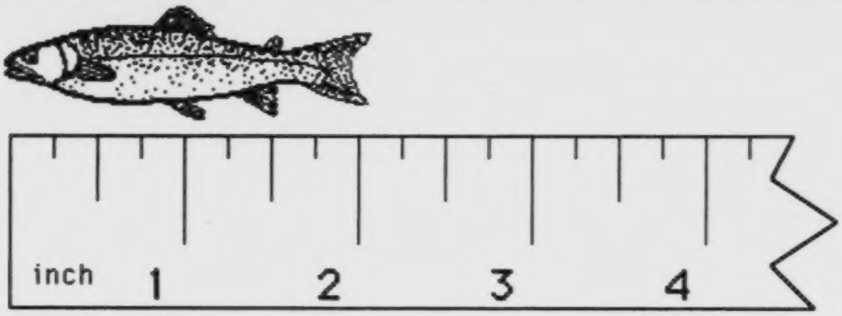




Now write the number 5 in the clouds as

- **a "one more" addition**
- **a "one less" subtraction**
- **either a double plus one or a double minus one**

12. Your ruler got broken, but your friend says you can still use it to measure the fish. Circle the correct statement.



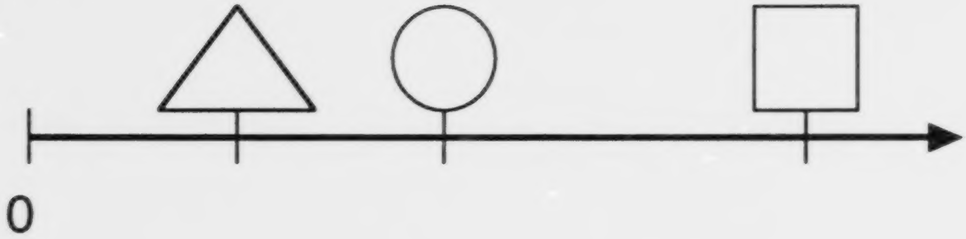
The fish is about 3 inches long .

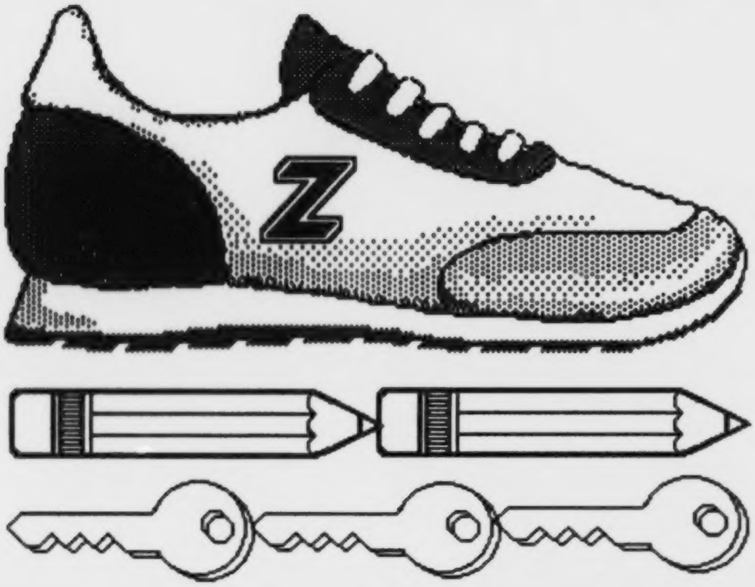
The fish is about 2 inches long .

You can't tell how long the fish .

13. If the numbers 4, 7, and 12 are placed in the circle, the square, and the triangle on the number line, which number would belong in the circle ○ ?

______ belongs in the circle





14. As I (you) read each statement, draw a circle around it if it is correct about the size of the shoe.

A. The shoe is shorter than two keys.

B. The shoe is longer than three pencils.

C. The shoe is longer than 2 keys.

15. There are many ways to write each numeral.
5 can be written as 2 + 3
5 can be written as 4 + 1
5 can be written as 7 - 2
You may know some other ways to write 5.

A. Now write 8 in three different ways.

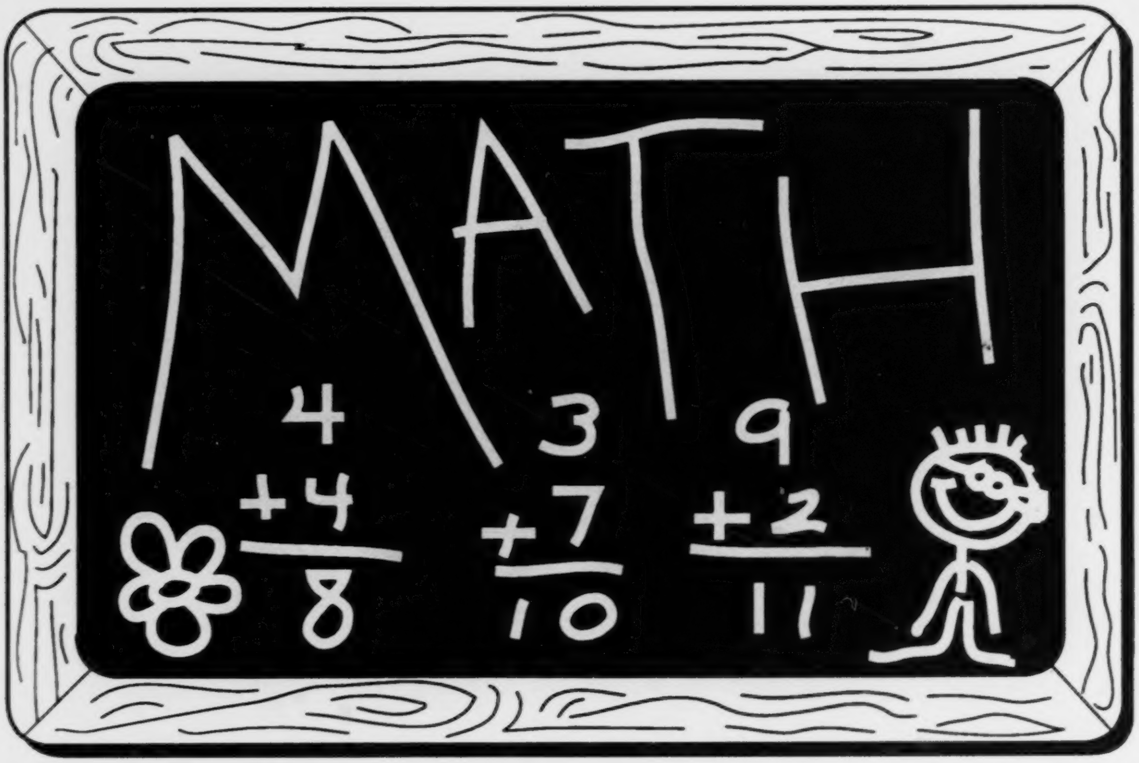
B. Now write 19 in three different ways.

16. Your friend is in kindergarten. He wants to know what to do if there are two numbers and a "+" (plus sign) between them. What would you tell him?

Competency-Based Education Assessment Series

First Grade Mathematics

Type 2 Assessment


MATH
4
+4
8
3
+7
10
9
+2
11

Developed by

Margaret Kasten

Patricia McNichols

Anne Mikesell

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency based assessment is based on a checklist format. The following information is intended to be a guide, not a rigid set of directions for use of this instrument. Many decisions about the administration and use of this assessment are best made at the local level. District and teacher discretion should be used in determining the best use of the checklist. The checklist provided contains only the first grade critical objectives from the Model Competency-Based Mathematics Program. Districts wishing to include additional objects as a part of this assessment should add those objectives to the checklist and develop district level tasks that could be used to determine if students are making satisfactory progress.

A task coded to each objective is included. Teachers could use these or similar tasks as the basis for professional decisions regarding student understanding and proficiency in mathematics. While specifics relative to the administration of the tasks should be determined at the district level, teachers may want to read the tasks to the students. Specific information relataive to the administration and interpretation of individual tasks follows.

Task One could be used to verify student ability to sort objects on multiple attributes. The task that is presented is a relatively simple using only two attributes, color and shape, a more difficult task of the same nature could be developed by adding another attribute. Size would be a possible attribute to include. Consideration of student responses should yield information allowing teachers to determine student achievement relative to objective 1-1-2.

Instead of using the printed black, gray, and white shapes in this instrument (which may not copy uniformly on some copiers), teachers could cut shapes substituting red for gray, blue for black, and yellow for white.

If this task is being given one-on-one, it might be possible to use actual objects, title cards ("triangle," "red," etc.) and string loops with children. Attribute blocks could be used to replicate this task. Other sets of objects could be used to create an *equivalent* task if there are at least two *different* attributes and some overlap of attributes. One commonly used material with young children is dyed macaroni in various shapes (shells, elbows, wheels, etc.) and in several colors. Make sure the same shape occurs in two or more colors and the same color occurs in two or more shapes. Pattern blocks are not suitable for this activity, since all of the triangles are green, all of the trapezoids are red, etc.

It is not a measure of children's abilities to sort materials if, in this task, they are asked to sort materials they have practiced sorting numerous times in class. They may only be demonstrating that they remember seeing the objects sorted and not understand the concept of sorting.

Task Two asks that students make up two story problems. This task is directly related to a student's understanding of the concept of addition and subtraction from situations in the environment, including joining, separating, and comparing of sets. It is also related to a student's ability to describe the operations of addition and subtraction in words. An acceptable answer for the addition problem might be something like: " Mary saw three butterflies and Sue saw two butterflies. How many butterflies did they see together?" The subtraction problem may be more difficult for students to generate. There is not an inherent "take away" situation when viewing the two sets, and since students often view subtraction as "take away," this may confuse some students. Nevertheless, a problem like, " There were three butterflies on the window. Two of the butterflies flew away. How many were left?" would be correct. Another model that students might employ would be comparison. The problem might say " Mary saw three butterflies and Sue saw two butterflies. How many more butterflies did Mary see than Sue?"

Task Three addresses the important problem solving skill dealing with appropriate notation and methods for symbolizing problem statements and solution processes. It is important to note that students do not need any formal understanding of the division in order to "symbolize" this problem. Technically, a student who plants one row of twelve has answered the question correctly. However, most students will use combinations of two and six or three and four. Four rows of three and three rows of four (or two rows of six and six rows of two) are separate and different ways in the view of most first grade children and should be judged correct.

Children may be given manipulatives to use for this item. Depending on the testing situation (paper/pencil vs. one-on-one interview), children might be asked to record their answers by drawing the patterns of "acorns" they made with manipulatives.

This task requires that children attend to three concepts simultaneously. The first is that of arranging the acorns in "rows." The second is to have the total number of acorns be 12. The third is that each row contain the same number of acorns. Some children may correctly complete one or two of these, but miss the third. For example, a child might make several rows of 12. Another child might have a total of 12 acorns, but have two rows of five and a row of 2. Some children arrange 12 acorns in circles or squares or in "clusters." Further questioning or use of similar tasks can help determine if the error is caused by misunderstanding of the directions for the task or by lack of comprehension of one of the concepts being assessed.

Task Four asks students to identify two specific shapes on three dimensional objects. Answers will vary depending on the specific items available (round topped wastebaskets vs. rectangular topped wastebaskets, etc.). It may be necessary to ask a child to explain his/her answer when it seems incorrect. One child may see a *rectangular* second hand on a clock that is a circle, for example.

To prevent the confusion of too many items to choose, the teacher may want to select 10-15 items to be placed in a convenient location in the classroom to be used in answering this question. Children should have the opportunity to go to that area to examine the objects if desired. This makes items easier to score. However, it limits children's ingenuity in finding circles, rectangles, or other shapes.

Teachers may want to change this item slightly by asking the students to think about their rooms at home, or the cafeteria, or some other location. Other variations on the item can be obtained by using different shapes (such as triangles or squares) or by asking students to identify all the shapes on a given figure.

Task Five asks students to cut out and paste pictures of coins. If possible, real or realistic plastic coins should be used for this sort. The activity asks students to count collections of coins which are all of the same type. An extension question might be "How much money is in all four regions added together"?

Task Six asks that students think about what might happen if a chip is drawn. They should be sure that the chip drawn will be black or white or gray. They know that they will not draw a chip that is red. They may believe that they are more likely to get a gray chip than a black or white chip but they should understand that they can't be sure which of the three colors they will get if they draw a chip.

One way to make this task more realistic to children is to place four colored counters in a bag in front of the class so that children can watch. You could substitute red, yellow, and blue for white, black, and gray.

Task Seven provides opportunities for students to measure with both standard and non-standard units. The difference between measuring the bat and tennis racket is primarily the orientation of the graphic. Students may have more difficulty deciding just what to measure with the tennis racket. If possible, children should be given real objects to measure. As with the pictured items, it is desirable to provide at least one item that challenges students to decide exactly what to measure.

If you are scoring this activity as children complete it and not scoring the whole class at a later time, you may want to give some children large paper clips and some children standard paper clips. Their answers will, of course, be different. However, be sure not to mix two sizes of paper clips for one child, as an important feature of measurement is that the units used to measure with must be all the same size.

Teachers may want to include a third activity and have students measure the length of the pencil they are using.

Task Eight asks students to write a number sentence describing a given situation. Using the graphic students are asked to write sentences reflecting both an addition and a subtractions situation. This task involves elements of several of the critical objectives: 1-2-1, 1-3-5, 1-3-6, and 1-5-3.

In the first task, most children will write either "5 + 2 = 7" or "2 + 5 = 7". However, a few children might see two groups of cats and write something like "3 + 2 + 2 = 7".

In the second task, chldren are asked to compare the quantities and answer the question, "how many more?" While adults typically see this as a subtraction problem and would symbolize it as "5 - 2 = 3", an equally common and equally valid child's response might be to think 2 butterflies and *how many more* make 5, and write "2 + 3 = 5."

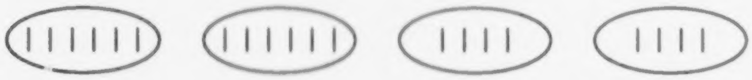
Task Nine also reflects several objectives. Students have to "decompose" the number 13, they may use a strategy for addition such as "doubles plus one" which certainly requires children to think about methods for symbolizing the problem. Students may solve this by subtracting, by drawing chips on top of the paper and counting, or by addition.

Task Ten, like Task Five, would be even more effective if done with real coins. It is important to try to get students to estimate rather than count. They should be developing strategies that will help them make estimates from a relatively brief visual inspection. If this task is administered in an interview format, the student could be asked to explain his/her strategy for each of the collections of coins (not just to select one to explain). In this way, it would be possible to determine if the student were actually estimating (not just counting the complete total then giving the answer which is closest). It could also reveal lack of understanding not uncovered by the answer to the task itself. For example, in the second set of coins the student might say it is closer to 25¢ because there are only 8 coins and 25 is the smallest number.

Task Eleven is an example of an instructional assessment task. This particular task could be used to verify that students can use the "counting on" addition strategy. In this task, getting the correct "answer" alone is not sufficient. The task specifically asks the student to model the "counting on" strategy. The design of similar tasks for other strategies would not be difficult. It is important for students to begin to develop a repertoire of strategies for addition and subtraction. It will be necessary to help them understand that different strategies will be appropriate with different types of problems.

Task Twelve, the Wigiwump activity can be easily changed by changing the number of legs on the wigiwumps or by changing the total number of legs. This task will help teachers determine student proficiency as decomposing and combining numbers.

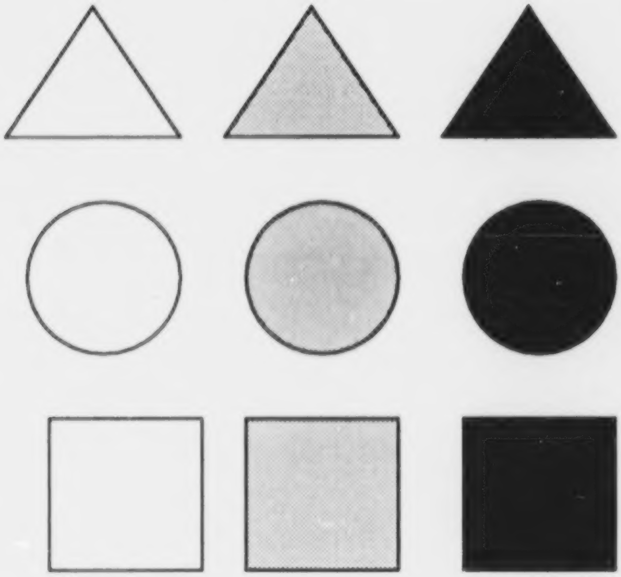
As with Task Three, students must balance two concepts at a time. The number of legs on ***each*** Wigiwump must fit the parameters of the problem (6, 4, or 2) **and** the total number of legs must be 18. Some children will have one Wigiwump with 18 legs. If they persist with this representation after the directions are read a second or third time, it probably indicates an inability to decompose a number. Some children may choose to be more symbolic in their representation of this problem. Instead of actually drawing the Wigiwumps, they may just write 4 + 4 + 4 + 4 + 2 = 18 or



Task Thirteen should help teachers determine a student's depth of understanding of the operations of addition and subtraction as well as contribute to an understanding of whether or not a student has be able to apply the concepts to situations in the environment .

Students will need an additional sheet of plain paper for drawing the pictures to go with their story problems.

First Grade Critical Objectives		satisfactory progress	Notes
1-2	sort objects on multiple attributes		
2-1	select appropriate notation and methods for symbolizing the problem statement and the solution process		
3-2	decompose, combine, order, and compare numbers		
3-5	develop the concept of addition and subtraction from situations in the environment, including jointing, separating, and comparing sets of objectives		
3-6	learn strategies for addition of numbers such as a. counting all b. counting on c. one more, one less d. two more, two less e. doubles f. doubles plus or minus one and doubles plus or minus two g. make ten h. using ten frames		
3-7	describe the operation of addition and subtraction in words		
4-3	identify two-dimensional shapes on three-dimensional objects		
5-3	model a problem situation using a number phrase or sentence		
6-1	recognize and count a collection of coins using pennies, nickels, dimes, and a quarter and determine its value		
6-2	measure lengths, using non-standard units, centimeters, and inches		
7-3	estimate the value of a given collection of coins		
8-5	identify events that are sure to happen, events that are sure not to happen, and those we cannot be sure about.		



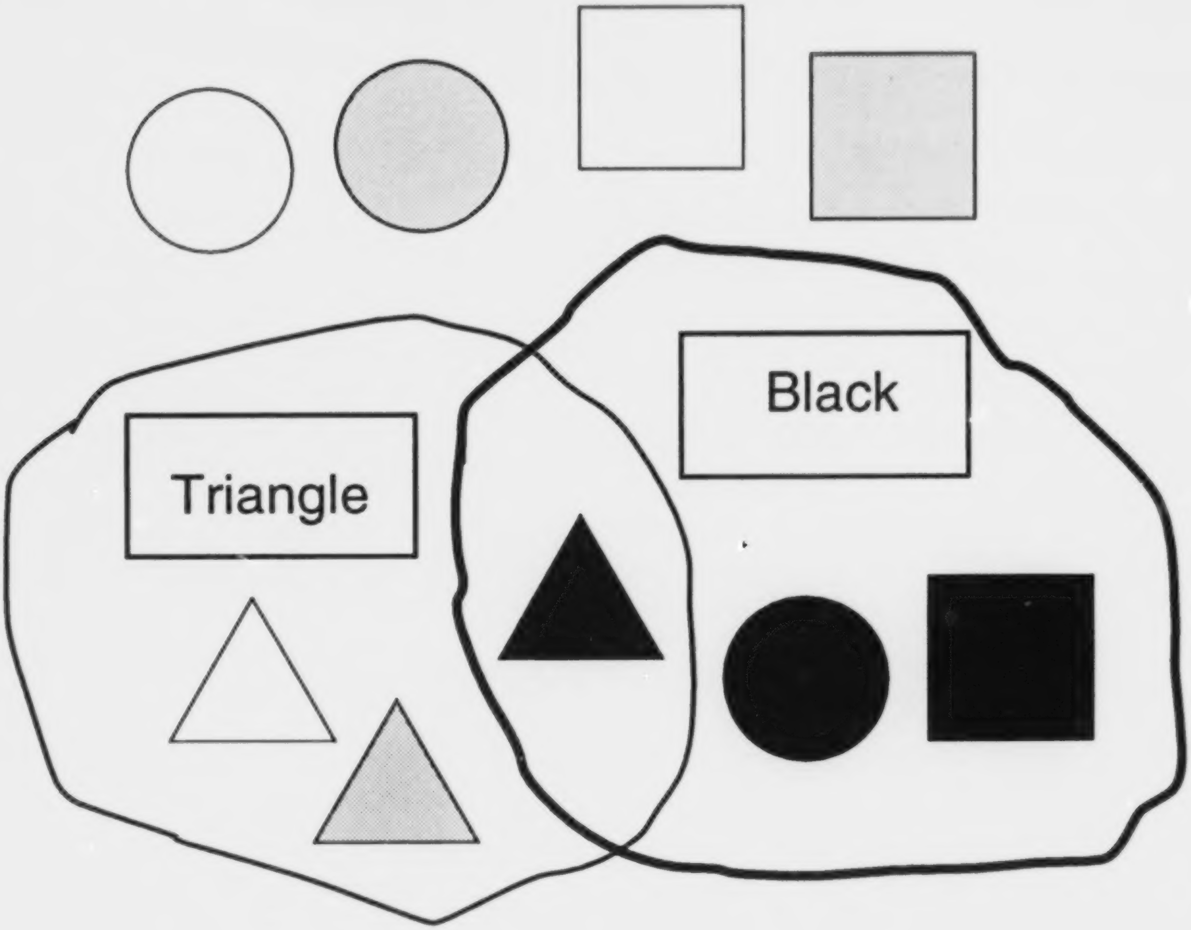
Here are some blocks with 3 shapes (triangle circle, and square) and 3 colors (white, gray, and black).

Nicky's teacher writes the 3 shapes and 3 colors on cards:

White	**Gray**	**Black**
Circle	**Triangle**	**Square**

and then the teacher arranges 2 loops of string on the table. She puts the card that says"Triangle" in one loop and the card that says "Black" in the other loop. She asks Nicky to put all the triangle shapes in the correct loop and all the black shapes in the other loop. Look at the picture on the next page and answer the questions.

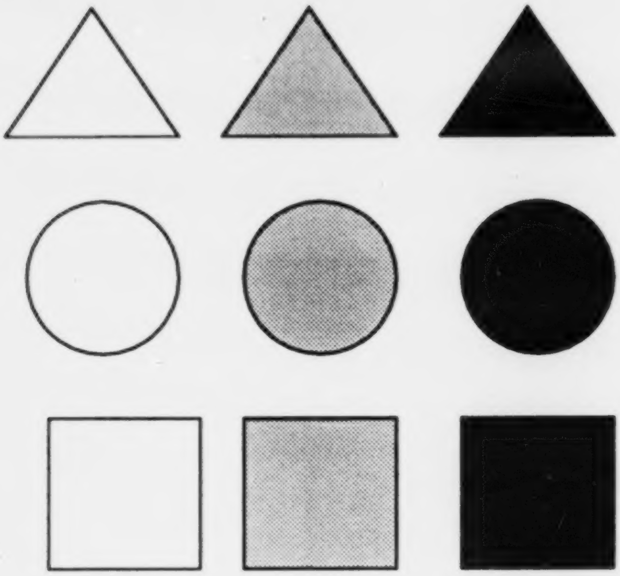
Nicky's arrangement:

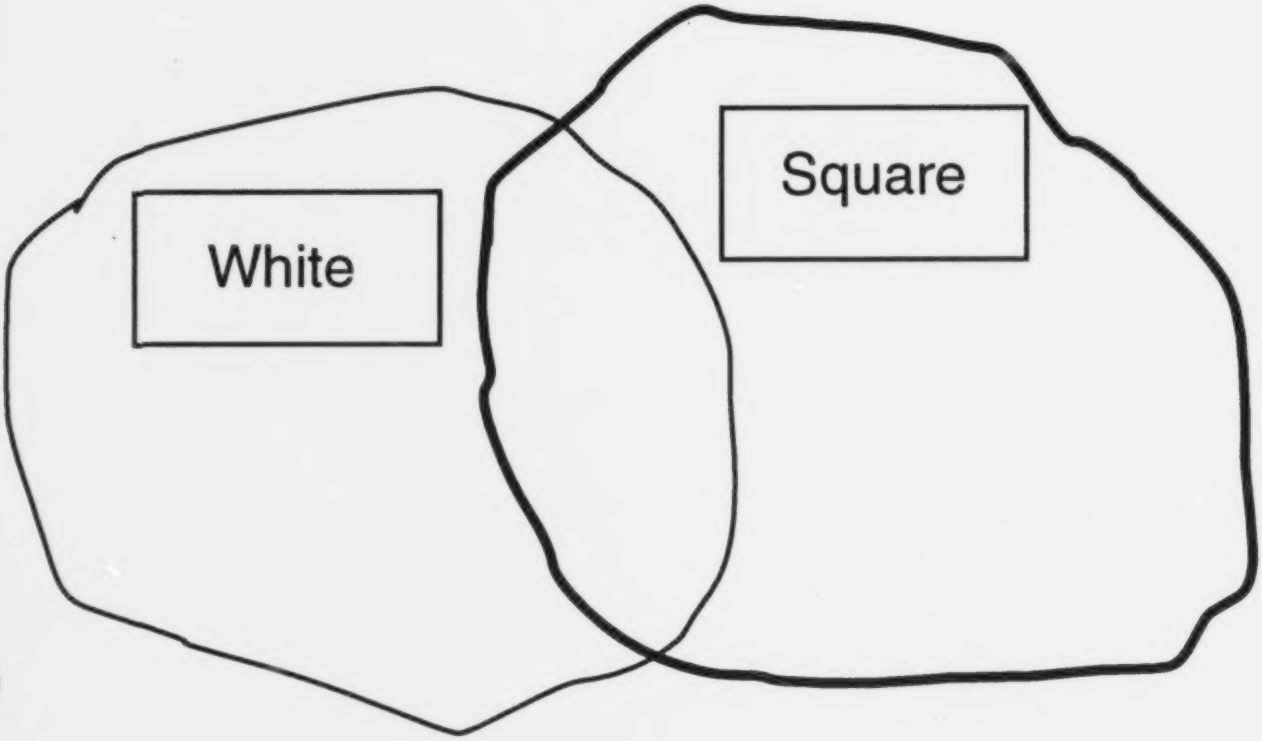


Did Nicky put the blocks in the right places?_______________

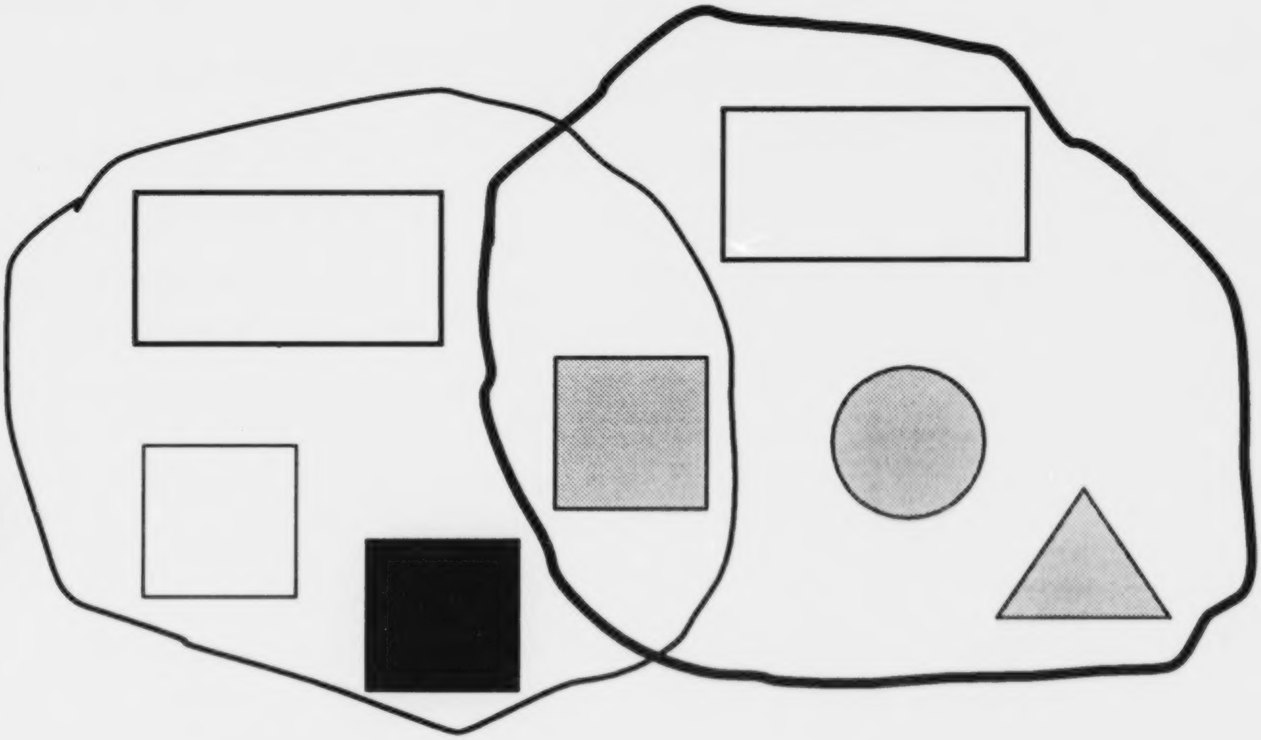
Why are four shapes not inside either loop?

Cut out the shapes and arrange them in the loops. Paste them on.

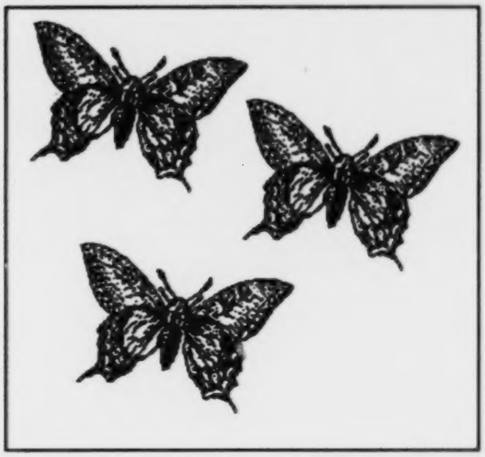


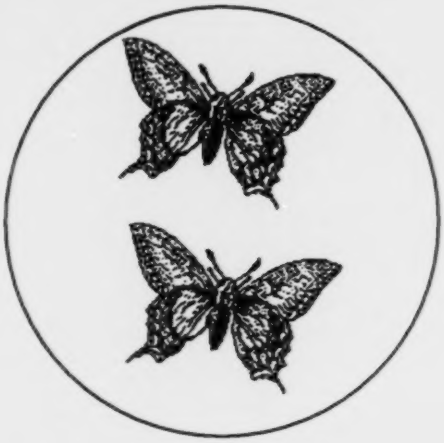


Sammy picked two cards, put in the correct blocks and THEN turned the cards upside down.



Write the correct words on the cards.



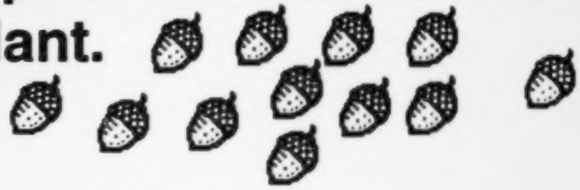


Make up 2 story problems using the two sets of butterflies shown.

Write an addition story problem about the butterflies here:

Write a subtraction story problem about the butterflies here:

Your class wants to plant oak trees. You have 12 acorns to plant.



Show one way your class could plant the 12 acorns in rows with the same number of acorns in each row.

Now show a different way your class could plant the 12 acorns in rows with the same number in each row.

Some objects that we see every day are made up of many shapes.

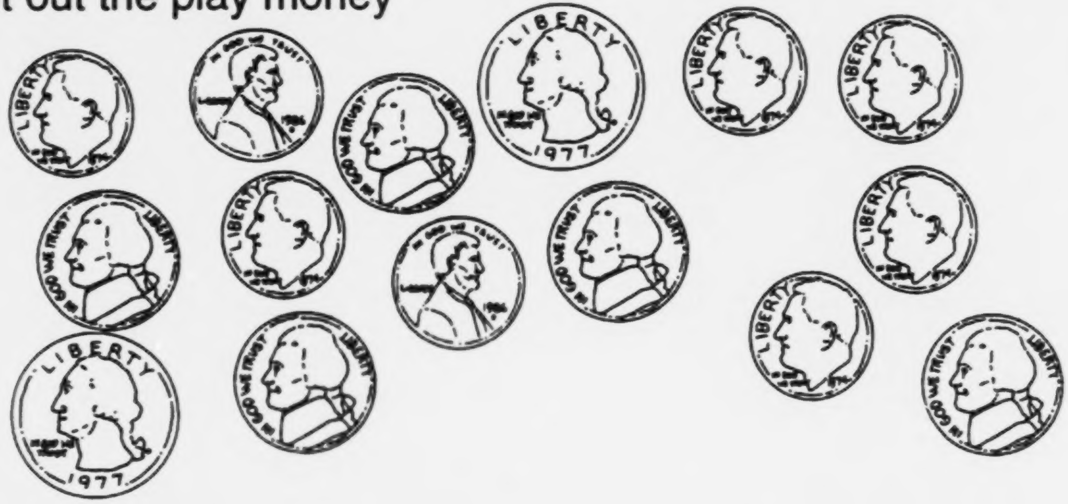
Name 3 things in the classroom that have circles as part of their shapes.

1.
2.
3.

Name 3 things in the classroom that have rectangles as part of their shapes.

1.
2.
3.

Cut out the play money



Paste each piece of money in the correct region below

Pennies 1¢

Nickels 5¢

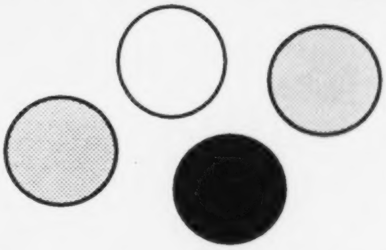
Dimes 10¢

Quarters 25¢

Put an X on the box that contains the most money.

Task Five

There are four counters in the bag: one white, one black, and two gray.

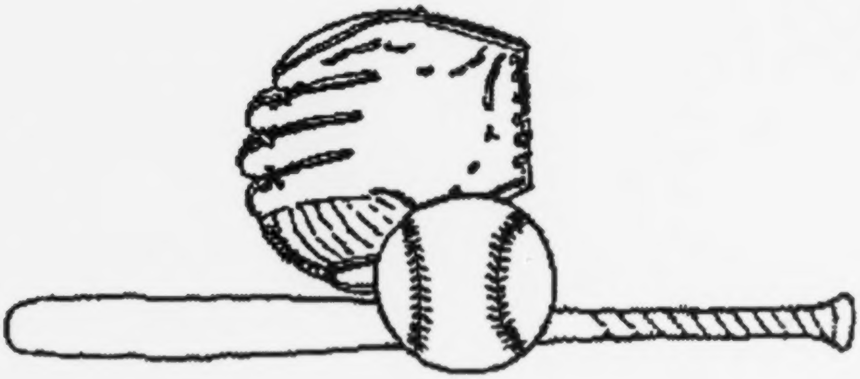


If you pick one counter from the bag without looking:

Tell one thing that you are sure is true about how the counter will look.

Tell one thing you are sure will not be true about how the counter will look.

Tell one thing that may or may not be true about the counter you picked.



Measure the picture of the bat.

Use the paper clips your teacher will give you.

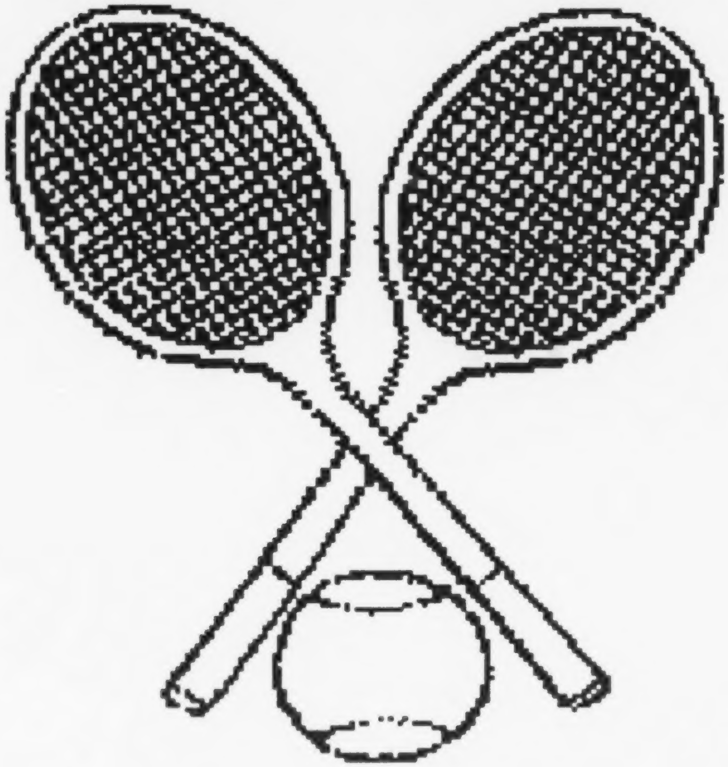
Use a centimeter ruler.

Use an inch ruler.

The bat is __________ paper clips.

The bat is __________ centimeters.

The bat is __________ inches.

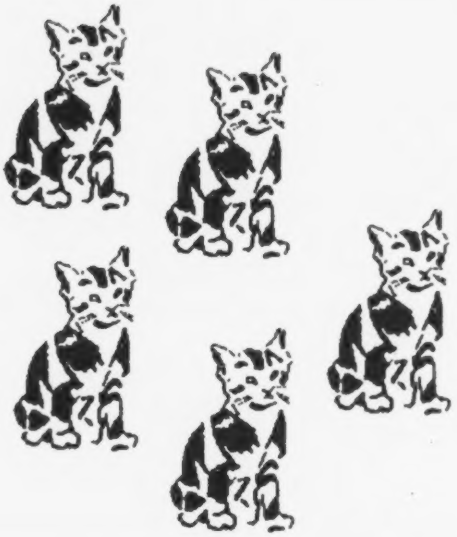


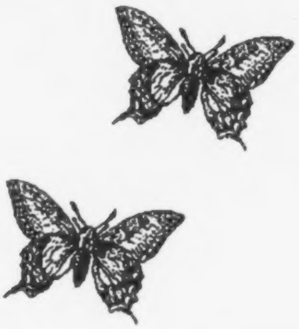
Measure the length of the tennis racket in the picture. Use the paper clips your teacher will give you. Use a centimeter ruler. Use an inch ruler.

The racket is ___________ paper clips.

The racket is ___________ centimeters.

The racket is ___________ inches.



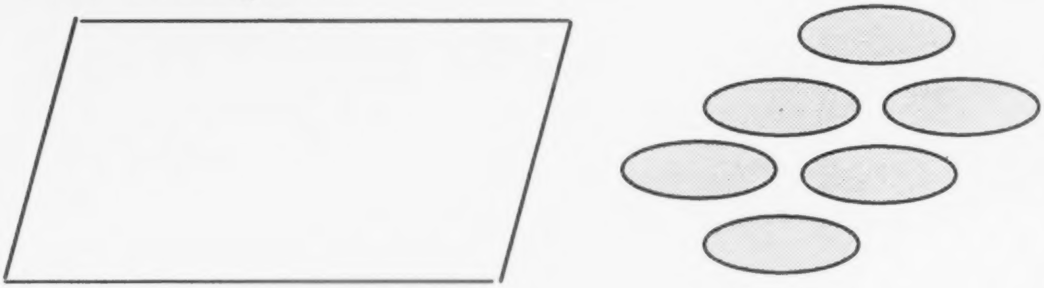


Write a number sentence that tells how many butterflies and cats there are all together.

Write a another number sentence that tells how many more butterflies there are than cats.

Task Eight

Sue has 13 chips. A piece of paper is covering up some of the chips.

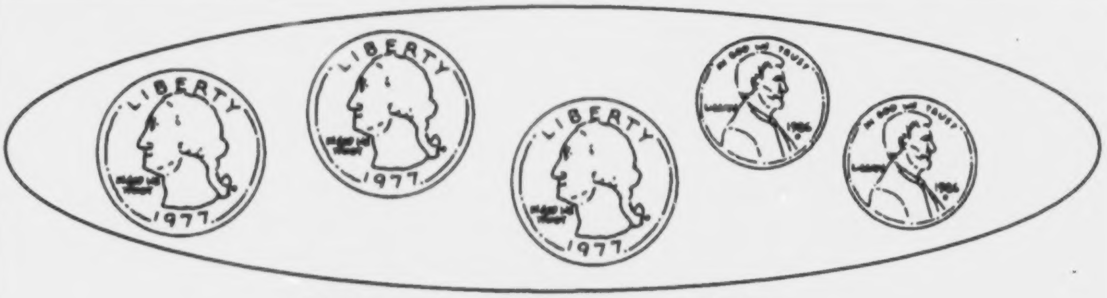


How many chips are under the paper?

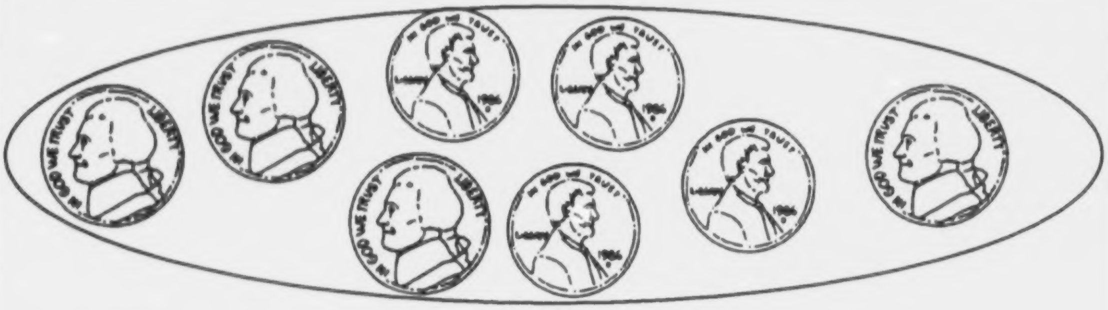
__________ chips

Explain how you figured out how many chips are under this paper.

Look at the collections of coins below. Without counting would you say each collection was about 25¢, 50¢ or 75¢? Circle the answer for each set.


LIBERTY
1977
LIBERTY
1977
LIBERTY
1977

25¢ 50¢ 75¢

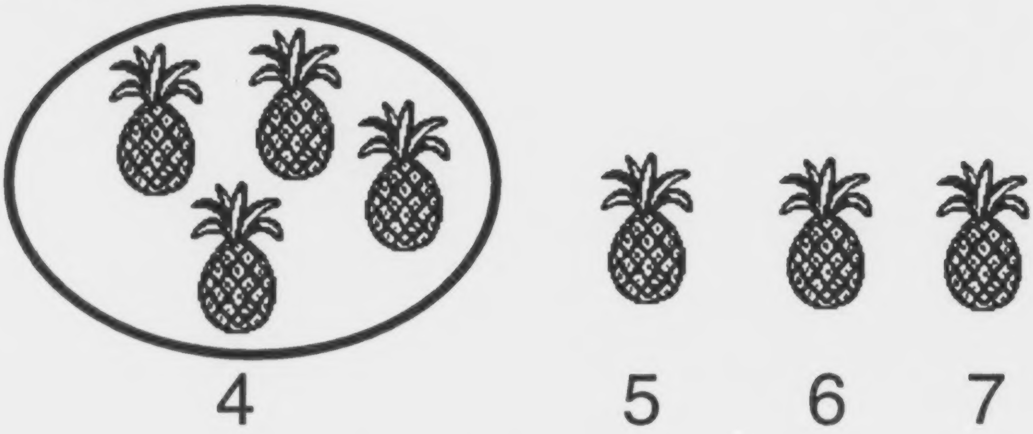


25¢ 50¢ 75¢

Pick one set and explain how you knew what to estimate without counting.

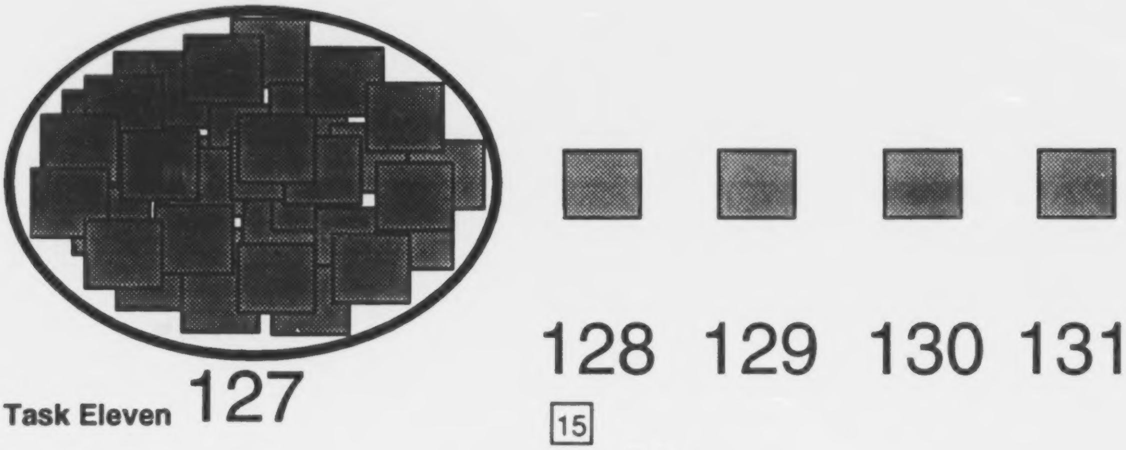
One way to do addition is to COUNT ON.

To find 4 + 3 start with 4 and count 3 more.



So, 4 + 3 equals 7.

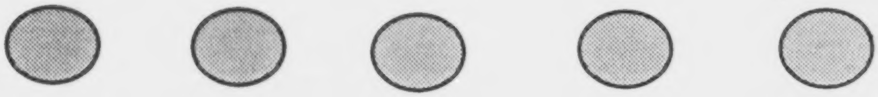
This was easy, but it works with bigger numbers too, like 127 + 4.



Now you know that 127 + 4 = 131.

One more example: 346 + 5

1 2 3 4 5



346 347 348 349 350 351

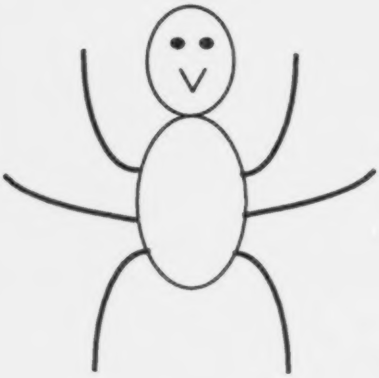
So 346 + 5 = 351

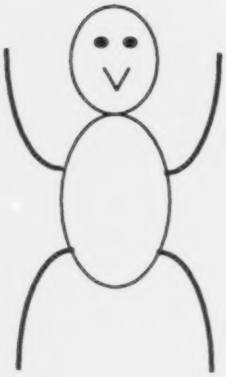
Now you do it, COUNT ON to find

177 + 4

789 + 12

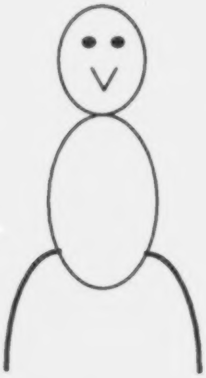
Some Wigiwumps have 6 legs.





Some Wigiwumps have 4 legs.

Some Wigiwumps have 2 legs.



All Wigiwumps have either 6 or 4 or 2 legs. Wigiwumps cannot have a different number of legs than 6 or 4 or 2.

Draw some Wigiwumps that have a total of 18 legs.

How many Wigiwumps did you draw?___

Now draw a different number of wigiwumps that have a total of 18 legs.

Choose either "Addition" or "Subtraction" for this project. Tell which of the two operations you chose____________.

Write a story problem that would use the operation you chose.

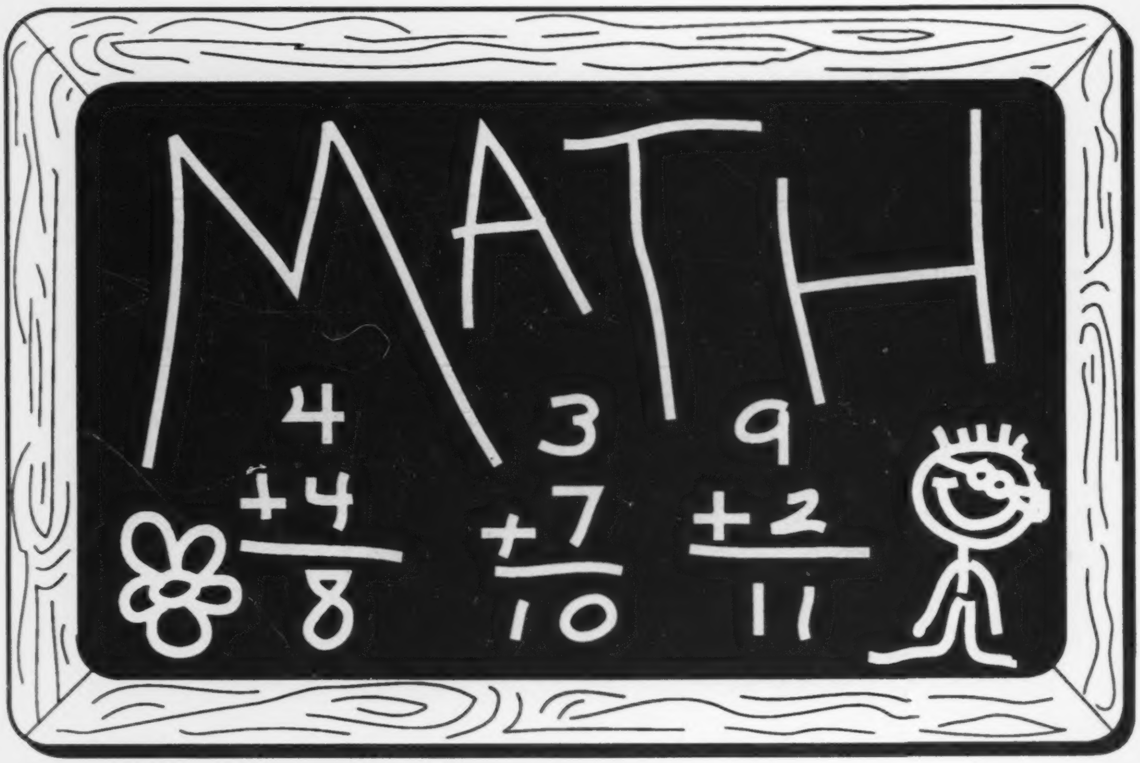
On your drawing paper, draw a picture that goes with your story problem.

Describe in words what your operation means.

Competency-Based Education Assessment Series

First Grade Mathematics

Type 3 Assessment


MATH
4
+4
8
3
+7
10
9
+2
11

Developed by

Margaret Kasten

Patricia McNichols

Anne Mikesell

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About this Instrument

This model competency test is an example of assessment that is integrated into the instructional process. The assessment is done as an on-going part of instruction. The instructional activity provides information about how the student applies mathematical skills, understandings, and thinking. Objectives from multiple strands are embedded within and assessed through the activities in which the students are engaged as part of instruction. Adequate time and appropriate format for assessing the objectives is included in the design of the instructional unit. Multiple forms of assessment are routinely used throughout the year - for example, student products, teacher observation, interviews, self-assessment, and journal writing. Periodically assessment will focus on gathering information about how well each student is progressing relative to the local competency-based mathematics program goals and objectives.

Instructional units that provide a context for assessing targeted objectives are identified and included within the year plan. Instructional activities and assessment techniques are matched to the purposes and needs of competency-based assessment as determined by the local program and assessment guidelines.

The instructional tasks within which the district-wide, grade level assessment will occur should involve situations that will reflect ways in which mathematics is learned and applied. These tasks will be richer and more in depth than those included on traditional, paper-pencil tests. The student products and records selected will be collected and evaluated in a standardized manner. The tasks should allow student to display the full range of performance and be evaluated using a scoring process that reflects the nature of the objectives and what students should know and be able to do. At a suitable point in time, a thorough review of the records and work collected for each student can be used to determine how well the student in progressing in relation to grade-level outcomes.

The suggested questions and student sheets to support those questions which follow serve as an example of how this type of assessment might be carried out. The model a is not intended to be fully developed instructional unit that can be inserted into a yearlong instructional plan as is, but rather it represents a guide for identifying and embedding competency-based assessment within instruction.

A number of publications such as the National Council of Teachers of Mathematics resource, Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions (Stenmark, 1991) are available.

The following story line can be used to create activities to assess the critical objectives for the first grade found in the Model Competency-Based Mathematics Program. The following checklist is provided to show the objectives being assessed in each activity.

First Grade Critical Objectives		satisfactory progress	Activity Number
1-2	sort objects on multiple attributes		2, 3
2-1	select appropriate notation and methods for symbolizing the problem statement and the solution process		1, 2
3-2	decompose, combine, order, and compare numbers		1
3-5	develop the concept of addition and subtraction from situations in the environment, including jointing, separating, and comparing sets of objectives		1, 4, 6
3-6	learn strategies for addition of numbers such as a. counting all b. counting on c. one more, one less d. two more, two less e. doubles f. doubles plus or minus one and doubles plus or minus two g. make ten h. using ten frames		4
3-7	describe the operation of addition and subtraction in words		4
4-3	identify two-dimensional shapes on three-dimensional objects		1
5-3	model a problem situation using a number phrase or sentence		1
6-1	recognize and count a collection of coins using pennies, nickels, dimes, and a quarter and determine its value		6
6-2	measure lengths, using non-standard units, centimeters, and inches		5
7-3	estimate the value of a given collection of coins		6
8-5	identify events that are sure to happen, events that are sure not to happen, and those we cannot be sure about.		1,3

Overview of the Field Day Project

To set the stage, teachers should discuss the upcoming "Field Day" activities with students. Some activities that are common in field day plans include an egg carry, tire roll, sack race, softball throw, long jump, and 50-yard dash. Since many first graders may be unfamiliar with these activities, the teacher may want to show a related video or have some students demonstrate some of the activities. The best problem-solving occurs when children can put themselves into the problem situation. When they have limited understanding of the context of the problem, they are unable to apply their own full range of mathematical understanding to it.

Ideally, if the school has an actual field day, this project should be done in conjunction with it, with first graders taking on some of the actual tasks of planning for their class's participation.

For purposes of this sample assessment, it is assumed that the following things need to be done to get ready:

- decide what color shirt to wear so that the class will look like a team
- sell cookies to raise money to buy matching hats
- find out who is going to be in each event
- invite parents and friends
- do lots of mathematics to answer questions.

If an actual field day is being planned, some of the tasks may be changed to meet the needs of the school's plans. Another activity (holiday program, etc.) could be substituted for the field day if it fits better with an individual school's operation.

Cookie Project

Within the cookie project, there are many decisions that need to be made. Some are a matter of choice, while others have right and wrong answers. Some decisions are dependent on previous decisions.

Students will be assessed on their abilities to:

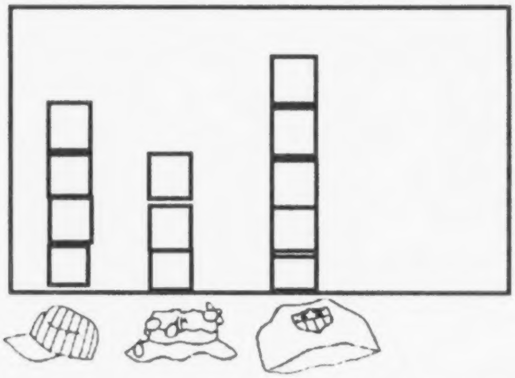
- Identify needed information to solve a problem.
- Use problem-solving strategies such as role play to find a solution, make a table to sort information, look for a pattern to predict a solution.
- Skip count by twos.
- Develop the concept of addition and subtraction from situations in the environment, including joining, separating, and comparing sets of objects.

A sequence of student assessment pages is included. The questions are not numbered, since it is not expected that they will be used as a "test" given at one or two sessions. Rather, it is expected that they will be used at relevant times throughout the project. Depending on classroom conditions, it may be possible to assess students on the tasks included here in a one-on-one interview or in an interview with a small group of 2 or 3 students. It is important to assess ***every*** student on each of the activities. Therefore, using the questions for whole class discussion with a different child answering each one is not acceptable. If these are used as pencil/paper tasks for students, teachers should **read all directions and questions** to students as many times as necessary for students to understand what they are to answer.

For each task, alternatives and teaching ideas are included in this description. Within each "project," tasks are numbered for reference to this commentary.

Cookie Project - Task One. If you are actually going to buy hats, it would be preferable to have some sample hats provided by the vendor(s). Children could choose the actual hat they prefer. After each child has indicated his/her choice, the choices could be graphed using Post-it™ notes or pictures of the hats.

The teacher should provide bulletin board or wall space for a Post-it™ graph of hat choices.



Cookie Project - Task Two. If you are actually going to purchase hats, adjust the price to the price of the hat chosen by your class (use only even dollar amounts).

Read the directions to students. Read "Do you need to know . . ." followed by each entry then ask children to circle **Yes** or **No**.
Example: "Do you need to know how many boys are in our class? Circle 'Yes' if we need to know this in order to find the cost of buying a hat for everyone in our class or 'No' if we do not need to know this."

Cookie Project - Task Three. Use the same price as in Task Two. For this task, it would be appropriate to have children work in cooperative groups to figure out the answer to the question. Manipulatives should be available for the groups' use. Make sure children understand that later everyone in the group should be able to explain how they got the answer and how they know it is correct. As students work in groups, the teacher could use an anecdotal record to look for evidence of understanding. Specific strategies, comments made by children, etc. should be noted. You will not necessarily have evidence of each element by each child, but will gain valuable insight into how children are thinking about the mathematics. A sample recording sheet might look like:

Cookie Project - Task 3
Anecdotal Record of Understanding

Name	Counts number in class as:	Evidence that student knows $2 needed for ea.	Method for calculating total	Comments by student in group

Cookie Project - Task Four. This task is important in that it has children take a look at the decisions that need to be made in any sales situation. It is one often overlooked in mathematics tasks. Often all of the amounts are predetermined by the problem, leaving children with the impression that these are items over which no one has any control. Additionally, some children may have had experience with selling lemonade in which they got to keep all of the money and did not have to pay for the supplies. Children may have siblings who have sold candy or Girl Scout Cookies in which they had no control over the price and did not realize that the items had to be paid for before they were sold. Understanding this concept helps the child understand the problem-solving process necessary. As students work through this task, read each item and ask children if they need this information. If so, then they should circle it.

Cookie Project - Task Five. Prior to answering the questions that make up this task, children could work in cooperative groups to try to solve these problems. For assessment purposes, using a checklist and observing each group is one way to determine each child's understanding of the task and its solution. Another alternative is, after children have solved the problem in groups, have each child complete the student sheet for Task Five to demonstrate his/her individual understanding of the work done by the group.

Cookie Project - Task 5
Problem-Solving Checklist

Cite specific evidence relevant to each problem aspect:

Name	Understands profit of $1 for each bag cookies	Understands 2 bags needed to buy one hat	Understands how to get total for class	Other relevant comments

Cookie Project - Task 6. This task is similar in structure to Task Five and can be completed in the same ways. Before attempting this task, it is appropriate to have a discussion about how much it would cost to make the cookies ourselves. Starting with Ms. Sugar's price and asking if making the cookies ourselves would probably be more expensive or less expensive can be valuable in helping children understand where prices come from. It is important to discuss what goes into the price of making cookies (although actual calculations of costs of butter, sugar, etc. are too complex). Other relevant discussions concern setting a price for the cookies. What are the consequences of setting too high a price? Too low? As with some previous tasks, one way to solve these problems would be to have students work in groups initially but complete the problem sheets individually later.

Cookie Project - Task Seven. This task can be introduced almost anywhere during the cookie project. It assesses a student's ability to separate a number into two numbers and compare them (more, fewer). If students wish to use manipulatives during this assessment, they should be permitted to do so. If manipulatives are used, it is important that their use arises out of the child's thoughts about solving the problem. Thus, it would be appropriate, for example, for a student to have access to a tub of tiles from which the child would have to count out the number needed to solve the problem. It would be ***inappropriate*** to give children 12 tiles each and tell them to divide these into two piles--Derek's and Thomas's--so that Thomas has more than Derek.

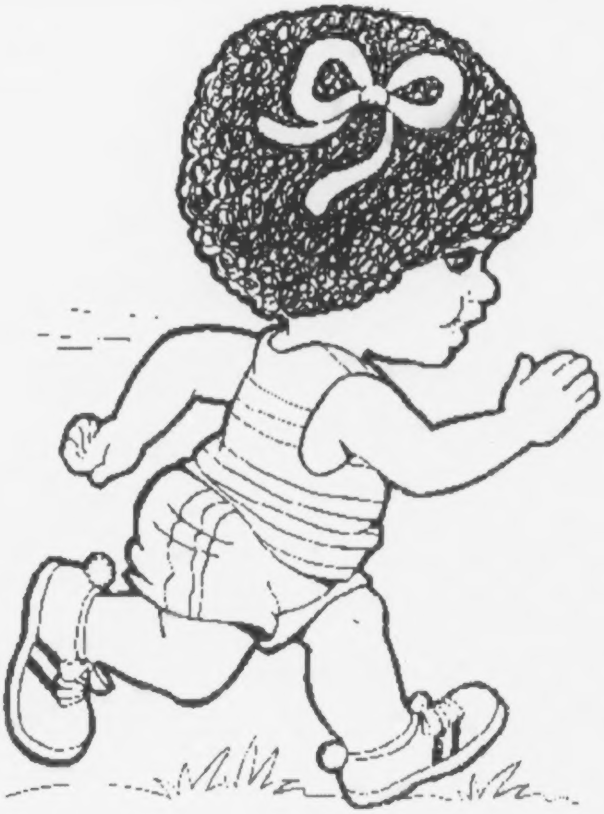
Cookie Project - Task Eight. Actual containers should be available for students to observe. Other containers could be substituted for the coffee can and cracker box. The containers should represent a variety of shapes (circle, square, rectangle, etc.). In class discussion, in order to keep to the theme of the field day plans and cookie sale, class discussion should include many attributes of the containers. Are they strong enough so they won't break or tear with cookies in them? How many cookies will each hold? Will the cookies get broken?

Cookie Project - Task Nine. The best way to do this task is to have actual bags of cookies (each with 5 chocolate chip cookies and 5 sugar cookies). Choose one to sit in front of the class.

Read the task directions then read each statement separately. After each statement, ask students to decide if they are sure the statement is true or if they cannot be sure.

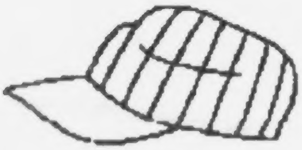
Name______________

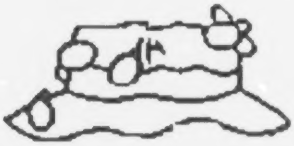
What could happen on field day?

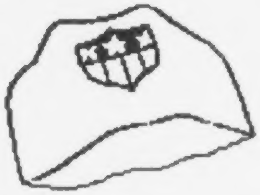


We want to get hats to match our shirts. We need to make the money to buy the hats. Each class will have it's own project to earn money for the hats. We get to sell cookies to earn our money. We have some choices to make for this project.

Some are easy choices. Circle the hat you like best.







Write your name on a Post-it™ note and put it on the graph aove the picture of the hat you chose.

The hats our class chose cost $2.00 each.
We want to figure out how much it will cost to buy a hat for each member of our class.
Do we need to know:

How many boys are in our class?	Yes	No
How many students are in all four first grade classes?	Yes	No
How many girls are in all four first grade classes?	Yes	No
How many students are in our first grade class?	Yes	No

How much money would you need to buy a $2.00 hat for each student in your class?

____

Explain how you figured out your answer and how you know your answer is right.

We could buy cookies from Ms. Sugar's Bakery and sell them to our parents and friends. Then we will not have to make the cookies ourselves.

Which things do we need to know in order to plan to earn enough money for hats?

How much we must pay Ms. Sugar for the cookies we buy.

Whether our class likes brownies better than chocolate chip cookies.

How much we will charge for the cookies we sell.

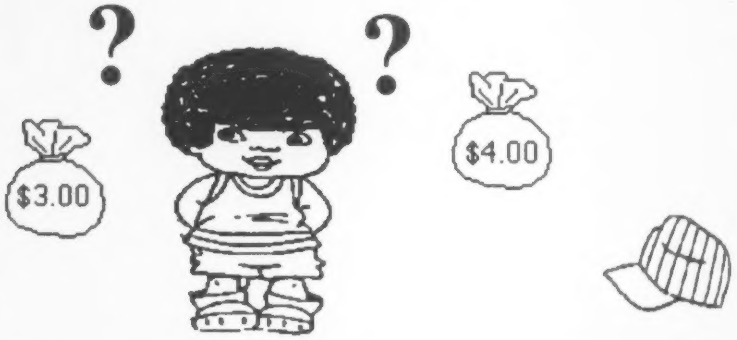
How many cookies Ms. Sugar can make in one hour.

What kind of cookies Ms. Sugar makes.

How many cookies to buy from Ms. Sugar.

How many cookies our class must sell.

We have to pay Ms. Sugar $3.00 for each bag of cookies we buy.
We charge $4.00 for every bag of cookies we sell.
How many bags must we sell to buy one hat? ____________________



How many bags of cookies must we sell to buy a hat for every student in the class?___________________________

Yesterday we found out that if we bought cookies from Ms. Sugar for $3.00 and sold them for $4.00, we would earn $1.00 for each bag.

If we make the cookies ourselves, it will cost us $2.00 for each bag instead of the $3.00 Ms. Sugar charges.

If we still want to make the same amount on each bag, how much should we charge for a bag of cookies we make ourselves? ____________

If we make our own cookies, and charge $4.00 for each bag, how much will we earn on each bag we sell?

____

How many bags would we need to sell in order to buy a hat for each member of our class?

____

Draw a picture, write a number sentence and give the answer that goes with this story.

Derek has 12 cookies in his bag. He wants to share with Thomas and he wants Thomas to get more cookies than he keeps for himself. How many cookies might he have given to Thomas?

Draw your picture here:

Write your number sentence here:

Derek could give Thomas________________ cookies.

Louie thinks that the cookies should be sold in containers that are stronger than bags. Your teacher has brought in 2 containers for you to think about.

A. An empty coffee can
B. An empty saltine cracker box

What shapes do you see when you look at the coffee can?

What shapes do you see when you look at the saltine cracker box?

Every bag of cookies has five chocolate chip cookies and five sugar cookies.

If you pick one cookie out of a bag without looking at it, what can you be sure about?

The cookie will not be oatmeal.	**I am sure**	**I cannot be sure**
It will be a sugar cookie or a chocolate chip.	**I am sure**	**I cannot be sure**
It will be a sugar cookie.	**I am sure**	**I cannot be sure**

Shirt Project

The Shirt Project is much shorter than the Cookie Project, consisting of only of three tasks. These tasks focus primarily on using data analysis to make decisions. As with other elements of the Field Day Project, if another similar decision is more appropriate (e.g. choosing a team name from among four choices--with the other classes having the same four choices), substituting a "real" decision is more meaningful for children.

Task 1: Begin with a whole class discussion so that children understand the task. If you have four first grades, you could give names of the classes who get to choose first, third, and fourth. Make sure children realize through this discussion that, once a color is chosen, no other class can choose it. Therefore, your class might not get their first choice of color.

In cooperative groups, have children come up with a plan for choosing a color. Each group should present their plan to the whole class, clarify any parts that are unclear, and answer questions. Some plans children might come up with are:

- Everyone picks his/her favorite color cube and places it in a stack of the same color. The tallest stack wins.

- Everyone writes his/her favorite color on a Post-it™ and puts it on a graph to see which color gets the most votes.

- Use a spinner with 4 colors and spin to see which color to pick.

Good questions to ask might include:

- What should you do in case of a tie?
- How many of the people in the class will be happy with the color chosen?

Probably no one will suggest having everyone write down 2 colors and graphing all the choices--so the teacher could suggest this as a method. Although the following tasks use this method, if time permits, the favorite method of the class could be used as well as the method in Tasks 2 and 3. The class could discuss why the results were the same (or different) with the two methods.

Task 2: First use the sample chart of student choices. Read the background and directions for the task before giving them the question sheet.

Task 3: Make a chart with the names of all class members. Have each child put an X under the **two** colors they want to choose. Have children use this chart to answer the questions.

If children have difficulty expressing their ideas in writing, the final section of this task can be done in an interview format.

The school will provide us with shirts. Each of the four first grades will wear a different color shirt. The colors are RED, GREEN, YELLOW, and BLUE.

The four first grade teachers pulled numbers out of a hat to see which class gets to choose its shirt color first, second, third, and fourth.

Our class gets to choose second.

How should we decide which color shirt to wear?

What should we do if the class that chooses first takes the color we want?

Five students made an X under the two colors that they wanted for shirts. Look at the chart they made.

Name	Red	Blue	Green	Yellow
Sue	X	X		
Tom	X			X
Jan	X		X	
Roz	X	X		
Dan	X	X		

What color should this group order to make the most people happy?

____

What color should they get if their favorite color is taken?______________

Field Day Events

Setting the Stage:

Read the background information to students. Substitute your actual field day events if appropriate. The question should be used for class discussion. Summarize children's ideas for ways to use mathematics on the chalkboard or a story board.

Task 1: Discuss the three events so that children understand them.

Fill out the chart of events with children by placing
Very Important or Not Important
in the squares as you discuss the events. You should begin with a blank chart as shown below and end with the completed chart:

Event	Speed	Strength
Softball Throw		
50 Yard Dash		
Long Jump		

Event	Speed	Strength
Softball Throw	Not Important	Very Important
50 Yard Dash	Very Important	Not Important
Long Jump	Very Important	Very Important

In cooperative groups, give children the chart of children's talents. Explain: This activity will help you learn how to decide who should be the representative in each event. For some events, speed is the most important. In other events, strength is more important. Last year, only 6 students want to be in these events. They rated themselves on strength and speed.

Direct groups to look at the chart of what is needed and the chart of children's talents to decide who they think would be best for each event.

Task 2: Children should make a chart of their own abilities. Each cooperative group should make a separate chart.

For "Speed" children can rate themselves as:
Slow
Average
Very Fast
For "Strength" children can rate themselves as:
Weak
Average
Very Strong
Write these words on a chart or chalkboard for children to see.

After completing their group's chart, each child should individually answer the question about the Long Jump.

Task 3: Explain the chart of times from last year's Field Day. Explain that they are going to need to add several numbers. As an example, to find Rodney's total time for all three events, you would need to add 3 + 5 + 1 to get 9.

Have students complete Task 3 on paper or in interview format.

Task 4: Give children a copy of the chart to complete this task. It will probably be necessary to discuss what determines the "best" time (smallest number). Usually, the larger the score the better.

Task 5: Substitute the name of your school, correct dates, etc. If you prefer, you could have cut out invitations on colored paper. The important things are that

- the two invitations are different sizes
- in each invitation, length and width are different
- children gain experience with measuring instruments in both standard and metric measurement

Task 6: If you prefer, you can use real or play money instead of the pictures of coins.

The second part of Task 6 is an estimation task. If this is done in an interview format, it would be good to ask students to tell you how they made their determinations and to record their answers as documentation of their understanding of money and estimation.

CHOOSING WHO WILL BE IN EACH EVENT

There are lots of events at the Field Day, there is the egg carry and the tire roll and the sack race. Everybody will compete in these events...if they want to. But there will be three events in the Field Day Contest where only three students from our class will participate. Those events are the softball throw, the long jump and the 50 yard dash. The whole class wants to choose the best representatives.

Can you think of ways that you might use mathematics to help you make the best choices?

Event	Speed	Strength
Softball Throw	Not Important	Very Important
50 Yard Dash	Very Important	Not Important
Long Jump	Very Important	Very Important

Last year only 6 students wanted to be in these events. The rated themselves.

Name	Strength	Speed
Jameel	Very Strong	Very Fast
Ken	Very Strong	Average
Leesa	Average	Average
Ada	Very Strong	Slow
Jon	Average	Very Fast

Working in your group and look at the two charts from last year.

Decide who you think should have been in each event.

Softball
Throw________________________

Why?

Long
Jump__________________________

Why?

50 Yard
Dash_________________________

Why?

Have your group rate themselves and fill out the chart.

Name	Speed	Strength

Who in your group should do the long jump?______________________________

You need to add the numbers under each race to get a person's total time. So Rodney's total time is

3 + 5 + 1 or a total of 9 minutes

Your new friend has never heard of addition.
Explain or show how to find
3 + 5 + 1, by "counting all."

How would you tell your new friend what addition means?

Name	Egg Carry Time	Tire Roll Time	SackRace Time
Rodney	3 minutes	5 minutes	1 minutes
Barbara	4 minutes	4 minutes	3 minutes
Peggy	4 minutes	6 minutes	3 minutes
Damen	3 minutes	4 minutes	2 minutes
Keenesha	2 minutes	4 minutes	3 minutes
Sammie	3 minutes	4 minutes	2 minutes

This chart shows how fast each child completed each event last year. Use this chart to answer the questions on the next page.

Who had the best total time in all 3 events?

Name: ______________________________

Total Time: ________________________

How much faster was Barbara than Peggy in the Tire Roll?

She was __________ minutes faster.

Make up an addition problem using the information in the chart.

Make up a subtraction problem using the information in this chart.

Measure the two sample invitations to see what size paper the teacher should cut so you can make invitations for your family.

Invitation A:
_________ inches long
_________ inches wide

Invitation B:
_________ inches long
_________ inches wide

Now measure the invitations in centimeters:

Invitation A:
_________ centimeters long
_________ centimeters wide

Invitation B:
_________ centimeters long
_________ centimeters wide

A

PLEASE COME
TO FIELD
DAY

MAY 18
9:00 A.M. TO 2:30 P.M.

AT

VIENNA ELEMENTARY SCHOOL

B

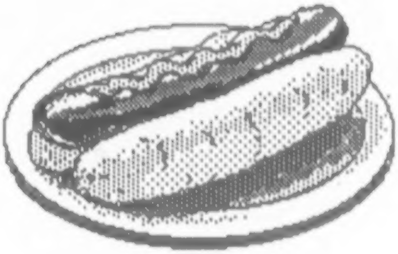
PLEASE COME
TO FIELD
DAY

MAY 18
9:00 A.M. TO 2:30 P.M.

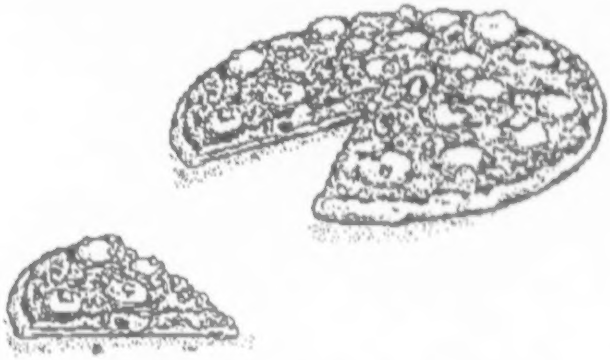
AT

VIENNA ELEMENTARY SCHOOL

You can buy your lunch at field day.

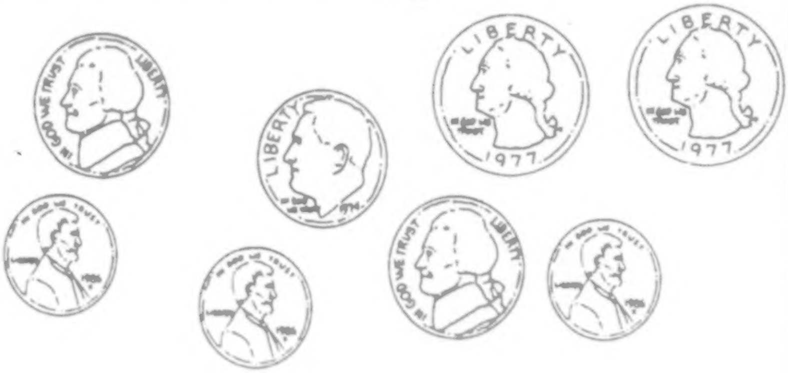


One hot dog
35 cents



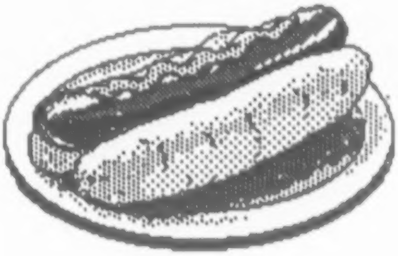
One slice of pizza
25 cents

You have this much money:



Can you buy 2 hot dogs?________

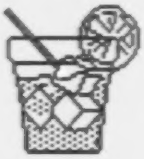
Can you buy 3 slices of pizza?_______



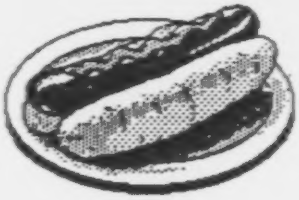
If Mary buys 2 hot dogs and Sue buys 3 hot dogs, how many hot dogs do they have together?__________

Try this without counting the money.

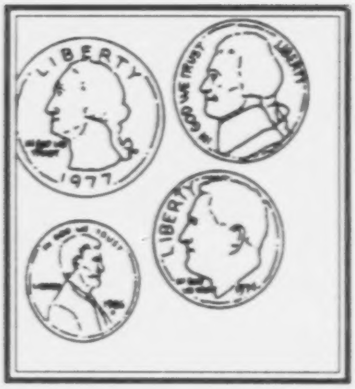
Put an X on all of the sets of money that could be used to buy a hot dog and a lemonade.

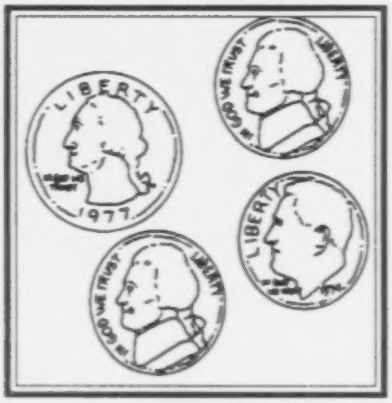


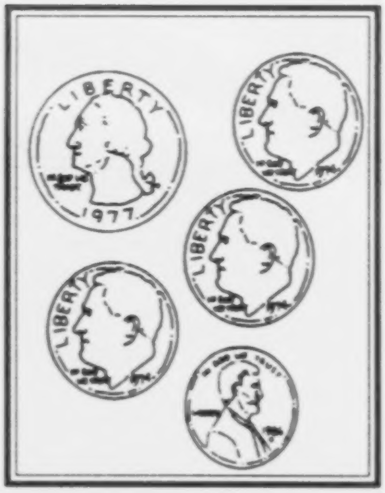
25¢

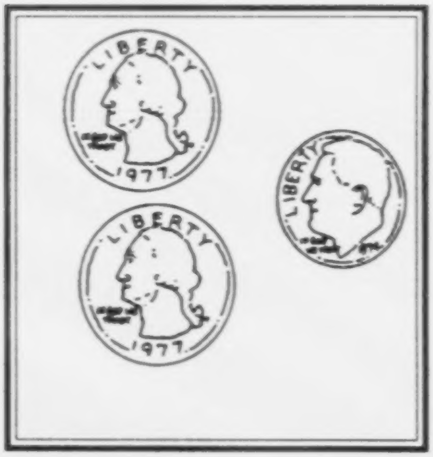


25¢





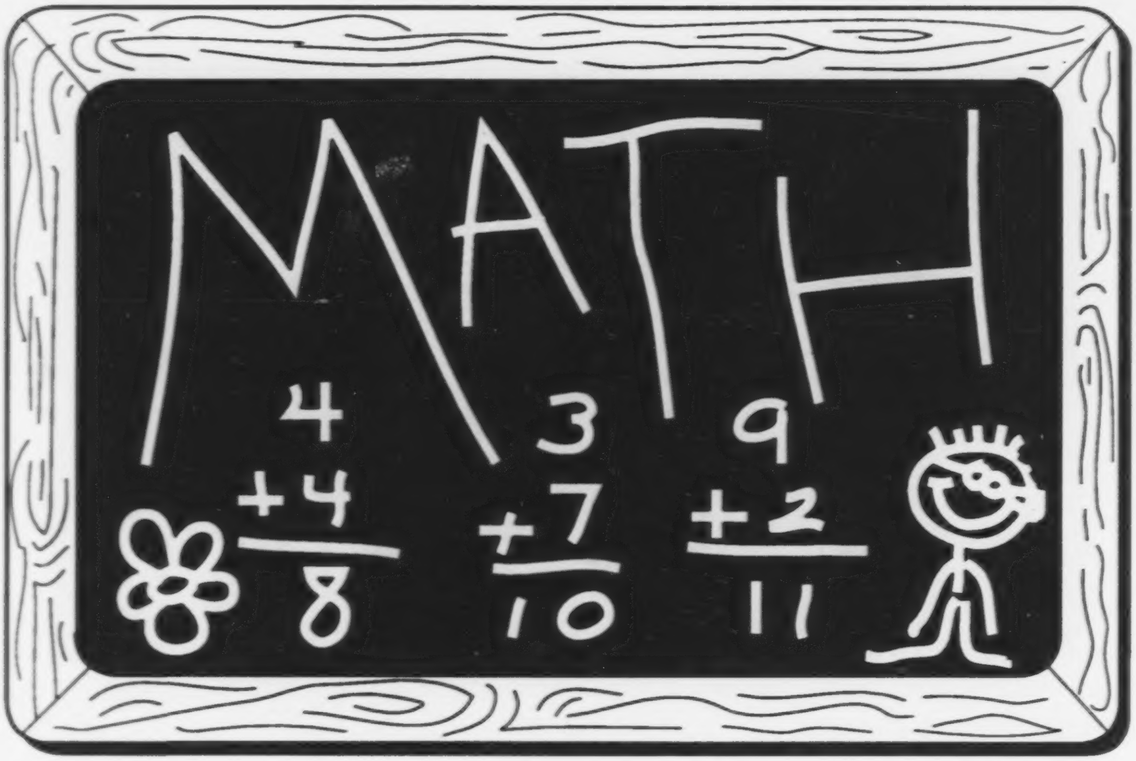




Competency-Based Education Assessment Series

Second Grade Mathematics

Type 1 Assessment


MATH
4
+4
8
3
+7
10
9
+2
11

Developed by

Gen Davis

Margaret Kasten

Trish Koontz

Anne Mikesell

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency test has a traditional design. The following information is intended to be a guide, not a rigid set of directions for the use of this instrument. It is not intended to be a timed test and district or teacher discretion should be used when setting the testing schedule. The instrument is designed to be primarily a pencil and paper, tradition style assessment. However if students routinely use counters, manipulatives, measuring tools, and calculating devices as an instrumental part of instructional activities, they should be available for use during the assessment. Also those items which ask about three-dimensional objects should be accompanied with access to those objects and not just pictures of those objects.

There are 20 items on the test with a suggested point value of 28. Most items require students to select a correct answer from a group of possible answers. Several items ask students to "explain" or "sort." Scoring of these items is discussed in the Teacher Notes Sections. Additional scoring information is provided on the information sheet at the back of this document.

What follows is a discussion on each item or item group. The items have been developed to measure progress on second grade objectives from the Model Competency-Based Mathematics Program. Each discussion includes the strand, a reference to the second grade critical objective being measured, and a suggested point value. A small scale sample of the item or items being discussed is included. Student materials follow the discussion, as does the standard information sheet.

TEACHER NOTES: Items 1 and 2
(1 point each)

Strand 1: Patterns, Relations and Functions

Objective 1.2 : recognize patterns in numbers and number combinations.

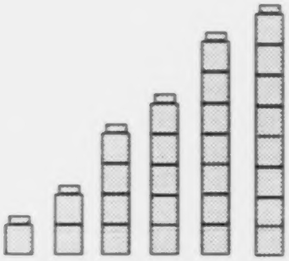
In this assessment, commonly used numeric patterns such as counting by twos, fives and tens can be assessed. Slightly more difficult patterns would be counting by numbers such as threes or sevens.

Other interesting patterns for children to explore might look like: (1,2,4,5,7...). This pattern grows by "add one, add two, add one, add two..." For these more unusual patterns, it is instructive for the children to model such patterns with manipulative materials such as blocks. In this example, both the numeric pattern and the pictorial representation of the pattern is given. Children need many varied opportunities to make connections between concrete patterns and their numeric representations.

1. Which number would you use to fill in the blank? Circle the correct answer.

3 6 9 __ 15

a) 10
b) 11
c) 12
d) 13



2. Here is a picture of the pattern 1, 2, 4, 5, 7, 8. How many units are in the next tower?

a) 9
b) 10
c) 11
d) 12

TEACHER NOTES: Item 3 (1 point)

STRAND 2: Problem Solving Strategies

Objective 2.1 -identify needed information to solve a problem.

Extra information with the same unit is more difficult for children to determine as "extra information", than information with the dissimilar units. The more opportunities children have to role play or act out given problems, the more able they are to determine needed and extra information in problems.

3. There are 8 frogs in the first grade classroom. There are 7 snakes in the second grade classroom. The third grade classroom has 5 frogs. How may frogs are there in the third grade classroom?

TEACHER NOTES: Items 4 and 5 (1 point each)

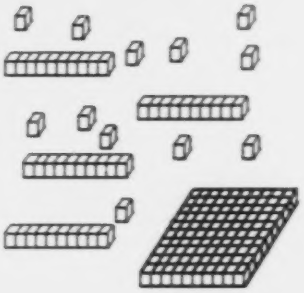
Strand 3: Number and Number Relations

Objective 3.1 -develop the concept of place value with concrete models of hundreds, tens, and ones.

There are many models that can be used to represent ones, tens and hundreds. Some of these models include, beans, money, macaroni, or bundles of objects. More structured models include Base Ten Blocks and graph paper representations of them.

It is important for children to develop flexibility with base ten quantities. For example, representing numbers where more than ten ones are given, or more than ten tens are given, challenge children to regroup and trade quantities to produce the compact base ten number. Also notice that base ten blocks should be randomly placed so that hundreds are not always placed together in the left column, etc. The base ten value of the individual blocks is inherent in the size and not position of the block.

4. Here are some Base Ten Blocks. How many are there? Circle the correct answer.



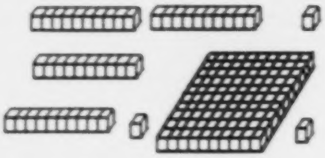
a) 1412

b) 142

c) 152

d) 412

5. How many Base Ten Blocks are here? Circle the correct answer.



a) 143

b) 8

c) 314

d) 413

TEACHER NOTES: Item 6 (1point)

Strand 3: Number and Number Relations

Objective 3.4 -illustrate fractional parts of whole objects or sets of objects.

It is important for children to have many varied opportunities to explore congruency (equal size pieces) so that they do not select such answers such as "A". This exploration is easiest when begun with circular regions, progressing to rectangular regions and then to other polygons.

TEACHER NOTES: Item 7 (3 points)

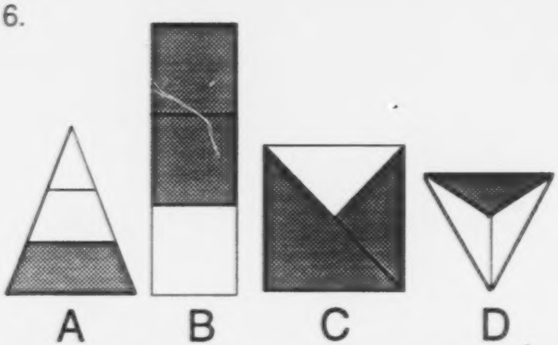
Strand 3: Number and Number Relations

Objective 3.7- learn strategies for the addition and subtraction of numbers such as (a) compatible numbers, (b) compensatory numbers, (c) borrow and pay back (subtraction), (d) regrouping and (e) using a calculator.

Children need numerous varied experiences exploring number relationships in order to discover their own strategies for operating on numbers. Allowing children to share their discovered strategies with the teacher and other children will strengthen their flexibility with number relationships. It is difficult to directly teach such strategies in this way, as they will often become meaningless rote rules to apply. Multiple strategies may apply to any given problem. It is the child's view of what strategy to apply that is more important than how the textbook suggests.

One point should be given for the answer 222 and two points for any correct explanation.

6.



Which shape shows one-third ($\frac{1}{3}$) colored in? Circle the correct answer.

a) A

b) B

c) C

d) D

7. **Kim explained how she did this problem to the class:**

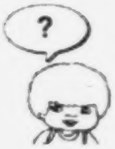
$$\begin{array}{r} 29 \\ +\ 6 \\ \hline \end{array}$$

Kim said, "I changed the problem to 30 + 5 and got 35."

Try this one using a strategy like Kim's or a strategy of your own.

$$\begin{array}{r} 199 \\ +\ 23 \\ \hline \end{array}$$

Explain your way.



TEACHER NOTES: Item 8 (1 point)

Strand 3: Number and Number Relations

Objective 3.9 -translate real-life situations involving addition and/or subtraction into conventional symbols of mathematics.

Children need opportunities to role play or act out many problems. Of the many types of subtraction problems, examples in which missing addends need to be determined are the most difficult. It is important to include missing addend problems along with the more traditional take-away and comparison problems.

8. Anthony has saved $7.00 to buy a soccer ball costing $15.00. How much more money does he need to save to buy the soccer ball? Circle the correct answer.

a) $15.00

b) $12.00

c) $22.00

d) $ 8.00

TEACHER NOTES: Item 9 (2 points)

Strand 4: Geometry

4.5 compare three-dimensional objects describing similarities and differences using appropriate standard and non-standard language.

It is important for children to explore three dimensional objects held in many ways or orientations. Children need to see a square base pyramid standing on its point (vertex) as well as on its base. The orientation does not change the name of a geometric term. One of the many ways children can explore similarities and differences among three dimensional shapes is to explore and compare their sides (faces).

9. Use the things the teacher gives you to fill in the blanks with the correct letters.

What shape is the bottom of the can?

____

What shape is the bottom of the box?

____

a) Sphere

b) Rectangle

c) Circle

d) Cube

TEACHER NOTES: Item 10 (2 points)

Strand 5: Algebra

Objective 5.3-model a problem situation using numbers and/or letters.

Children need opportunities to role play or act out many problems. Encourage children to explore many approaches to the same problem. Of the many types of subtraction problems, examples in which missing addends need to be determined are the most difficult. It is important to include comparison problems that can be solved as missing addend problems.

Notice for this example, both "A" and "D" are acceptable number sentences. Children do not need to identify both answers as correct.

TEACHER NOTES: Items 11 and 12 (3 points each)

Strand 6: Measurement

Objective 6.1-explore length, capacity, and weight by selecting and using appropriate metric and conventional units such as centimeter, inch, liter, cup, pint, quart, kilogram, and pound.

Children need hands-on experiences with measurement tools to determine the appropriateness of their uses.

These types of assessments should be observed during active exploration in class--not as specific items in a formal test setting.

10. Elizabeth has 7 rings. Her friend Jenny has 9 rings. How many more rings does Jenny have than Elizabeth?
Fill in the blank and circle a number sentence that could help you solve this problem.

Jenny has ____ more rings than Elizabeth.

a) $7 + \square = 9$

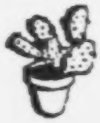
b) $9 + \square = 7$

c) $7 + 9 = \square$

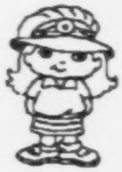
d) $9 - 7 = \square$

11. What would you use to find out:
(Put the letter in the blank)

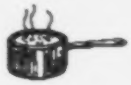
How much this plant weighs?______

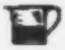


How tall the girl is?________

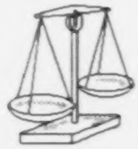


How much water to fill the pot?______

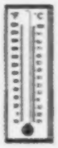




A.



B.

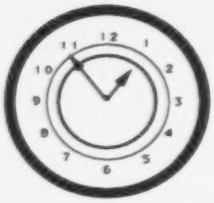


C.

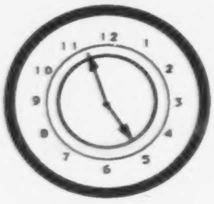
D.

13. Your friend needs to leave your house at 11:05. Which clock shows this time? Circle the correct answer.

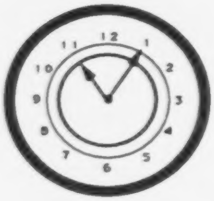
a)



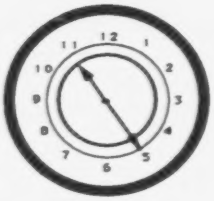
b)



c)



d)



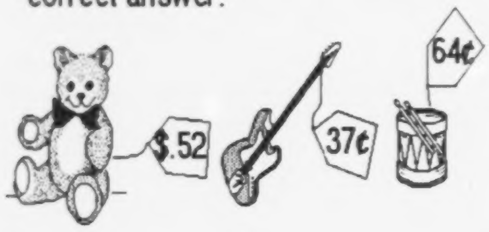
TEACHER NOTES: Item 13 (1 poiont)

Strand 6: Measurement

Objective 6.3- tell time to the nearest five-minute interval on digital and dial time-pieces.

It is very helpful for children to individually explore the movement of the hour hand in relation to the movement of the minute hand on a geared clock.

14. Darcy has $1.75. She used front end estimation to see if she had enough money to buy these toys. Which of the following problems shows her thinking? Circle the correct answer.



a) 5¢	b) 2¢	c) $.50	d) $.50
3¢	4¢	.40	.30
6¢	8¢	.70	.60
14¢	14¢	$1.60	$1.40

TEACHER NOTES: Item 14 (1 point)

Strand 7: Estimation and Mental Computation

Objective 7.5-make estimates in addition and subtraction, using front-end digits.

There are many ways to estimate an answer in a given situation. The front end estimation process is one of these processes. This strategy can be used with other strategies when solving a given problem.

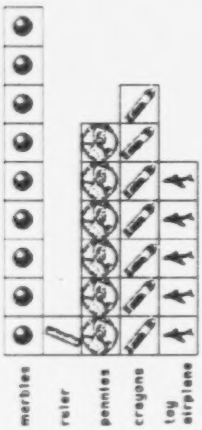
TEACHER NOTES: Items 15 and 16 (1 point each)

Strand 8: Data Analysis and Probability

Objective 8.3-explore picture and bar graphs (scales by one) by making identifications, comparisons, and predictions.

Children need opportunities to collect, organize and display real information in picture and bar graph form. It is important for children to title the graph and label the items. Experience makes this process very easy and enjoyable.

Andrew made a graph of the things in his shoe box. Here is his graph. Circle the correct answers in 15 and 16.



15. What does he have the most of?

a)

b)

c)

d)

16. What does he have exactly five of?

a)

b)

c)

d)

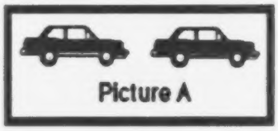
TEACHER NOTES: Item 17 (1 point)

Strand 4: Geometry

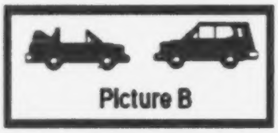
Objective 4-investigate symmetry and congruence.

TEACHER NOTES:
It is important for children to explore the concepts of symmetry in real life contexts such as nature, art, poetry, architecture, etc. These concepts may be explored within the child's natural experiences and environment as well as from trade books and selected literature.

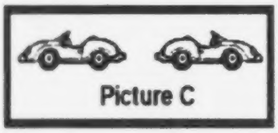
17. Mrs. Miller asked her class to find a picture of something that shows symmetry. Three of her students showed her these pictures. Which one shows symmetry? Circle the picture that shows symmetry.



Picture A



Picture B



Picture C

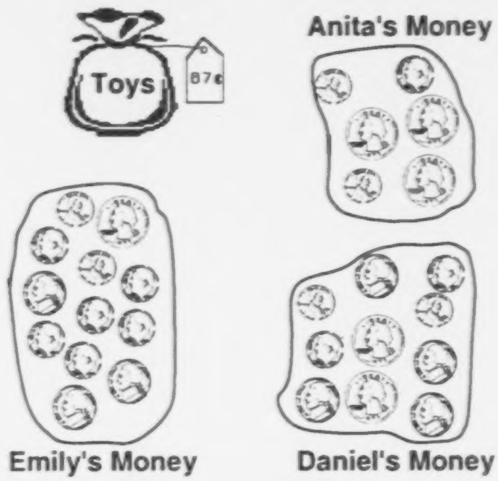
TEACHER NOTES: Items 18 and 19 (2 points)

Strand 6: Measurement

Objective 6.2-count collections of coins including pennies, nickels, dimes, quarters and half dollars and compare values.

It is very helpful for children to explore their own approaches and strategies for counting money. Being able to group coins in ways that make counting easier is very important. For example, grouping nickels to make tens is helpful or putting a nickel with a quarter, etc. For this item and in all activities in which money is used, it is strongly recommended that real coins be used for children to manipulate, count and compare.

Anita, Emily and Daniel saved some money to buy a bag of toys. Here is a picture of what each of them has saved.



18. Who has exactly 87 cents to buy the bag of toys?____________________

19. Who else can buy this bag of toys?____________________

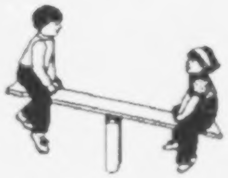
TEACHER NOTES: Item 20 (1 point)

Strand 7: Estimation and Mental Computation

Objective 7.7-estimate length, capacity and mass.

It is important for children to develop useful benchmarks about measurement. For example, knowing that a centimeter is about the length across their little finger or that a meter is as high as most doorknobs are from the floor, is most helpful in understanding and estimating units of measurement.

20. Two second graders are on a seesaw Together they can weigh about:



(Circle the correct answer.)

a) 20 pounds

b) 90 pounds

c) 250 pounds

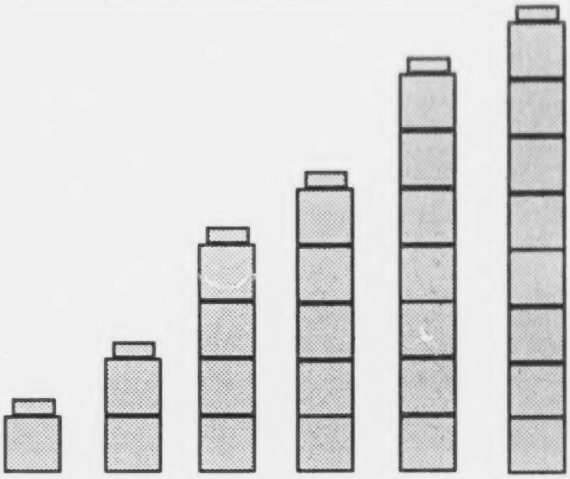
Information Sheet

Item	Answer	OBJECTIVE	OUTCOME LEVEL	CRITICAL OBJECTIVE
1.	c	2-1-2	K,C	yes
2.	b	2-1-2	PS	yes
3.	5 frogs	2-2-1	K	yes
4.	152	2-3-1	C	yes
5.	143	2-3-1	C	yes
6.	d	2-3-4	C	yes
7.	Teacher notes	2-3-7	C, PS	yes
8.	d	2-3-9, 2-2-7	K,PS	yes
9.	c,b	2-4-5	K	yes
10.	a or d	2-5-3, 2.2, 2.7	K,C,PS	yes
11.	b,d,a	2-6-1	K	yes
12.	answer varies	2-6-1	C	yes
13.	c	2-6-3	K	yes
14.	d	2-7-5, 2-2-7	K	yes
15. & 16.	c,d	2-8-3	C, PS	yes
17.	c	2-4-1	K,C	no
18. & 19	Anita, Emily	2-6-2	K,C	no
20.	b	2-7-7	K,C	no

1. Which number would you use to fill in the blank? Circle the correct answer.

3 6 9 __ 15

a) 10
b) 11
c) 12
d) 13



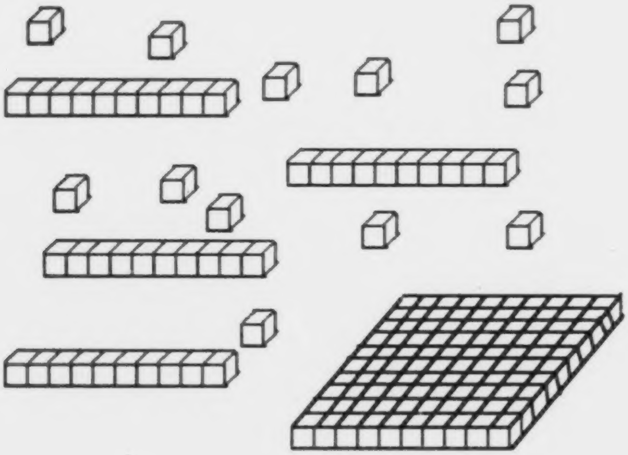
2. Here is a picture of the pattern 1, 2, 4, 5, 7, 8. How many units are in the next tower?

a) 9
b) 10
c) 11
d) 12

3. There are 8 frogs in the first grade classroom. There are 7 snakes in the second grade classroom. The third grade classroom has 5 frogs. How many frogs are there in the third grade classroom?

____

4. Here are some Base Ten Blocks. How many are there? Circle the correct answer.



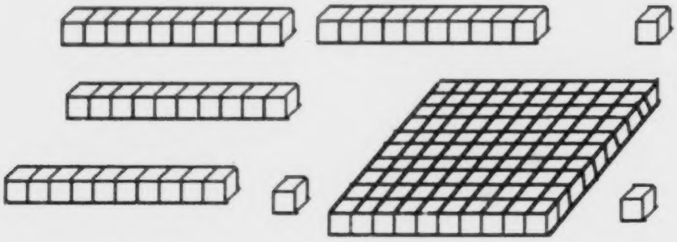
a) 1412

b) 142

c) 152

d) 412

5. How many Base Ten Blocks are here? Circle the correct answer.



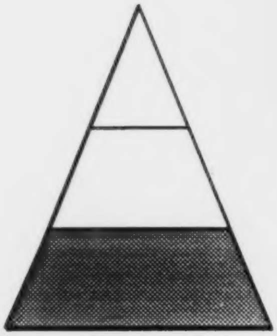
a) 143

b) 8

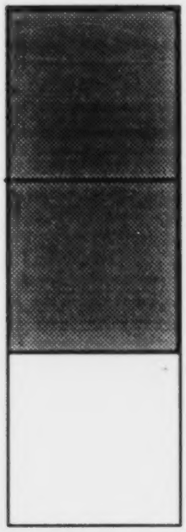
c) 314

d) 413

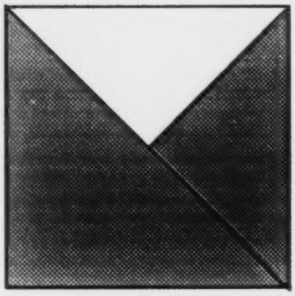
6.



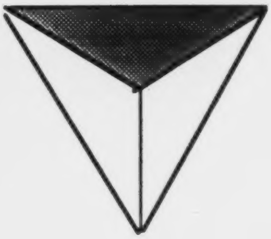
A



B



C



D

Which shape shows one-third ($\frac{1}{3}$) colored in? Circle the correct answer.

a) A

b) B

c) C

d) D

7. Kim explained how she did this problem to the class:

$$\begin{array}{r} 29 \\ +\ \ 6 \\ \hline \end{array}$$

Kim said, "I changed the problem to 30 + 5 and got 35."

Try this one using a strategy like Kim's or a strategy of your own.

$$\begin{array}{r} 199 \\ +\ \ 23 \\ \hline \end{array}$$

Explain your way.

8. Anthony has saved $7.00 to buy a soccer ball costing $15.00. How much more money does he need to save to buy the soccer ball? Circle the correct answer.

a) $15.00

b) $12.00

c) $22.00

d) $ 8.00

9. **Use the things the teacher gives you to fill in the blanks with the correct letters.**

What shape is the bottom of the can?

____

What shape is the bottom of the box?

____

a) Sphere

b) Rectangle

c) Circle

d) Cube

10. Elizabeth has 7 rings. Her friend Jenny has 9 rings. How many more rings does Jenny have than Elizabeth?

Fill in the blank and circle a number sentence that could help you solve this problem.

Jenny has ______ more rings than Elizabeth.

a) 7 + ☐ = 9

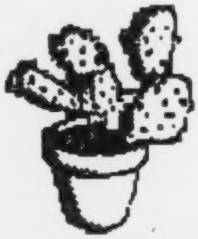
b) 9 + ☐ = 7

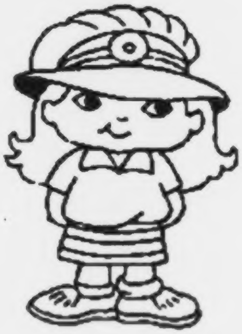
c) 7 + 9 = ☐

d) 9 - 7 = ☐

11. What would you use to find out:
(Put the letter in the blank)

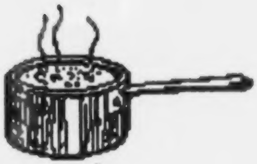
How much this
plant weighs?______

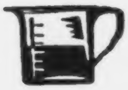




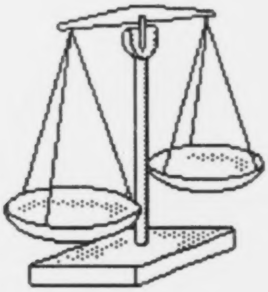
How tall the
girl is?_________

How much water to
fill the pot?______

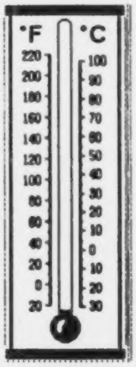




A.



B.



°F
°C
220
200
180
160
140
120
100
80
60
40
20
0
20
100
90
80
70
60
50
40
30
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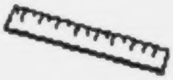
10
0
10
20
30

C.



D.

12. Your teacher will give you some things used to measure. Sort these things into three groups.

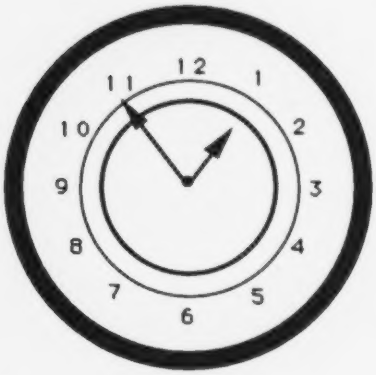
a) measure length

b) measure weight

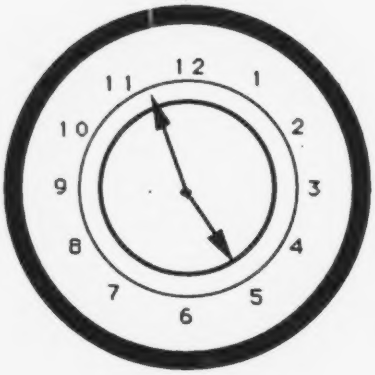
c) measure how much something will hold

13. **Your friend needs to leave your house at 11:05. Which clock shows this time? Circle the correct answer.**

a)


12
1
2
3
4
5
6
7
8
9
10
11

b)


12
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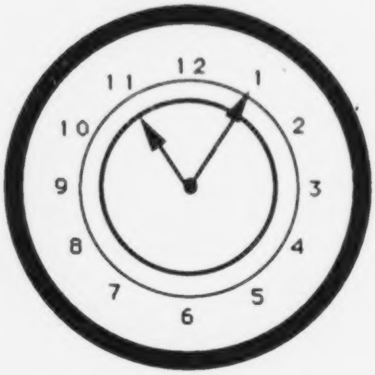
10
11

c)


12
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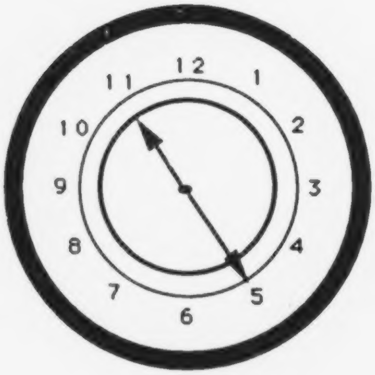
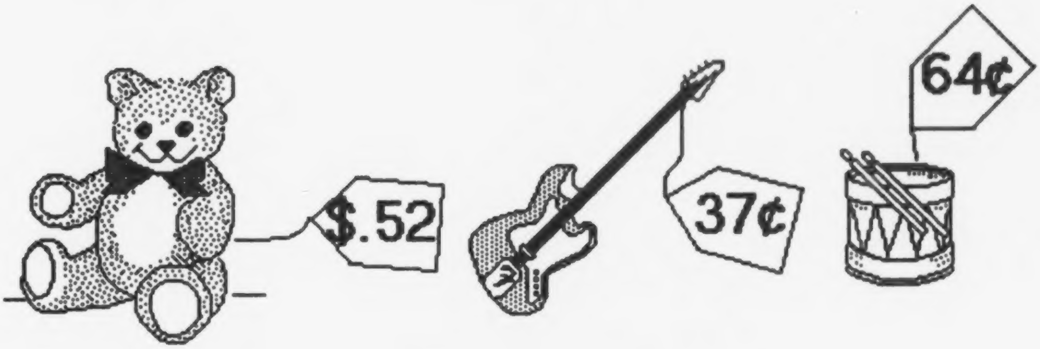
9
10
11

d)


12
1
2
3
4
5
6
7
8
9
10
11

14. Darcy has $1.75. She used front end estimation to see if she had enough money to buy these toys. Which of the following problems shows her thinking? Circle the correct answer.



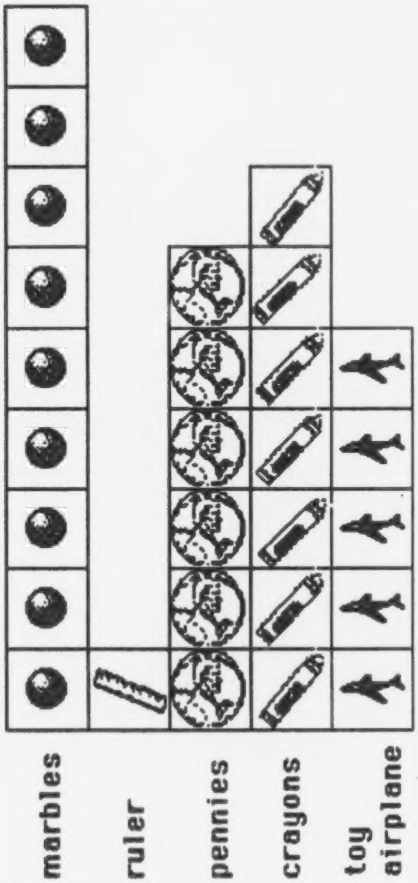
a)
5¢
3¢
6¢
14¢

b)
2¢
4¢
8¢
14¢

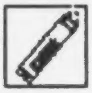
c)
$ .50
.40
.70
$ 1.60

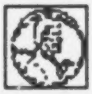
d)
$ .50
.30
.60
$ 1.40

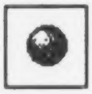
Andrew made a graph of the things in his shoe box. Here is his graph. Circle the correct answers in 15 and 16.

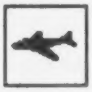


15. What does he have the most of?

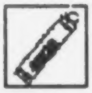
a) 

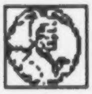
b) 

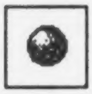
c) 

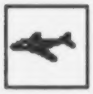
d) 

16. What does he have exactly five of?

a) 

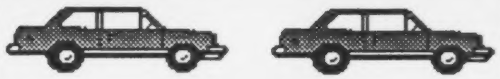
b) 

c) 

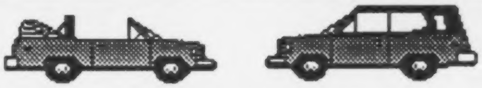
d) 

17. Mrs. Miller asked her class to find a picture of something that shows symmetry. Three of her students showed her these pictures. Which one shows symmetry? Circle the picture that shows symmetry.

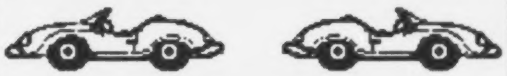
7



Picture A

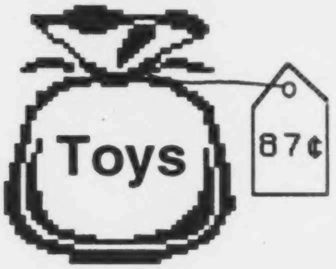


Picture B

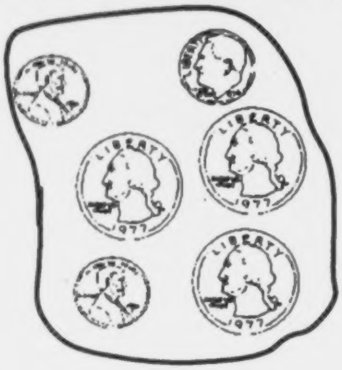


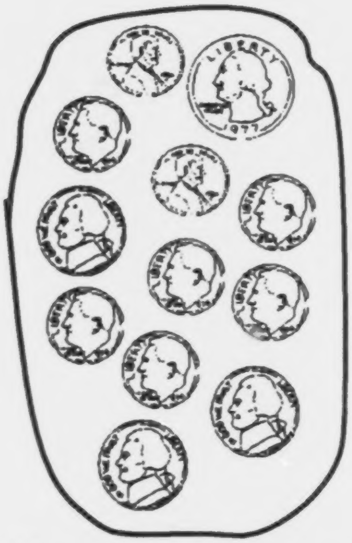
Picture C

Anita, Emily and Daniel saved some money to buy a bag of toys. Here is a picture of what each of them has saved.

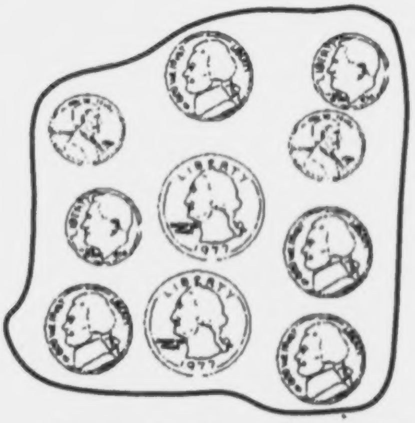


Anita's Money





Emily's Money

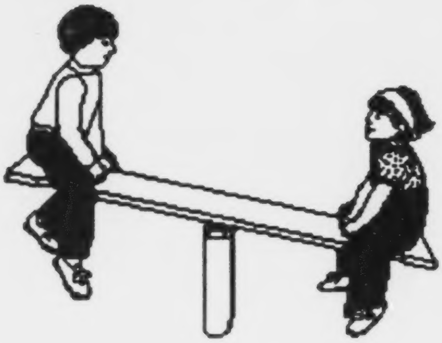


Daniel's Money

18. Who has exactly 87 cents to buy the bag of toys?____________________

19. Who else can buy this bag of toys?____________________

20. Two second graders are on a seesaw. Together they can weigh about:



(Circle the correct answer.)

a) 20 pounds

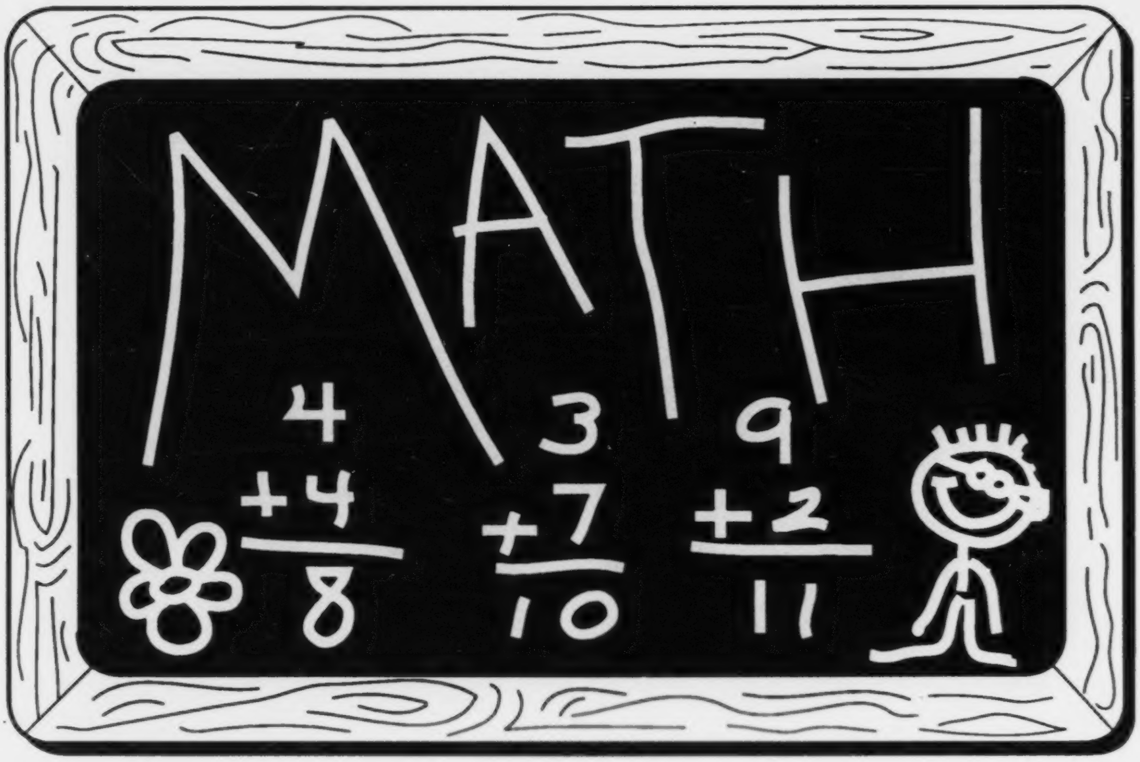
b) 90 pounds

c) 250 pounds

Competency-Based Education Assessment Series

Second Grade Mathematics

Type 2 Assessment



Developed by

Gen Davis

Margaret Kasten

Trish Koontz

Anne Mikesell

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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Pat McNichols	Lucas County Schools
Steve Meiring	Ohio Department of Education
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Ray Trenta	Akron City Schools

About This Instrument

This model competency test is intended to go beyond the traditional multiple choice assessment. The following information is intended to be a guide, not a rigid set of directions for the use of this instrument. It is not intended to be a timed test and district or teacher discretion should be used when setting the testing schedule. The test is a mixture of open form items and items requiring extended responses. Some items are direct, straight-forward measures of particular objectives. Some items assess combinations of more than one objective. The test has a limited number of multiple choice and short answer items and includes items that should be scored with a rubric and items to be evaluated by teacher observation or interview.

The instrument is intended to assess a deeper level of understanding than that being assessed on the Type 1 instrument, therefore, scoring on this instrument is a bit more complicated than the scoring on the Type 1 instrument. There are 17 items on the test with a suggested point value of 52 points.

What follows is a discussion of each item or item group. Each discussion includes the strand, the specific objective being assessed and suggested point values. A small scale sample of the item or items being discussed is included in the TEACHER NOTES. Student materials follow this discussion as does the standard Information Sheet.

Sample rubrics specific to some items are included. When an item requires a rubric and one is not included the following general rubric may be adapted.

0 points	No answer, only recopying of problem, or uninterpretable response
1 point	Some interpretation of the problem is evident
2 points	Appropriate strategy, but the answer is wrong or incomplete
3 points	Correct strategy yielding correct answer that is justified when appropriate

TEACHER NOTES: Items 1 and 2
(4 points each)

Strand 1: Patterns, Relations and Functions

Objective 1.2-recognize patterns in numbers and number combinations.

In this assessment, commonly used numeric patterns such as counting by twos, fives and tens can be assessed. Slightly more difficult patterns would be counting by numbers such as threes or sevens.

Other interesting patterns for children to explore might look like: (1,2,4,5,7...). This pattern grows by "add one, add two, add one, add two..." For these more unusual patterns, it is instructive for the children to model such patterns with manipulative materials such as blocks. In this example, both the numeric pattern and the pictorial representation of the pattern is given. Children need many varied opportunities to make connections between concrete patterns and their numeric representations.

A rich source of patterns can be found in the hundred chart. As an alternative to coloring in the discovered patterns, children could cut them out and glue them on paper.

To obtain an accurate assessment, children must explain their own pattern in their own words without teacher intervention. Prompting and leading children's responses may produce an inaccurate assessment.

SAMPLE RUBRIC:

0 points	No obvious pattern identified
1 point	Accurate visual pattern with no explanation given
2 points	Accurate visual pattern with visual explanation A child may describe their visual pattern as a checker board or one with stripes. Another common response is for children to color in a vertical line and to explain this pattern as, "...numbers with four in it".
3 points	Accurate visual pattern with immature mathematical explanation A child may describe their visual pattern as having "numbers with three on the right".
4 points	Accurate visual pattern with mature mathematical explanation A more mathematical response would be for the children to write, "counting by tens with fours in the ones place".

1. Look at this number chart. Find an interesting number pattern and color it.

	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

How would you explain your number pattern to a friend?

2. Find another interesting number pattern and color it.

	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

How would you explain this number pattern to a friend?

TEACHER NOTES: Item 3 (3 points)

Strand 2: Problem Solving Strategies

Objective 2.1-identify needed information to solve a problem.

The more opportunities children have to role play or act out given problems, the more able they are to determine needed and extra information in problems.

SAMPLE RUBRIC:

0 points	No answer or uninterpretable response
1 point	Some interpretation of problem with some mention of money
2 points	Clear statement of one of the two pieces of information needed to solve the problem Either the child states that you need to know the cost of the items or how much money they have.
3 points	Clear statement of both pieces of information needed to solve the problem

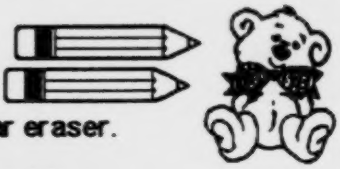
TEACHER NOTES: Item 4 (1 point)

Strand 3: Number and Number Relations

Objective 3.1-develop the concept of place value with concrete models of hundreds, tens, and ones.

There are many models that can be used to represent ones, tens and hundreds. Some of these models include, beans, money, macaroni, or bundles of objects. More structured models include Base Ten Blocks and graph paper representations of them.

3. At the store you want to buy 2 pencils and a teddy bear eraser.



What do you need to know to be sure you have enough money to buy these?

4. There are 140 pieces of candy in this jar. You need to put 10 pieces of candy in a bag. How many bags do need to bag all of the candy? Circle the correct answer.

a) 140
b) 40
c) 14
d) 10

It is important to connect estimation with base ten patterns of grouping. Regular estimation activities will provide opportunities to assess children's understanding and approaches to place value concepts. This particular multiple choice example can be used after such exploration.

TEACHER NOTES: Item 5 (4 points)

Strand 3: Number and Number Relations

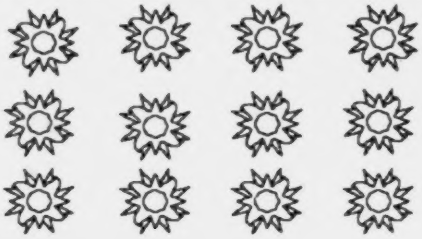
Objective 3.4- illustrate fractional parts of whole objects or sets of objects.

It is important for children to have many varied opportunities to explore congruency for parts of a set or parts of a whole. This exploration is easiest when begun with manipulatives in real life situations.

SAMPLE RUBRIC:

0 points	No obvious understanding of fractional part of set or whole
1 point	Some understanding of fractional part of a set or whole with no mathematical explanation given
2 points	Accurate understanding of fractional part of a set or whole including incomplete mathematical explanation For example, when a child was asked to color one third of twelve suns yellow, he colored four suns yellow and wrote, "I counted them."
3 points	Accurate understanding of fractional part of a set or whole with clear mathematical explanation for one part of the example

5. Color one-third of the suns yellow



Explain how you did this.

Color one-half of the yellow suns with a red crayon. Explain how you did this.

4 points Accurate understanding of fractional part of a set or whole with clear mathematical explanation of both parts of the example

After coloring in four suns to show one third of the set, a child will color two of the yellow suns red and write something such as, "2 + 2 = 4, so I colored 2 of them." or "I colored one red in each group of two."

TEACHER NOTES: Item 6 (3 points)

Strand 3: Number and Number Relations

Objective 3.7-learn strategies for the addition and subtraction of numbers such as (a) compatible numbers, (b) compensatory numbers, (c) borrow and pay back (subtraction), (d) regrouping and (e) using a calculator.

Children need numerous varied experiences exploring number relationships in order to discover their own strategies for operating on numbers. Allowing children to share their discovered strategies with the teacher and other children will strengthen their flexibility with number relationships. It is difficult to directly teach such strategies in this way, as they will often become meaningless rote rules to apply. Multiple strategies may apply to any given problem. It is the child's view of what strategy to apply that is more important than how the textbook suggests.

This particular assessment is best accomplished through interviews and observation as children work cooperatively with others.

6. **Carmen was given three problems to solve. Carmen wrote each problem in a different way. Help Carmen solve each problem by filling in the box.**

$$\begin{array}{r} 5 \\ +6 \\ \hline 11 \end{array} \rightarrow 5 + 5 + \boxed{} = 11$$

$$\begin{array}{r} 34 \\ -9 \\ \hline 25 \end{array} \rightarrow \begin{array}{r} 34 \\ -\boxed{} \\ \hline 24 \end{array} + 1 = 25$$

$$\begin{array}{r} 199 \\ +23 \\ \hline 222 \end{array} \rightarrow \begin{array}{r} 200 \\ +\boxed{} \\ \hline 222 \end{array}$$

TEACHER NOTES: Item 7 (4 points)

Strand 3: Number and Number Relations

3.9 translate real-life situations involving addition and/or subtraction into conventional symbols of mathematics.

Children need opportunities to role play or act out many problems. By solving word problems in such active ways, children will not fall into the trap of looking for key words which are often misleading in problems. Many children will answer this question by subtracting (15 - 7 = 8) reinforced by the words "left" and "gave".

Use the rubric in the "About this Instrument" section of this test.

TEACHER NOTES: Item 8 (3 points)

Strand 4: Geometry

Objective 4.5-compare three-dimensional objects describing similarities and differences using appropriate standard and non-standard language.

It is important for children to explore three dimensional objects held in many ways or orientations. Children need to see a tetrahedron standing on its point (vertex) as well as on its base. The orientation does not change the name of a geometric term. One of the many ways children can explore similarities and differences is by sorting activities. Children should be given opportunities to select their own geometric sorting categories and to see other children's categories. If children sort by a non-geometric category such as color, simply ask them to try another sort.

7. Andrew gave Maria 7 candy kisses. He had 15 candy kisses left. How many candy kisses did Andrew have before he gave Maria any?

Andrew had ___ candy kisses.

Explain your answer.

Many three dimensional common items (cans, boxes, cones, etc.) should be available.

TEACHER NOTES: Item 9 (3 points)

Strand 5: Algebra

Objective 5.3- model a problem situation using numbers and/or letters.

Children need opportunities to role play or act out many problems. Encourage children to explore many approaches to the same problem and many ways of recording solutions to the problems including use of numbers and letters.

SAMPLE RUBRIC:

0 points	No answer or uninterpretable response
1 point	Some interpretation of problem, one correct solution given
2 points	More than one equation given but not all correct
3 points	More than one equation given with all correct

TEACHER NOTES: Item 10 (3 points)

Strand 5: Algebra

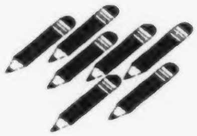
Objective 5.3- model a problem situation using numbers and/or letters.

Children need opportunities to role play or act out many problems. Encourage children to explore many approaches to the same problem and many ways of recording solutions to the problems including use of numbers and letters.

9. Mr. McGregor has 23 animals. He only has horses and cows.

Write different number sentences to tell how many horses and cows he could have.

10. Michael has fewer colored pencils than Janet. Janet has seven colored pencils. How many pencils could Michael have? You may use counters.



Write your answers here:

Explain how you got your answers.

SAMPLE RUBRIC:

0 points	No answer or uninterpretable response
1 point	Some interpretation of problem in that only one correct solution given with or without an explanation
2 points	More than one equation given but not all solved correctly or not explained clearly.
3 points	More than one equation given with all solved correctly with appropriate explanation

TEACHER NOTES: Item 11 (3 points)

Strand 6: Measurement

Objective 6.1-explore length, capacity, and weight by selecting and using appropriate metric and conventional units such as centimeter, inch, liter, cup, pint, quart, kilogram, and pound.

Children need hands-on experiences with measurement tools and units to determine the appropriateness of their uses. Understanding the differences among the categories of length, capacity, and weight is achieved through meaningful experiences with measurement. In addition, continual estimation activities with these tools and units help children to develop a sense of relative size and amount. Children need many varied hands-on experiences with measurement tools and to determine the appropriateness of their uses.

11. You are planning to build a playground for your pet gerbil. Using the units: inch, centimeter, cup, liter, quart, kilogram, pound and milliliter, tell about how big you would make:

Length of tunnel:____________

How much the water dish holds:____

How much the cage weighs:_______

SAMPLE RUBRIC:

0 points	No answer, uninterpretable response, or incorrect unit for category to be measured For example, a child might answer "3 inches" when asked a question concerning capacity or weight.
1 point	Some interpretation of problem in that an appropriate unit within a category was selected, but the unit is inappropriate for the item to be measured. For example, a child may select to measure in "meters" when to measure the length of an insect.
2 points	Appropriate category and unit selected but magnitude is inappropriate For example, a child may write "50 cups" for how much the water dish holds.
3 points	Appropriate category, unit and magnitude are all appropriate

TEACHER NOTES: Item 12 (2 points)

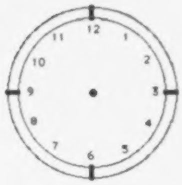
Strand 6: Measurement

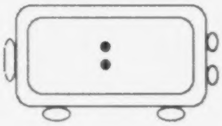
Objective 6.3 tell time to the nearest five-minute interval on digital and dial timepieces.

It is very helpful for children to individually explore the movement of the hour in relation to the movement of the minute hand on a geared clock.

12. This parking meter will run out at 3:05.

Show Clarabelle the Clown what time that will look like on these two clocks.





SAMPLE RUBRIC:

0 points	No answer or inappropriate response
1 point	Incorrect time recorded but digital and dial time correspond or one time recorded correctly
2 points	Correct time recorded on both digital and dial clock

TEACHER NOTES: Item 13 (3 points)

Strand 7: Estimation and Mental Computation

Objective 7.5-make estimates in addition and subtraction, using front-end digits.

There are many ways to estimate an answer in a given situation. The front end estimation process is one of these processes. This strategy can be used with other strategies when solving a given problem. Use the general rubric to evaluate the explanation.

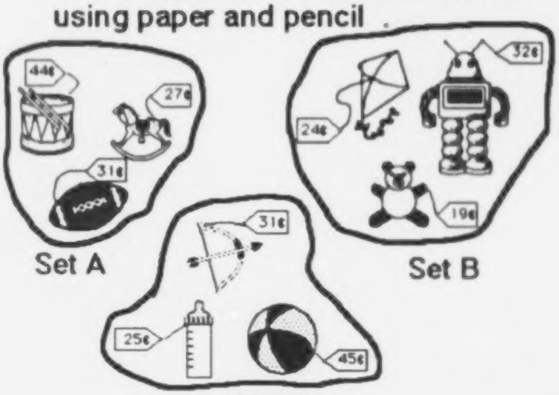
TEACHER NOTES: Item 14 (3 points)

Strand 8: Data Analysis and Probability

Objective 8.3-explore picture and bar graphs (scales by one) by making identifications, comparisons, and predictions.

Children need many opportunities to collect, organize, display and analyze real information in picture and bar graph form. In analyzing the graph, concepts such as range, mode and frequency should be emphasized. It is important for children to title the graph and label the items. Experience makes this process very easy and enjoyable.

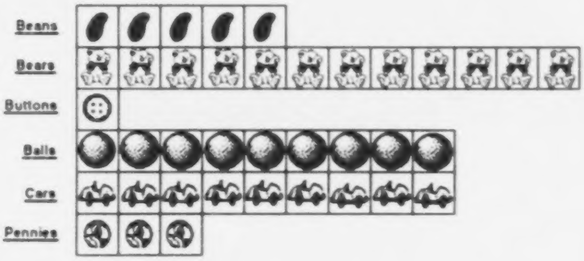
13. **Explain which set of toys Wendy could buy if she has 80¢ to spend. Tell how you could find out without using paper and pencil .**



Which set?____

Explain your answer

14. **Mrs. Gibson's kindergarten class voted for their favorite thing to count. Here is the graph they made:**



- **What is their favorite counter?**
- **What is the difference between the most favorite and the least favorite counters? Tell how many.**
- **Using the information in this graph, tell what you think other kindergarten children like best as counters.**
- **How many children voted in Mrs. Gibson's class?**

SAMPLE RUBRIC:

0 points	No answer, uninterpretable or incorrect response
1 point	Some interpretation of graph that produces an incorrect response For example, when asked to determine the difference between the most favorite and the least favorite item, the child finds the difference between two other items instead.
2 points	More accurate interpretation of the graph with minor computational error or incomplete answer Minor computational errors might include adding total number of voters and being off by one or two. Incomplete answers might include "12,1" for answering the difference between the most favorite and least favorite counter.
3 points	Accurate interpretation of graph with correct responses

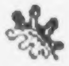
TEACHER NOTES: Item 15 (3 points)

Strand 4: Geometry

Objective 4.- investigate symmetry and congruence.

It is important for children to explore and talk about the concept of congruency observed in real life contexts such as in nature, art, poetry, architecture, etc. Children need opportunities to discover that two shapes can be congruent even if one of the shapes is rotated and in a different orientation than its congruent partner. This discovery can be concluded when children place one shape over the other to "prove" congruency. Use the general rubric to evaluate the explanation.

15. Mrs. Jackson has these pictures. She wants you to tell which two are congruent. Explain your choice.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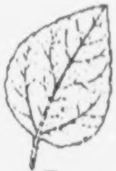
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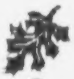
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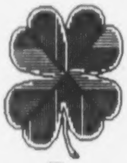
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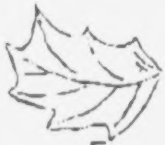
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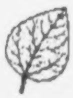
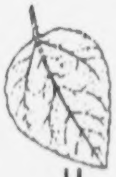

F.

G.


H.

Which two are congruent? ____and____

Explain:

TEACHER NOTES: Item 16 (3 points)

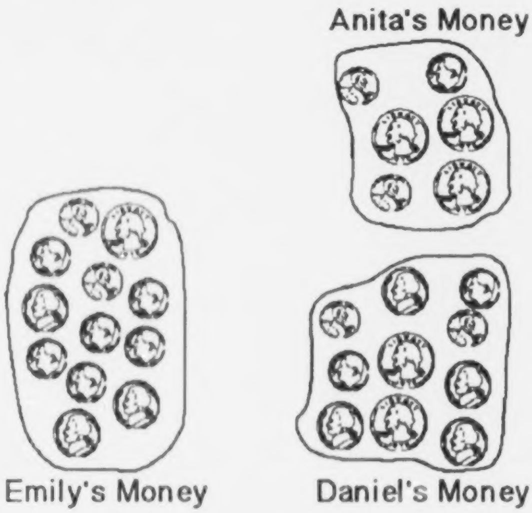
Strand 6: Measurement

Objective 6.2-count collections of coins including pennies, nickels, dimes, quarters and half dollars and compare values.

It is very helpful for children to explore and talk about their own approaches and strategies for counting money. Being able to group coins in ways that make counting and counting-on easier is very important. For example, grouping nickels to make tens is helpful or putting a nickel with a quarter, etc. In a given set of coins, figuring out which coins to begin the counting sequence is very important. Many counting-on strategies need to be explored. Counting-on by tens is different when a child begins with 25 cents rather than beginning with ten cents. Many experiences are needed for these concepts to be understood. For this item and in all activities in which money is used, it is strongly recommended that real coins be used for children to manipulate, count and compare.

Refer toType 1 Assessment: Objective 6.2. Supply the child with three sets of coins to match the three separate amounts given in this example. Ask the child to demonstrate and explain how to count and compare the money amounts. Note confidence displayed in counting/grouping the money and the various counting and counting-on strategies used by the child.

16. Count out your coins to match the money that Anita, Emily and Daniel have.



Explain how you know how much money each of the three children has.

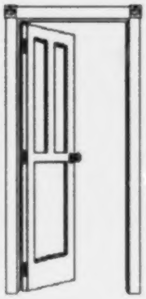
TEACHER NOTES: Item 17 (3 points)

Strand 7: Estimation and Mental Computation

Objective 7.7- estimate length, capacity and mass.

For children to be able to confidently estimate with measurement, it is important for them to develop useful benchmarks about measurement. For example, knowing that a centimeter is about the length across their little finger or that a meter is as high as most doorknobs are from the floor, is most helpful in understanding and estimating units of measurement. Experience with estimating and then actually measuring with appropriate tools is essential.

17. Explain how you could determine the height of the doorway of your classroom. The only thing you may use to figure this out is your own shoe.



Information Sheet

ITEM	ANSWER	OBJECTIVE	OUTCOME LEVEL	CRITICAL OBJECTIVE
1-2	see rubric	2-1-2	C, PS	yes
3	see rubric	2-2-1	PS	yes
4	c	2-3-1	C, PS	yes
5	4 suns, see rubric	2-3-4	C	yes
6	1, 10, 22	2-3-7	C, PS	yes
7	22	2-3-9, 2-2-7	PS	yes
8	answers vary	2-4-5	C	yes
9	see rubric	2-5-3, 2-2-2, 2-2-7	PS	yes
10	see rubric	2-5-3, 2-2-2, 2-2-7	PS	yes
11	inches,cups or ml, kg or lbs.	2-6-1	K,C	yes
12	see rubric	2-6-3	K	yes
13	b	2-7-5, 2-2-7	C, PS	yes
14	bears, 11, answers will vary, 39	2-8-3	C, PS	yes
15	b and h	2-4-1	K, C	no
16	answers vary	2-6-2	K, C	no
17	answers vary	2-7-7	PS	no

1. **Look at this number chart. Find an interesting number pattern and color it.**

	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

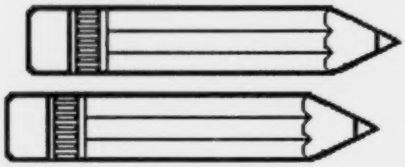
How would you explain your number pattern to a friend?

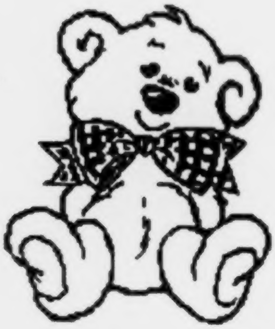
2. **Find another interesting number pattern and color it.**

	1	2	3	4	5	6	7	8	9
10	11	12	13	14	15	16	17	18	19
20	21	22	23	24	25	26	27	28	29
30	31	32	33	34	35	36	37	38	39
40	41	42	43	44	45	46	47	48	49
50	51	52	53	54	55	56	57	58	59
60	61	62	63	64	65	66	67	68	69
70	71	72	73	74	75	76	77	78	79
80	81	82	83	84	85	86	87	88	89
90	91	92	93	94	95	96	97	98	99

How would you explain this number pattern to a friend?

3. At the store you want to buy 2 pencils and a teddy bear eraser.





What do you need to know to be sure you have enough money to buy these?

4. There are 140 pieces of candy in this jar. You need to put 10 pieces of candy in bags. How many bags do you need to bag all of the candy? Circle the correct answer.

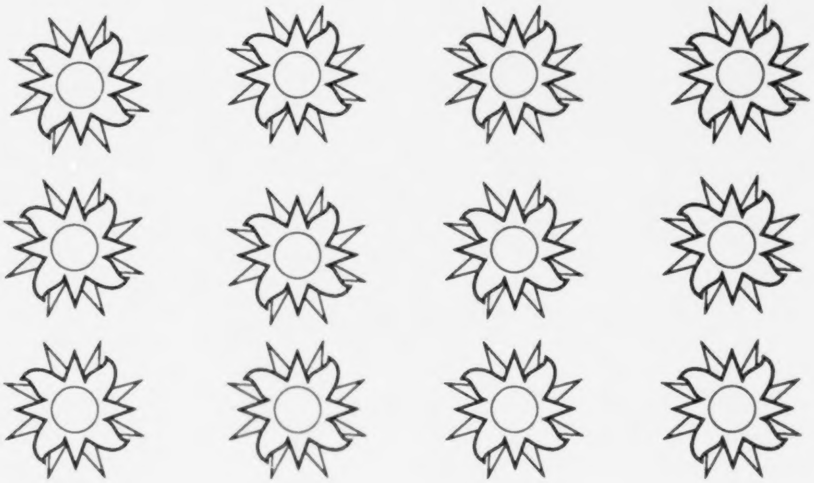
a) 140

b) 40

c) 14

d) 10

5. **Color one-third of the suns yellow.**



Explain how you did this.

Color one-half of the yellow suns with a red crayon. Explain how you did this.

6. **Carmen was given three problems to solve. Carmen wrote each problem in a different way. Help Carmen solve each problem by filling in the box.**

$$\begin{array}{r} 5 \\ +6 \\ \hline 11 \end{array} \rightarrow 5 + 5 + \boxed{} = 11$$

$$\begin{array}{r} 34 \\ -\ 9 \\ \hline 25 \end{array} \rightarrow \begin{array}{l} 34 \\ -\boxed{} \\ \hline 24 + 1 = 25 \end{array}$$

$$\begin{array}{r} 199 \\ +23 \\ \hline 222 \end{array} \rightarrow \begin{array}{r} 200 \\ +\boxed{} \\ \hline 222 \end{array}$$

7. Andrew gave Maria 7 candy kisses. He had 15 candy kisses left. How many candy kisses did Andrew have before he gave Maria any?

Andrew had ___ candy kisses.

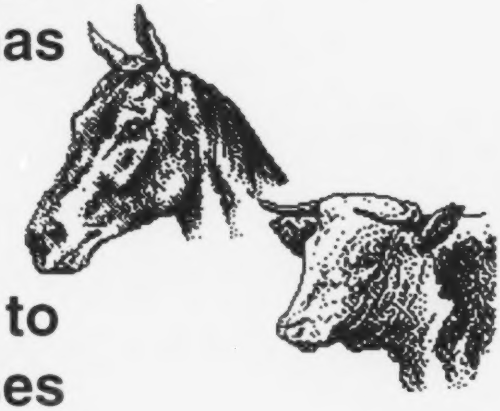
Explain your answer.

8. Look at the shapes your teacher has. Tell how you could sort these shapes into two groups.

What other way can you sort these shapes?

Pick any two shapes. Tell how they are the same and how they are different.

9. **Mr. McGregor has 23 animals. He only has horses and cows.**



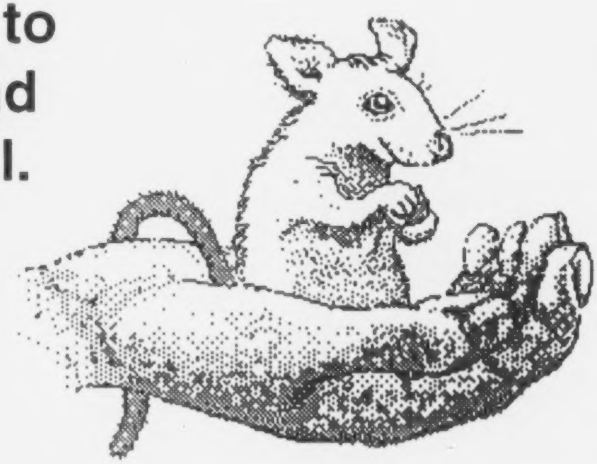
Write different number sentences to tell how many horses and cows he could have.

10. Michael has fewer colored pencils than Janet. Janet has seven colored pencils. How many pencils could Michael have? You may use counters.

Write your answers here:

Explain how you got your answers.

11. You are planning to build a playground for your pet gerbil. Using the units: inch, centimeter, cup, liter, quart, kilogram, pound and milliliter, tell about how big you would make:



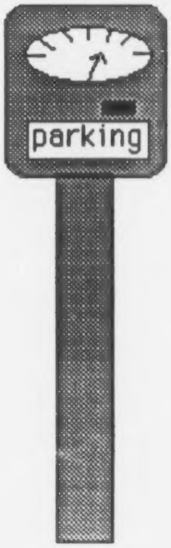
Length of tunnel:____________

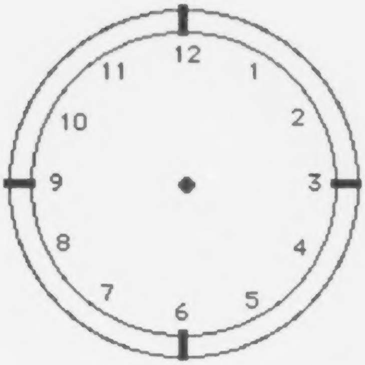
How much the water dish holds:____

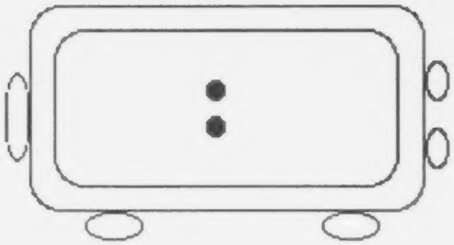
How much the cage weighs:_______

12. This parking meter will run out at 3:05.

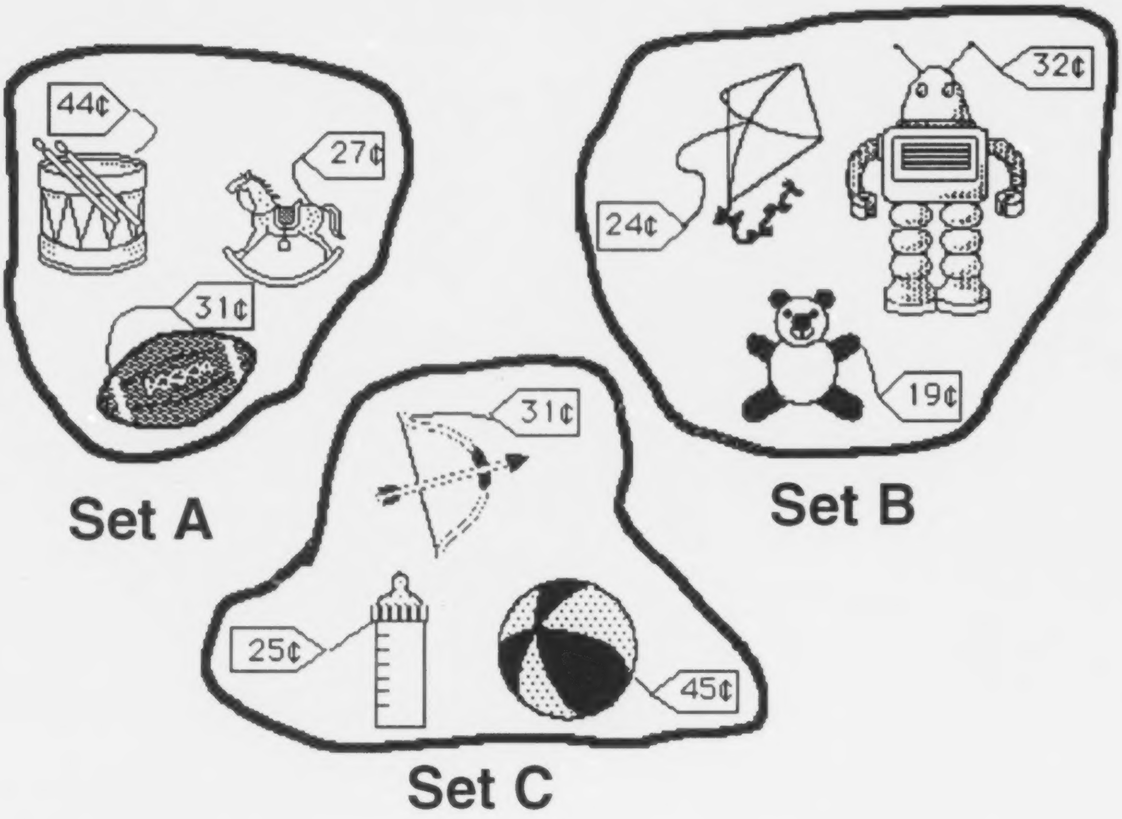
Show Clarabelle the Clown what time that will look like on these two clocks.




12
1
2
3
4
5
6
7
8
9
10
11



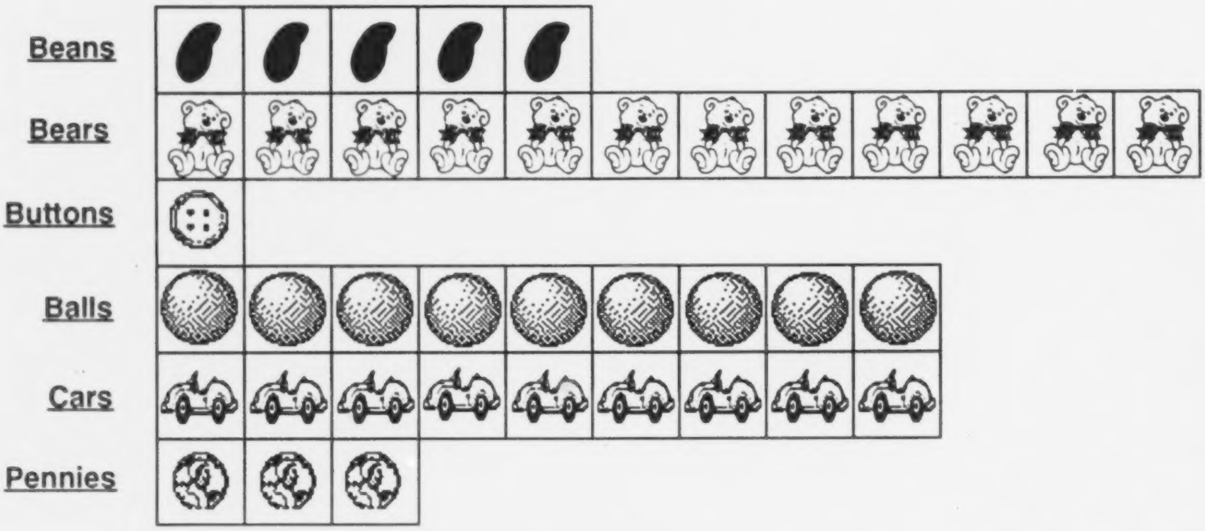
13. **Explain which set of plastic toys Wendi could buy if she has 80¢ to spend. Tell how you could find out without using paper and pencil.**


44¢
27¢
31¢
Set A
32¢
24¢
19¢
Set B
31¢
25¢
45¢
Set C

Which set?______

Explain your answer

14. **Mrs. Gibson's kindergarten class voted for their favorite thing to count. Here is the graph they made:**



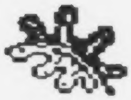
- **What is their favorite counter?**

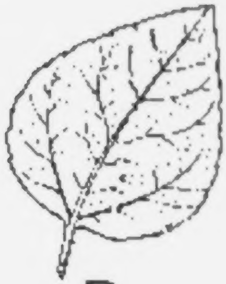
- **What is the difference between the most favorite and the least favorite counters? Tell how many.**

- **Using the information in this graph, tell what you think other kindergarten children like best as counters.**

- **How many children voted in Mrs. Gibson's class?**

15. Mrs. Jackson has these pictures. She wants you to tell which two are congruent. Explain your choice.


A.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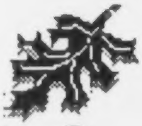
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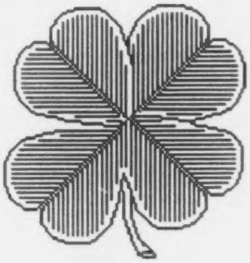
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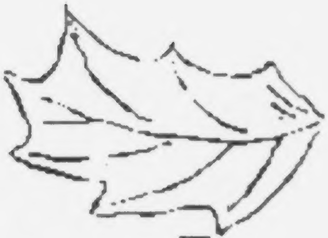
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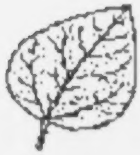
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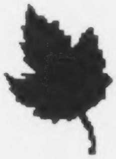
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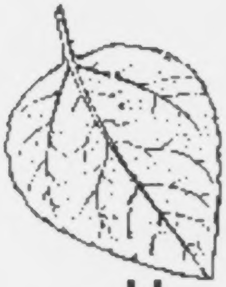
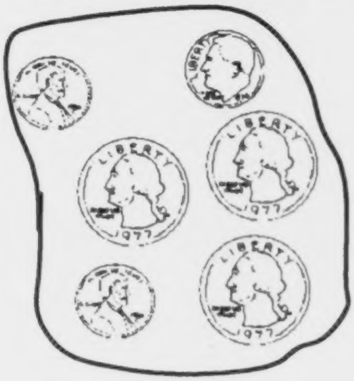

H.

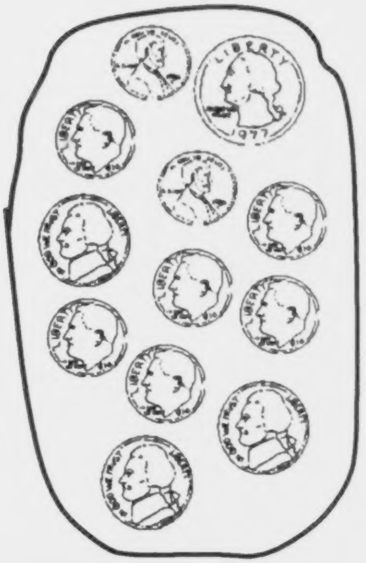
Which two are congruent? ______ and ______

Explain:

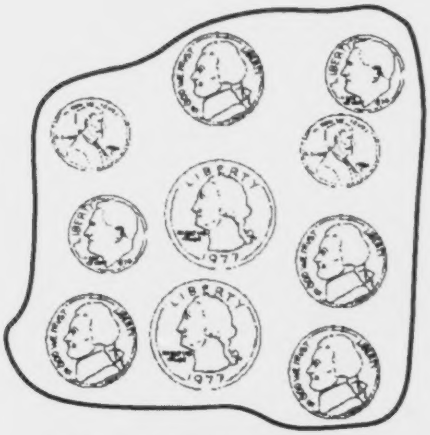
16. Count out your coins to match the money that Anita, Emily and Daniel have.

Anita's Money





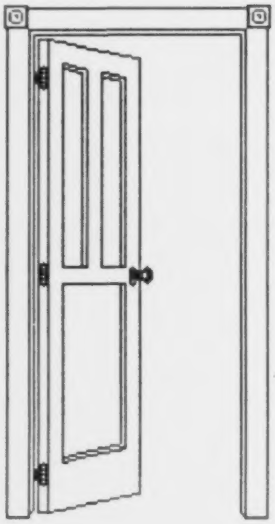
Emily's Money



Daniel's Money

Explain how you know how much money each of the three children has.

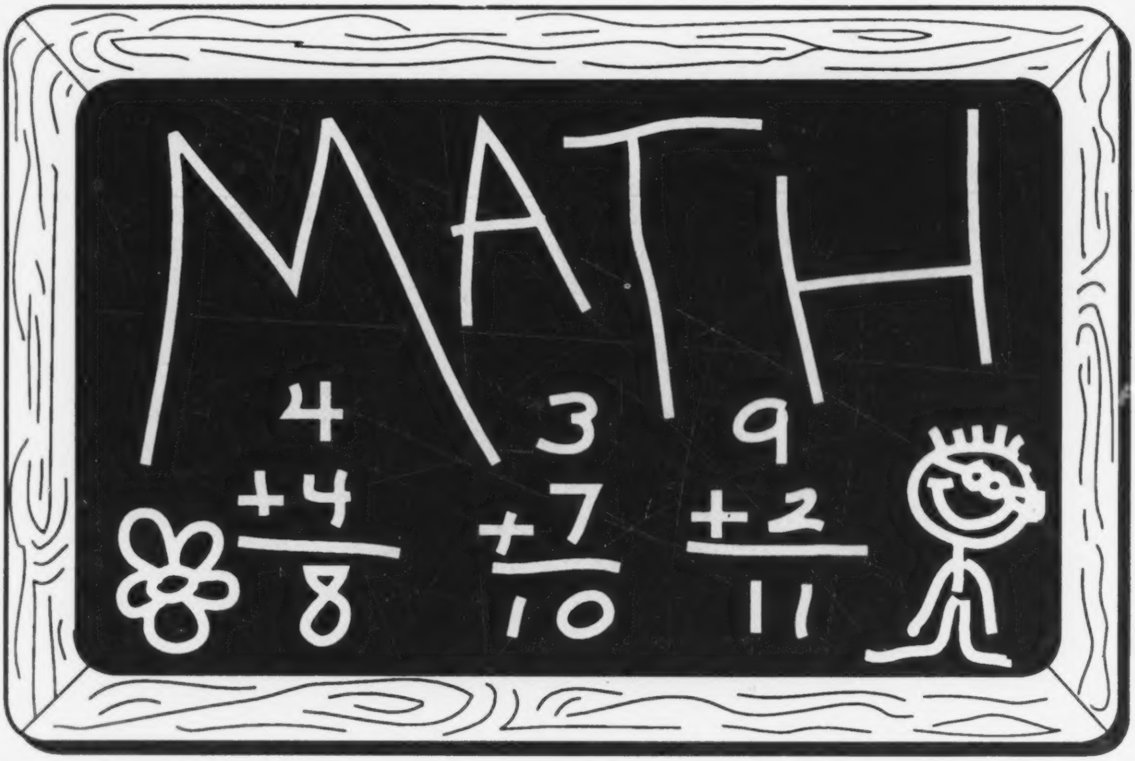
17. **Explain how you could determine the height of the doorway of your classroom. The only thing you may use to figure this out is your own shoe.**



Competency-Based Education Assessment Series

Second Grade Mathematics

Type 3 Assessment



Developed by

Gen Davis

Margaret Kasten

Trish Koontz

Anne Mikesell

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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Steve Meiring	Ohio Department of Education
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About This Instrument

This model competency test integrates assessment and instruction . The assessment is done as an on-going part of instruction. The instructional activity provides information about how the student applies mathematical skills, understandings, and thinking. Objectives from multiple strands are embedded within and assessed through the activities in which the students are engaged as part of instruction. Adequate time and appropriate format for assessing the objectives is included in the design of the instructional unit. Multiple forms of assessment are routinely used throughout the year - for example, student products, teacher observation, interviews, self-assessment, and journal writing. Periodically assessment will focus on gathering information about how well each student is progressing relative to the local competency-based mathematics program goals and objectives.

Instructional units that provide a context for assessing targeted objectives are identified and included within the year plan. Instructional activities and assessment techniques are matched to the purposes and needs of competency-based assessment as determined by the local program and assessment guidelines.

The instructional tasks within which the district-wide, grade level assessment will occur should involve situations that will reflect ways in which mathematics is learned and applied. These tasks will be richer and more in depth than those included on traditional, paper-pencil tests. The student products and records selected will be collected and evaluated in a standardized manner. The tasks should allow student to display the full range of performance and be evaluated using a scoring process that reflects the nature of the objectives and what students should know and be able to do. At a suitable point in time, a thorough review of the records and work collected for each student can be used to determine how well the student in progressing in relation to grade-level outcomes.

The suggested questions and student sheets to support those questions which follow serve as an example of how this type of assessment might be carried out. The model a is not intended to be fully developed instructional unit that can be inserted into a year-long instructional plan as is, but rather it represents a guide for identifying and embedding competency-based assessment within instruction.

A number of publications such as the National Council of Teachers of Mathematics resource, Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions (Stenmark, 1991) are available.

The following story line can be used to create activities to assess the critical objectives. The objective or objectives from the Model Competency Based Mathematics Program being assessed is referenced at the end of each question or series of questions. By using this simulation format, assessment can be continually integrated into the discovery learning activities.

Each student should be given a small tub which contains some plastic beads that can be strung on pipe cleaners to make bracelets. If it is difficult to obtain enough beads to make the bracelets, there are several different kinds of breakfast cereal "rings" that can be substituted for beads.

In addition to a series of questions designed to assess all of the critical objectives in a single context, a series of student sheets that might be used to record answers is included. These student sheets may be used in a variety of ways.

Simulation context:

A friend has asked me if our class would be willing to help her make prizes for an upcoming county fair. She would like some bead bracelets to be made. How many of you would like to help in the project? She would like each of the bracelets to have exactly 10 beads on a pipe cleaner. They should be bagged ten bracelets in each bag for delivery.

Questions:

Here is a tub of beads. Estimate how many are in your tub__________ If you need 10 beads for each bracelet, how many pipe cleaners do you need?____________ How many beads will you use to compete a bag of bracelets. (Note: In addition to providing practice with estimation, this activity can be used to develop the concept of place value Objective 3.1)

How can you count how many beads you used if you counted your bracelets?_______ If you counted your bags?__________ (Note: These questions address Objective 1.2: recognize patterns in numbers and number combinations.)

If you have 98 beads and your partner has 32, how many pipe cleaners will you need altogether to make as many bracelets as you can?_____________(Note: Objective 3.7: learn strategies for the addition and subtraction of numbers...)

My friend asked me if you would like to be paid with bracelets. Is that okay? ____ If she wanted to give each of you two bracelets, how many bracelets can the class keep? ______ (Objective 2.1: identify needed information to solve a problem)

Today my friend wants us to make a different type bracelet. Half of the beads need to be yellow and the other half orange. How would you make a bracelet if you used 8 beads altogether? _____ 12 beads altogether?_____ etc. (Objective 3.4: illustrate fractional parts of whole objects or sets of objects)

Today we are working in groups of five. Your group is to make all of the different bracelets you can think of that use just two colors and only 10 beads for each bracelet. Using these mailing labels, write a number sentence on each label and tag the bracelet. (Objectives 3.9 and 5.3: translate real-life situations involving addition and/or subtraction into conventional symbols of mathematics and Model a problem situation using numbers and/or letters)

If you were to package the bracelets in these decorative boxes, which box would you pick. How would you describe the box to an ill friend over the phone. (Encourage students to describe how many faces are on the box and what are these faces called.) (Objective 4.5 Compare three-dimensional objects describing similarities and differences using appropriate standard and non-standard language)

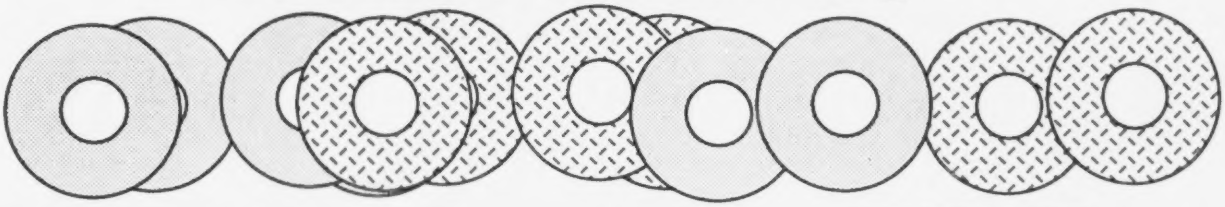
If we were to put these beads end to end how long would the chain be? _______ How many cups would 500 beads fill ? ______ How much would they weigh?______ What tool do we need to weight all of the bracelets we made as a class? _______ (Objective 6.1: explore length, capacity, and weight by selecting and using appropriate metric and conventional units...)

If it takes you five minutes to make a bracelet and you start at 4:00, what time will you be done making 5 bracelets?________ 10 bracelets? _________ (Objective 6.3: tell time to the nearest five-minute interval on digital and dial time-pieces)

Each of the five groups used the following amount of beads? 42, 73, 85, 51, and 44. Estimate how many pipe cleaners they used? (Objective 7.5: make estimates in addition and subtraction, using front-end digits)

Design a graph showing how many bracelets each group made. (Objective 8.3: explore picture and bar graphs (scales by one) by making identifications, comparison, and predictions)

Estimation and Counting



Estimate how many beads are in your tub. Put your estimate in the box.

☐

Think about your estimate. How many pipe cleaners will you need? Put your estimate in the circle.

○

How many beads will you need to complete a bag of bracelets?

____

How can you count how many beads you used if you counted your bracelets?

Use your bracelets and then ask three other students in class to help you fill out the chart. Do you see a pattern?

Name	Number of Bags	Number of Bracelets	Number of Beads

What is the pattern?

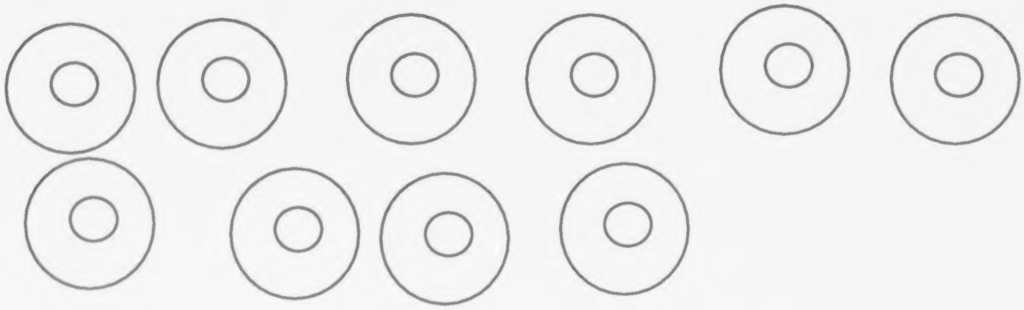
Take a vote:

IT IS OKAY TO TO BE PAID IN BRACELETS	IT IS NOT OKAY TO BE PAID IN BRACELETS

How can you keep track of the vote?

Are there questions you want to ask before you vote? If you have a question write it here:

If you use 10 beads and half are yellow, how many are yellow? You may use the picture to help you.



If you use 8 beads and half are yellow, how many are yellow? You may draw a picture to help you.

If you use 12 beads and half are yellow how many are yellow? You may draw a picture to help you.

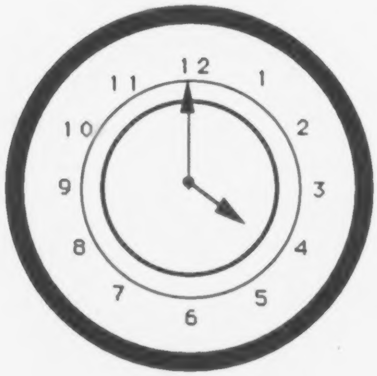
If it takes you five minutes to make a bracelet and you start at 4:00, what time will you be done making 5 bracelets?

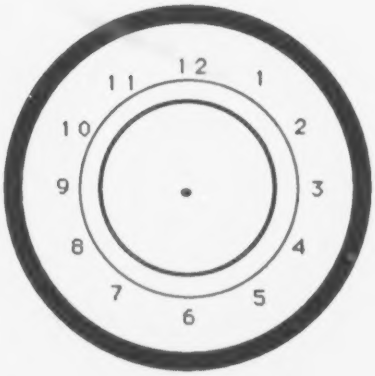
____

If you make 10 more bracelets what time will you finish?

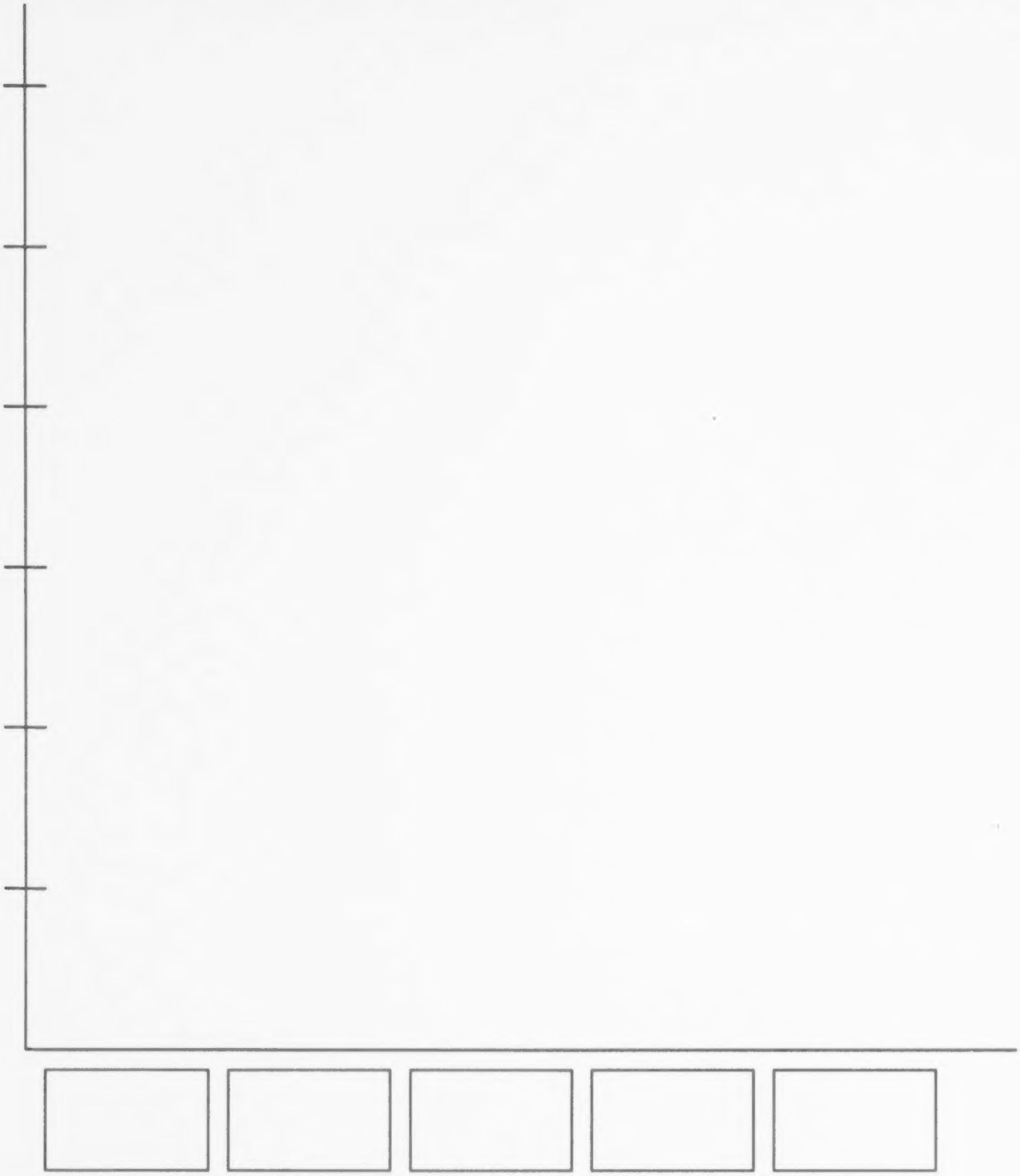
____

You may use the pictures of the clocks to help you.





Write the names of the groups in the boxes. Label the side of the graph. Show how many bracelets each group made.

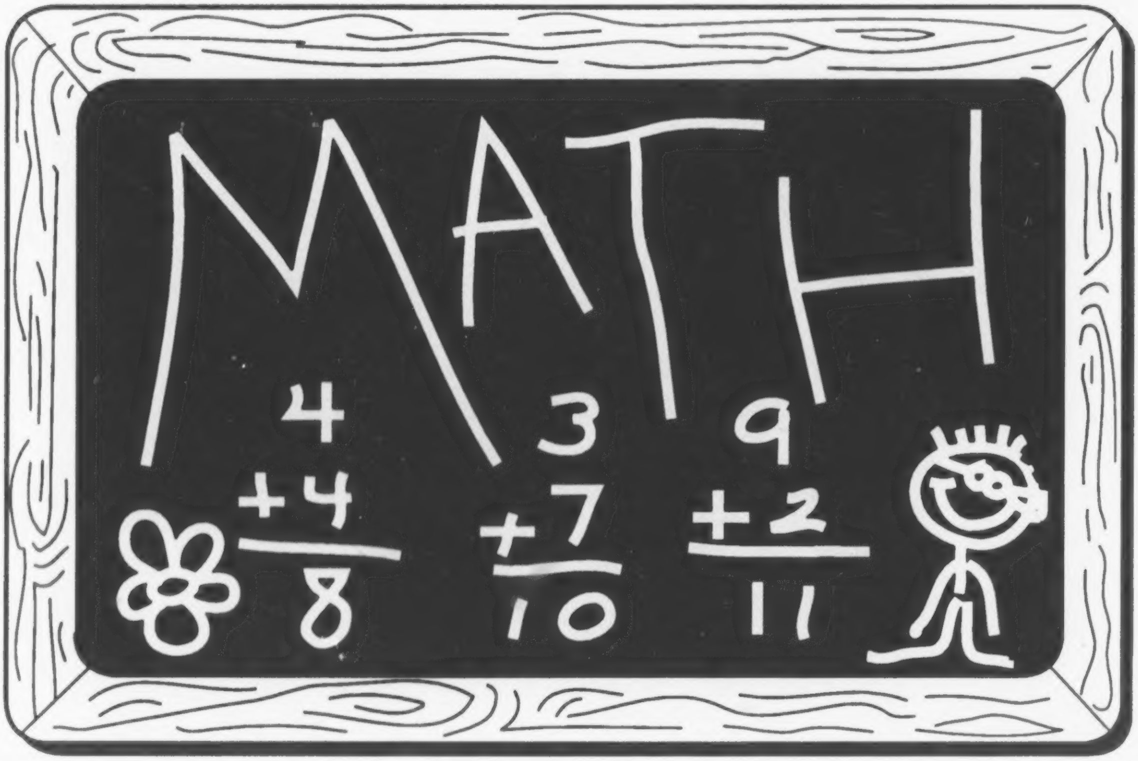


Make another graph that shows the same information in a different way.

Competency-Based Education Assessment Series

Third Grade Mathematics

Type 1 Assessment



Developed by

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The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency assessment instrument has a largely traditional design and is intended to be given in one period of approximately 45 to 60 minutes. However, it is also not intended to be a timed test. Therefore district or teacher discretion should be used when determing the schedule for assessment. This instrument is composed of 18 multiple choice, 4 short answer, and 2 rubric scored items. These 24 items have a suggested point value of 34 points.

If students routinely use counters, manipulatives, and calculating devices as an integral part of instructional activities, they should be available for use during the assessment. Also, those items which ask about three-dimensional objects should be accompanied by actual objects and not just pictures of those objects.

Each item on this assessment addresses either a critical objective for third grade. A number of items address objectives that are learning outcomes for the Fourth-grade Proficiency Test.

The Teacher Notes include discussion about each item. The items have been developed to measure progress on third grade objectives for the Model Competency-Based Mathematics Program. Discussion includes the strand, a reference to the Model objective being measured, a suggested point value, ideas for pre-assessment activities or comments about the objective and item, and suggested scoring guidelines or sample scoring rubric.

TEACHER NOTES: Item 1 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.3 - demonstrate an understanding of multiplication and division by developing models, using symbols, and describing applications in words

In this assessment, the understanding of the concept of multiplication/division is what is being addressed. Students should be given problems written in a variety of ways in order to expand this understanding. Problems involving facts as well as those involving remainders should be used. This is not an assessment of "knowing" the facts, but rather understanding what multiplication/division means. The use of manipulatives would be useful in the development of this understanding prior to assessment.

TEACHER NOTES: Item 2 (1 point)

Strand 2: Problem-Solving Strategies

Objective 2.5 - extend the guess and check procedure by recording guesses and checks to help make better guesses until the solution is reached

Students should be afforded recurring opportunities to practice and refine their guess and check problem-solving strategy and demonstrate their understanding of this strategy by explaining how to use the check to make a better guess. The correct answer to the problem was purposely not given, as the assessment is not for the student to check each of the given answers, but to determine if the student is using the check of the previous guesses to refine the next guess.

TEACHER NOTES: Item 3 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.8 - recall multiplication and division facts through 12 x 12 using ... anchor facts

Students should have opportunities to share how they remember "difficult" facts. The third grade objective includes the commutative property (turn around facts) and the distributive property (4 sevens is 2 sevens and 2 sevens; since 2 sevens is 14, then 4 sevens is 14 + 14 which equals 28) as well as anchor facts (facts "known" by the child) which is assessed by this item.

TEACHER NOTES: Item 4 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.9 - multiply using paper-and-pencil algorithms

This item deals with the multiplication algorithm and also is a precusor to algebraic concepts. As problems are presented to the students, the same digit (or number) within the problem should be represented by the same symbol, and different digits (or numbers) should be represented by different symbols.

For example:

□16	□16
x6	x6
2□96	○ 296

In the first example, both squares would be filled in with the same number, 4. In the second example, the square and the circle could be filled in with different numbers: the square could equal 2 and the circle would equal 1 **OR** the square could equal 7 and the circle would equal 4.

TEACHER NOTES: Item 5 (1 point)

Strand 6: Measurement

Objective 6.1 - continue explorations of length, capacity, and weight, and extend familiarity of units to include kilometer, meter, mile, yard, foot, gallon, gram, ounce, and fractional parts of each

Students need frequent opportunities to measure the same objects using different units of measure and then compare the numbers resulting from these measurements.

TEACHER NOTES: Item 6 (1 point)

Strand 8: Data Analysis and Probability

Objective 8.7 - translate freely among pictographs, tables, charts, and bar graphs

Students need many opportunities to read and interpret information from pictographs, tables, charts, and bar graphs, and then to translate that information into any of the other forms.

TEACHER NOTES: Item 7 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.3 - demonstrate an understanding of multiplication and division by developing models, using symbols, and describing applications in words

This item assesses whether the student understands the concept of division and is able to use conventional symbols to represent a given situation. The use of the word "phrase" should indicate that the answers are not given in complete sentences. A correlation should be drawn between the use of these words in mathematics and their use in language arts.

TEACHER NOTES: Item 8 (2 points)

Strand 6: Measurement

Objective 6.5 - illustrate the approximate size of units (inch, centimeter, meter, and yard)

Students need frequent opportunities to estimate and then measure lengths of various objects in and around the classroom. Through these activities, the students develop an understanding of the approximate size of each of these units. Many teachers use body parts to illustrate approximate size of different units.

Sample Rubric:
2 = the line drawn is approximately 1.5 cm to 2.5 cm in length
1 = the line is approximately 1 to 1.5 cm **OR** if the line is approximately 2.5 to 3 cm in length
0 = anything not covered by the above choices

TEACHER NOTES: Item 9 (1 point)

Strand 7: Estimation and Mental Computation

Objective 7.5 - add strings of numbers mentally by finding groups of tens

This assessment item uses combinations of numbers that form ten. This concept, in instruction, should also include problems that use combinations of numbers that form multiples of ten

$$2 + 17 + 18 + 23 + 5 =$$
$$(2 + 18) + (17 + 23) + 5 =$$
$$20 + 40 + 5 = 65$$

as well as some practice in finding three numbers that equal ten

$$2 + 6 + 3 + 5 =$$
$$(2 + 3 + 5) + 6 =$$
$$10 + 6 = 16$$

The above examples are **not** expected to be written. As the strand indicates, the work is to be done mentally and an answer given. However, to insure that the child is thinking correctly, an oral explanation should be required.

TEACHER NOTES: Item 10 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.1 - add and subtract numbers fluently using any strategy

The students are expected to answer both parts correctly to obtain a point. Instructional problems could also include a precursor of algebraic concepts by using different shapes for the missing numbers. See the explanation for item 4 for more information and example problems.

Suggested Scoring Guidelines: BOTH numbers (4 and 7) must be correctly written to obtain one point.

$$\begin{array}{r} 5\,2\,\boxed{4} \\ -\,1\,\boxed{7}\,8 \\ \hline 3\,4\,6 \end{array}$$

TEACHER NOTES: Item 11 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.16 - use the symbols <, ≤, >, ≥, and = in describing order as well as the terms "at least" and "at most"

This item is at the knowledge/skill level. Students should be familiar with the symbols as well as the terminology. Students can be taught to "read" the symbols in the normal left to right progression. The symbol < is read as "is less than" (the point is smaller than the wide end) and conversely the symbol > is read as "is greater than" (the wide part is bigger than the point). In beginning to teach the "at least" and "at most" terminology, it may be useful to use numbers and discuss the meaning of the terminology and then expand the idea to non-numerical references.

TEACHER NOTES: Item 12 (1 point)

Strand 7: Estimation and Mental Computation

Objective 7.2 - use front end digits to estimate addition with several addends

Students need a quick, mental way to "check" an answer to computational problems. Front end estimation provides a simple way for third graders to determine if their answers are "within the ballpark." There are also times when an estimated answer is all that is necessary. Students should get into the habit of always making a mental estimation of an answer so that when they find the actual answer they know whether it "makes sense" or not.

TEACHER NOTES: Item 13 (1 point)

Strand 6: Measurement

Objective 6.5 - illustrate the approximate size of units (inch, centimeter, meter, and yard)

Students need frequent opportunities to estimate and then measure lengths of various objects in and around the classroom. Taking 2 or 3 minutes between classes (or while waiting for lunch, or for the bus, or.....) to estimate the length of the window or the height of the chalkboard (or ....) in the given unit of measure and then measuring the object provides students with ongoing practice. These "measurement moments" done on a daily basis will help the students develop an understanding of the approximate size of each of these units and improve their linear estimation skills.

TEACHER NOTES: Item 14 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.11 - order fractions on the basis of concrete materials

Students should use fractional names with concrete fractional materials to compare and develop the concept of relative size of the various common fractions. As this concept is being developed, students also need to be introduced to the concept that a fraction's size varies as the size of the whole (area model) or the number of objects (set model) varies. Thus, half the days of April produces a much different answer than half a dozen eggs as does half a small pizza compared to half a large pizza. Order should be least to greatest sometimes and greatest to least at other times as students need to be aware that careful reading (or listening) is necessary. They should be capable of working both directions with the fractions.

TEACHER NOTES: Item 15 (4 points)

Strand: 1: Patterns, Relations, and Functions

Objective 1.3 - use patterns to make generalizations and predictions by ... identifying missing elements in a pattern and justifying their inclusion
Objective 1.4 - make a table of values to record the pairing of members of two sets, determine the relationship (rule) between each pair, and use the rule to generate additional pairs.

This item addresses a critical objective in the Model Competency-Based Mathematics Program and is usually assessed in the form of a problem solving or application situation. This form of assessment will give an indication of the student's understanding of concepts and his/her ability to make connections in mathematics, solve problems, and verify results as well as perform correct computations. Students need practice in determining the rule between pairs of numbers and in setting up tables to do this. Frequent playing of the game "What's My Rule" would help develop these skills prior to assessing the critical objective. (See the type three activity in this document called "A Function Machine" which will also help develop skills for this objective.)

Suggested Scoring Guidelines: 4 points possible with
1 point = the correct numerical answer (10) for part a
1 point = the correct numerical answer (10) for part b
1 point = an adequate description of the rule; i.e., "Robbie adds three each time"
1 point = a correct table showing the pairs of numbers

TEACHER NOTES: Item 16 (2 points)

Strand 3: Numbers and Number Relations and
Strand 5: Algebra

Objective 3.4 - translate real life situations involving multiplication and division into conventional mathematical symbols
Objective 5.3 - understand the use of letters in statements such as ab=12 or 3c=d and find a when b is given, etc.

In this item, the students are asked to write a number phrase (3 x 12) rather than a sentence since the first part is not to find an answer, but rather serve as a model for the second part. The students are being asked to show what they need to do to find an answer. In the Model Competency-Based Mathematics Program, algebraic statements using letters for variables are introduced in first grade as are the use of number phrases and number sentences.

Suggested Scoring Guidelines: 2 points possible with
1 point = any correct phrase (i.e., 12 x 3 or 3+3+3+3+3+3+3+3+3+3+3+3)
1 point = any correct phrase (i.e., 12 x N or N+N+N+N+N+N+N+N+N+N+N+N, 12•N, or 12N)

TEACHER NOTES: Item 17 (1 point)

Strand 6: Measurement

Objective 6.1 - continue explorations of length, capacity, and weight and extend familiarity of units to include kilometer, meter, mile, yard, foot, gallon, gram, ounce and fractional parts of each

A second grade objective has been expanded and smaller units have been introduced. Relationships among the various units need to be established through comparisons and actual measurement of objects and liquids (or sand).

TEACHER NOTES: Item 18 (1 point)

Strand 6: Measurement

Objective 6.2 - count collections of coins and bills which include dollar, five, and ten dollar bills and compare values

The objective asks students to compare values. This may be done by more than one method. The student may either first count the money and then compare **OR** the student may compare the individual amounts first and then compute the difference. In this particular problem, the second method is probably simpler. In comparing quarters, Jessica has 6 more than Tyler, so she has $1.50 more. Dimes are the same--neither child has more, so we can ignore them since we are being asked to determine who saved the most and how much more they saved (from the answers given). Continue in this manner to complete the comparisons to figure out how much more was saved by whom.

TEACHER NOTES: Item 19 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.16 - use the symbols <, ≤, >, ≥, and = in describing order as well as the terms "at least" and "at most"

This item is at the knowledge/skill level. Students should be familiar with the symbols as well as the terminology. Students can be taught to "read" the symbols in the normal left to right progression. The symbol < is read as "is less than" (the point is smaller than the wide end) and conversely the symbol > is read as "is greater than" (the wide part is bigger than the point). Students may also "discover" that the larger number is at the wide part and the smaller number is at the smaller part (point) through questions posed by the teacher to facilitate "discovery."

TEACHER NOTES: Item 20 (4 points)

Strand 2: Problem-Solving Strategies

Objective 2.5 - extend the guess and check procedure by recording guesses and checks to help make better guesses until the solution is reached

Students should be afforded recurring opportunities to practice and refine their guess and check problem-solving strategy and to demonstrate their understanding of this strategy by explaining how to use the check to make a better guess. This item assesses the guess and check strategy in a problem situation. The students' explanation of their thinking is needed to determine whether they have used the strategy requested.

Sample Rubric: 4 = a correct answer (4 girls) **AND** an adequate explanation using guess and check strategy are given
3 = a correct answer (4 girls) is given with an indication that guess and check was used but the explanation is inadequate
2 = an incorrect answer caused by computational error with an attempt at the use of guess and check strategy
1 = correct answer not showing use of guess and check, may or may not have an attempt at an explanation
0 = anything not covered by the above choices

TEACHER NOTES: Item 21 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.3 - from situations created in the classroom develop models of multiplication and division (arrays)
Objective 3.8 - recall multiplication and division facts through 12 x 12 using the distributive property

This item shows an array using the distributive property to enable children to remember the fact 7 x 8. Children should have prior experience in making arrays for their personally difficult facts that make use of the distributive property and a fact they know.

TEACHER NOTES: Item 22 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.10 (Grade 2) - find equivalent forms of numbers using hundreds, tens, and ones

This is a continuation of a second grade objective so it was written in the form of finding the incorrect answer among the choices given. Children must revisit previously taught concepts from a different perspective.

TEACHER NOTES: Item 23 (3 points)

Strand 4: Geometry

Objective 4.4 - describe a three-dimensional object from different perspectives, and Objective 4.6 - use mathematically correct names for common geometric figures

Children should be familiar with pattern blocks and tangrams and should be using correct mathematical names for the shapes. Correct spelling of the names is not necessary as long as the invented spelling would be recognizable.

Suggested Scoring Guidelines: 3 points possible with a half point for each answer:

- Chimney is a square (give credit for rectangle)
- Gable is a triangle
- Decoration over door is a parallelogram
- Door is a rectangle
- Window is a square (give credit for rectangle)
- Window box is a trapezoid

TEACHER NOTES: Item 24 (1 point)

Strand 3: Numbers and Number Relations

Objective 3.13: order whole numbers, fractions, and decimals (tenths and hundredths) on the number line

At the instructional level, this item should include whole numbers, fractions, and decimals on the same number line to give a visual comparison of fractions and decimals.

Information Sheet

Item	Answer	Objective	Outcome Level*	Critical Objective
1.	A	3-3-3	PS/A	Yes
2.	C	3-2-5	PS/A	Yes
3.	B	3-3-8	C	Yes
4.	B	3-3-9	C	No
5.	B	3-6-1	C	Yes
6.	C	3-8-7	K/S	Yes
7.	C	3-3-3	C	Yes
8.	See "Teacher Notes"	3-6-5	K/S	Yes
9.	C	3-7-5	K/S	Yes
10.	4, 7	3-3-1	C	No
11.	C	3-3-16	K/S	Yes
12.	A	3-7-2	K/S	No
13.	C	3-6-5	C	Yes
14.	C	3-3-11	C	No
15.	See "Teacher Notes"	3-1-3 3-1-4	PS/A	Yes
16.	See "Teacher Notes"	3-3-4 3-5-3	C	No
17.	B	3-6-1	C	No
18.	C	3-6-2	K/S	Yes
19.	C	3-3-16	K/S	Yes
20.	See "Teacher Notes"	3-2-5	PS/A	Yes
21.	B	3-3-3 3-3-8	C	Yes
22.	C	2-3-10	C	No
23.	See "Teacher Notes"	3-4-4 3-4-6	K/S	Yes
24.	B	3-3-13	K/S	No

* P S/A = Problem Solving/Application
C = Concept
K/S = Knowledge/Skills

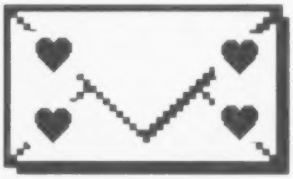
Directions: Circle the letter (A, B, or C) corresponding to the correct answer.

1. Betty wanted to paste hearts on the envelopes for the party. She decided to put four hearts on each envelope. She had 33 hearts. How many envelopes could she complete?

   A. 8

   B. 9

   C. 29



2. John had 10 coins (nickels and dimes) worth 75¢. He asked Sissie to guess how many nickels he had. She guessed two nickels and wrote 2, 8 and 90¢ in her table. Her next guess was seven nickels and she wrote 7, 3, and 65¢. A reasonable next guess would be

   A. 1 nickel

   B. 9 nickels

   C. 3 nickels

5¢	10¢	Total
2	8	90¢
7	3	65¢

3. If you know that 5 x 8 = 40, then 6 x 8 is an easy fact because

   A. it is one more six.

   B. it is one more eight.

   C. 6 + 8 is 14, and 40 + 14 = 54.

4. If □56
   X 6
   21□6 then what goes in □?

   A. □ = 2

   B. □ = 3

   C. □ = 4

5. If Jerry measures the table in front of the room using inches and Tom measures the same table using centimeters, then

   A. Jerry's measurement number will be greater than Tom's.

   B. Tom's measurement number will be greater than Jerry's.

   C. both measurement numbers will be the same.

6. As John walked to school he counted the animals and birds that he saw each week. Based on his record below, which of these statements is correct?

ANIMALS AND BIRDS SEEN WHILE WALKING

	DOGS	CATS	BIRDS	TOADS
WEEK 1	3	1	5	1
WEEK 2	2	2	2	0
WEEK 3	4	3	6	2
WEEK 4	1	4	3	1
WEEK 5	2	2	1	1

   A. Each week he saw more birds than cats.

   B. Altogether, in five weeks, he saw more dogs than cats.

   C. He saw more toads in week three than in any other week.

7. 28 children are to sit around seven tables in the room, with the same number of children at each table. Which of the following number phrases explains how to find the number of children at each table?

   A. 28 x 7

   B. 7 ÷ 28

   C. 28 ÷ 7

8. Without using a ruler, draw a line about two centimeters long in the space below.

9. Kami was very good at adding numbers in his head. He found that if he could find some numbers that added to 10, he did not get mixed up. What is one combination of numbers he might have added to get 10 in the following exercise?

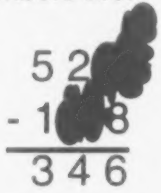
3 + 5 + 6 + 4 + 3 + 7

A. The first number and the third number.

B. The second number and the third number.

C. The first number and the last number.

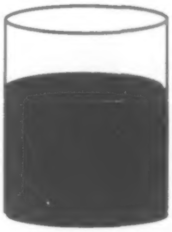
10. Mindy spilled ink on her brother's paper. Help her figure out what the ink-covered numbers are.



a) The number in the top line should be ________ .

b) The number the the second line should be ________ .

11. The glass of grape juice appears to be:



A. at most half full.

B. exactly half full.

C. at least half full.

12. Mindy needed to estimate a total for the following numbers: 92, 31, 25, and 76. She used front-end estimation. Her estimated sum would be

A. 210

B. 224

C. 230

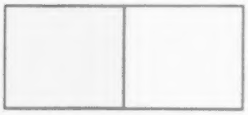
13. Which of the following would measure closest to a meter?

A. The height of the door to your classroom

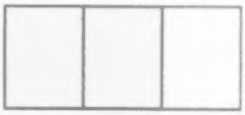
B. The length of a pencil

C. The height of your teacher's desk

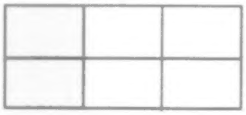
14. The order of the shaded part of the following pictures of fractions from least to greatest is



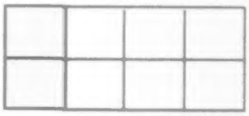
a



b



c

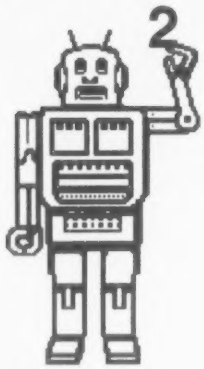


d

A) a b c d

B) d b c a

C) d c a b



15. When I feed Robbie the Robot a number she does stuff to it which causes the number to change before she spits it back. When I put in a two, she spits back a five. When I put in a five, out came an eight. When I put in a one, a four came out. Make a table to discover what Robbie is doing and answer the questions below.

Make your table here:

a) What comes out if you put in a seven? ______

b) What did you put in if a 13 comes out? ______

c) Describe what Robbie does to the number.

____

____

____

____

16. There are 12 seats on the school minibus. Write a phrase that tells how many children the bus can hold if three children sit on each seat.

____

Write a phrase that tells how many children the bus can hold if **N** children sit on each seat.

____

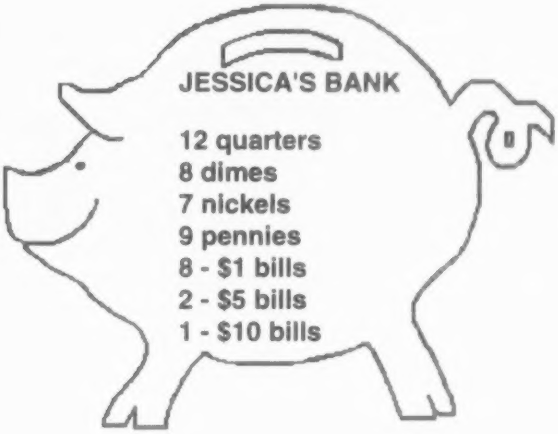
17. John has a quart of water. Which of the following would most likely be filled by his water with none left over?

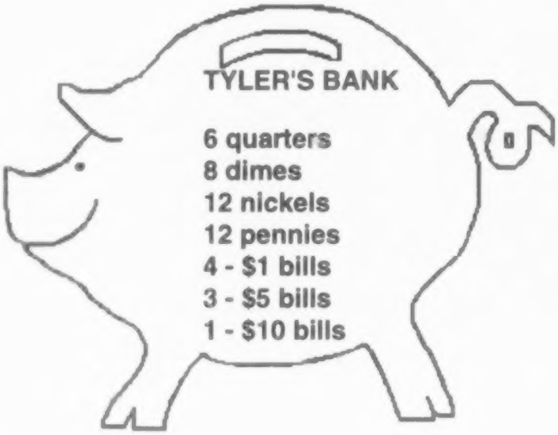
A. a cup

B. a small mixing bowl

C. a kitchen sink

18. Jessica and Tyler have been saving money for the last two months to purchase a pair of rollerblades.


JESSICA'S BANK
12 quarters
8 dimes
7 nickels
9 pennies
8 - $1 bills
2 - $5 bills
1 - $10 bills


TYLER'S BANK
6 quarters
8 dimes
12 nickels
12 pennies
4 - $1 bills
3 - $5 bills
1 - $10 bills

Who has saved the most money and how much more has he or she saved?

A. Jessica has saved 35¢ more than Tyler.

B. Tyler has saved 33¢ more than Jessica.

C. Jessica has saved 22¢ more than Tyler.

19. Which statement is correct?

A. 8 < 5 < 1

B. 7 > 2 > 4

C. 9 < 11 < 15

20. In one corner of the playground there were 16 children playing. Mrs. Jones noticed that there were three times as many boys as girls playing there. How many of the children were girls?

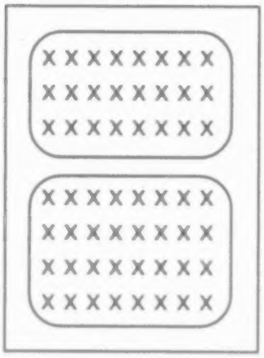
    Use guess and check to solve this problem. Describe what you are thinking.

21. The picture to the right illustrates that

    A. (2 x 8) + (3 x 8) = 7 x 8

    B. (3 x 8) + (4 x 8) = 7 x 8

    C. (4 x 8) + (5 x 8) = 7 x 8



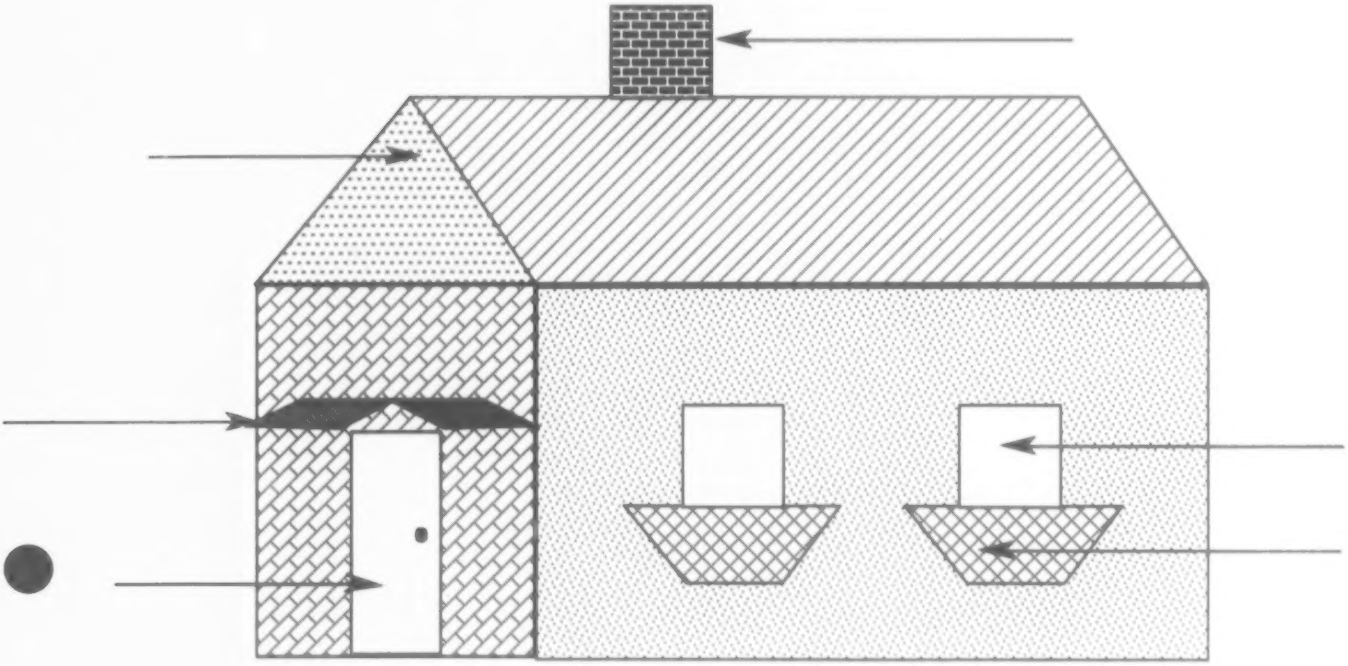
22. Which of the following is NOT a way to write 342?

    A. three hundreds, four tens, and two ones

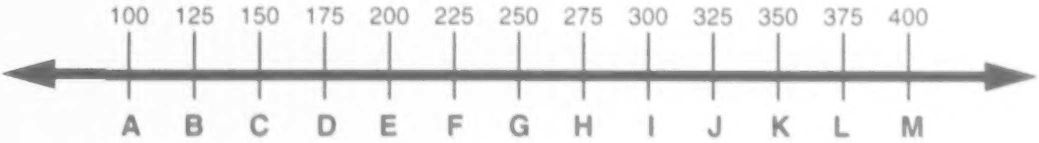
    B. thirty-four tens and two ones

    C. three hundreds and forty-two tens

23. On each arrow write the mathematical name that **best** describes the shape that the arrow points to.



24. The number 237 will fall between what two points on the number line?


100
125
150
175
200
225
250
275
300
325
350
375
400
A
B
C
D
E
F
G
H
I
J
K
L
M

A. **E** and **F**

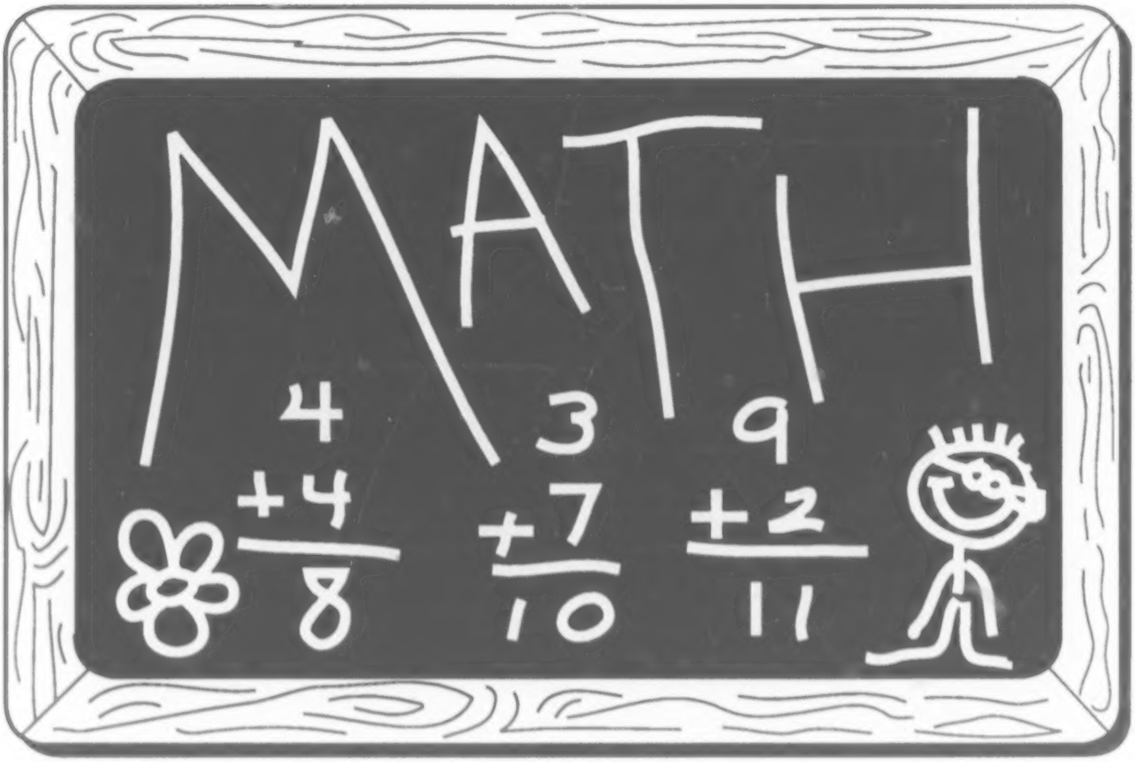
B. **F** and **G**

C. **G** and **H**

Competency-Based Education Assessment Series

Third Grade Mathematics

Type 2 Assessment



Developed by

Evelyn Altherr
Ethel Briggs
Margaret Comstock
Margaret Kasten
Anne Mikesell

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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Ray Heitger	Ottawa Hills City Schools
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Steve Meiring	Ohio Department of Education
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Jane Stockdale	Columbiana County Schools
Ray Trenta	Akron City Schools

About This Instrument

This model competency assessment instrument is intended go beyond the traditional multiple choice assessment. The length of time a student will need to complete the instrument may extend beyond the typical 50 minute class period. It is not intended to be a timed test and district or teacher discretion should be used when determining the schedule for assessment. The instrument is a mixture of open form items and items requiring extended responses. Some items are direct, straight-forward measures of particular objectives. Some items assess combinations of more than one objective.

The instrument is intended to assess a deeper level of understanding than that being assessed on the Type 1 instrument. Therefore scoring of this instrument is a bit more complicated than the scoring of the Type 1 instrument. While the entire instrument has a suggested point value of 50, please note that districts are encouraged to consider adapting any or all parts of the assessment, including scoring suggestions.

If students routinely use counters, manipulatives, measuring tools, and calculating devices as an instrumental part of instructional activities, they should be available for use during the assessment. Also, those items which ask about three-dimensional objects should be accompanied with access to those objects and not just pictures of those objects. For that purpose, patterns for making paper models of the necessary solid objects have been included at the end of this teacher section.

Each item on this assessment addresses a critical objective for third grade or a model objective related to a learning outcome for the Fourth-grade Proficiency Test.

The Teacher Notes that follow include discussion about each item. The items have been developed to measure progress on third grade objectives from the Model Competency-Based Mathematics Program. Discussion includes the strand, a reference to the third grade objective being measured, a suggested point value, ideas for pre-assessment activities or comments about the objective and item, and suggested scoring guidelines or sample rubric.

TEACHER NOTES: Item 1 (2 points)

Strand 3: Numbers and Number Relations

Objective 3.3 - from situations created in the classroom develop models of multiplication and division (arrays) and use invented and conventional symbols to represent multiplication and division ...

Children should have had ample opportunities to show multiplication and division in arrays prior to this assessment. An excellent activity to teach or to provide practice in this skill is the Type 3 activity, "Finding Factors."

Suggested Scoring Guidelines: One point for a correct sketch showing four rows of six chairs each and one point for a correct number sentence. Most students will answer 4 X 6 = 24 or 6 X 4 =24, 6+6+6+6 = 24 or 4+4+4+4+4+4 = 24 should also be accepted.

TEACHER NOTES: Item 2 (2 points)

Strand 6: Measurement

Objective 6.1 - continue explorations of length, capacity, and weight, and extend familiarity of units to include kilometer, meter, mile, yard, foot, gallon, gram, ounce, and fractional parts of each

Children need opportunities to weigh various objects about the classroom in both in U.S. Customary units (pounds and ounces) and in metric units (kilograms and grams). They should also try to find an object that weighs about a pound (an ounce, a kilogram, and a gram) to use as a reference for approximate weight of each of the units. Relationships among the various units need to be established through comparisons and actual measurement of objects and liquids (or sand).

Suggested Scoring Guidelines: One point for each item. Any reasonable items should be accepted.

TEACHER NOTES: Item 3 (2 points)

Strand 7: Estimation and Mental Computation

Objective 7.5 - add strings of numbers mentally by finding groups of tens.

The assessment problem uses combinations of numbers that form ten. This concept, in instruction, should also include problems that use combinations that form multiples of ten

2 + 17 + 18 + 23 + 5 =
(2 + 18) + (17 + 23) + 5 =
20 + 40 + 5 = 65

as well as some practice in finding three numbers that equal tens.

2 + 6 + 3 + 5 =
(2 + 3 + 5) + 6 =
10 + 6 = 16

The above examples are **not** expected to be written, as the strand indicates that the work is done mentally and an answer given. However, to insure an understanding of what the child is thinking, an oral explanation should be required.

Suggested Scoring Guidelines: One point for a sum of 31 and one point for showing mentioning BOTH 8 + 2 and 7 + 3.

TEACHER NOTES: Item 4 (2 points)

Strand 6: Measurement

Objective 6.1 - continue explorations of length, capacity, and weight, and extend familiarity of units to include kilometer, meter, mile, yard, foot, gallon, gram, ounce, and fractional parts of each

Students need opportunities to weigh the same object with different units of measure and then compare the numbers resulting from these weighings.

Suggested Scoring Guidelines: One point for saying Mickey will get the larger number and one point for any indication that this is because a gram is a smaller unit than a kilogram.

TEACHER NOTES: Item 5 (4 points)

Strand 2: Problem Solving

Objective 2.5 - extend the guess-and-check procedure by recording guesses and checks to help make better guesses until the solution is reached

Students should be afforded recurring opportunities to practice and refine their guess and check problem-solving strategy and demonstrate their understanding of this strategy by explaining how to use the check to make a better guess.

Sample Rubric: A district scoring rubric may be appropriately applied to score this item. If a district scoring rubric does not exist, the following may be used:

4 = Correct answer and explanation using a guess and check strategy are given.

3 = Correct answer with thorough explanation that does not use guess and check strategy or correct answer with an indication that guess and check was used with an inadequate explanation.

2 = Incorrect answer caused by computational error with an attempt at the use of guess and check strategy.

1 = Exactly one of: correction answer, adequate application of guess and check strategy, or explanation.

0 = Other

TEACHER NOTES: Item 6 (4 points)

Strand 6: Measurement

Objective 6.1 - continue explorations of length, capacity, and weight, and extend familiarity of units to include kilometer, meter, mile, yard, foot, gallon, gram, ounce, and fractional parts of each

Students should have ample opportunities throughout the year to measure items in a variety of units — both standard and non-standard. When measuring the same item in different units, the students should compare the numbers that result from the measurement.

Suggested Scoring Guidelines: One point for each unit suggested (standard or non-standard units such as paces or tennis shoes should be accepted) and two points for explanation of why the particular unit was chosen.

TEACHER NOTES: Item 7 (1 point)

Strand 3: Number and Number Relations

Objective 3.4 - translate real-life situations involving multiplication and division into conventional mathematical symbols

Students should see multiplication and division in a variety of examples. Thus, it makes sense to use word problems to practice multiplication and division. This strategy also provides increased practice in reading in mathematics.

TEACHER NOTES: Item 8 (2 points)

Strand 2: Problem Solving
Strand 3: Numbers and Number Relations

Objective 2.1 - select appropriate notation and methods for symbolizing the problem statement and the solution process
Objective 3.3 - from situations created in the classroom. . .describe multiplication and division in words

Students should be able to read word problems, determine what operation will lead to a solution, and translate the story into correct mathematical sentences using conventional symbols. Most computational drill should be in the form of word problems to provide practice in this problem-solving skill, to provide the child context for determining whether the answer makes sense, and to provide the child practice in reading in mathematics.

Suggested Scoring Guidelines: One point for correct answer and one point for the reason. The correct answer is C. It is different because it is not a division situation.

TEACHER NOTES: Item 9 (4 points)

Strand 7: Estimation and Mental Computation

Objective 7.3 - round factors and use multiples of ten to estimate products

Students should be able to round to the nearest ten mentally and then use these rounded numbers to estimate sums, differences, and products. They should compute the answer mentally and then tell what they were thinking in order to get their estimate.

Suggested Scoring Guidelines: One point for A, one point for saying estimate is less, and two points for an adequate explanation.

TEACHER NOTES: Item 10 (3 points)

Strand 8: Data Analysis and Probability

Objective 8.7 - translate freely among pictographs, tables, charts and bar graphs

This objective implies that the student should be able to extract and interpret data from each of the forms listed as well as be able to put that information into a different form. Instructional activities addressing this objective should go beyond this particular assessment item.

Suggested Scoring Guidelines: In scoring this item, there are no points given for the first part because any answer can be acceptable. However, this answer is important for determining the score of the last question.

Part A: No points

Part B: 2 points = A reasonable statement is given that either looks at the trends shown by the scores in the graph or is based on the time of year indicated by the graph. Example: since the weather is getting warmer and the days longer Julie spends more time playing outside and at baseball practice therefore she has less time to study **OR** it's the end of the school year, Julie is worried about failing so she is studying harder.

1 point = Any reasonable explanation not connected to the graph. Example: she is going to try harder **OR** she is busy **OR** she doesn't like this subject.

Part C: One point for a correct comparison of answer given for Part A to the answer for Part C. Example: If the child said in parts A and B that the May score would be 20 because she didn't study and then said the May score would be higher than the score in September, it would be an incorrect comparison worth no points. The Part C answer **must** take into consideration that Part A answer.

TEACHER NOTES: Item 11 (1 point)

Strand 3: Numbers and Number relations

Objective 3.16 - use the symbols $<$, $\leq$, $>$, $\geq$ and $=$ in describing order as well as the terms "at least" and "at most"

This item is at the knowledge/skill level. Students should be familiar with the symbols as well as the terminology. Students can be taught to "read" the symbols in the normal left to right progression. The symbols < is read as "is less than" (the point is smaller than the wide end) and conversely the symbol > is read as "is greater than" (the wide part is bigger than the point). Students may also "discover" that the larger number is at the wider part and the smaller number is at the smaller part (point) through teacher questioning to lead them to this "discovery." In beginning to teach the "at least" and "at most" terminology it may be useful to use numbers and discuss the meaning of the terminology and then expand the idea to non-numerical references (i.e., at least half full, at most half full, etc.)

Suggested Scoring Guidelines: All numbers must be placed correctly on the lines

$$12 < 16 < 27 < 52 < 94$$

TEACHER NOTES: Item 12 (2 points)

Strand 1: Patterns, Relations, and Functions

Objective 1.3 - use patterns to make generalizations and predictions by ... identifying missing elements in a pattern and justifying their inclusion
Objective 1.4 - make a table of values to record the pairing of members of two sets, determine the relationship (rule) between each pair, and use the rule to generate additional pairs

This item addresses a critical objective from the Model Competency-Based Mathematics Program and is usually assessed using a problem-solving or application situation.

This form of assessment will give an indication of the student's understanding of concepts and his/her ability to make connections in mathematics, solve problems, and verify results as well as perform correct computations. Students need practice in determining the rule between pairs of numbers and in setting up tables to do this. Frequent playing of the game "What's My Rule" would help develop these skills prior to assessing the critical objective. See the activity in the Type 3 Third Grade Model Competency Assessments called "A Function Machine" which also develops skills for this objective.

Suggested Scoring Guidelines: One point for the missing number (13) and one point for the rule (subtract two).

TEACHER NOTES: Item 13 (2 points)

Strand 3: Numbers and Number Relations

Objective 3.16 - use the symbols <, ≤, >, ≥ and = in describing order as well as the terms "at least" and "at most"

Suggested Scoring Guidelines: One-half point for correct placement of each number. One is in the "overlap" of the circle and square, one-half and zero are in the square, but not the circle, and two is in the circle but not the square.

TEACHER NOTES: Item 14 (2 points)

Strand 3: Number and Number Relations

Objective 3.8 - recall multiplication and division facts through 12 x 12 using ... anchor facts

Students should have opportunities to share how they remember "difficult" facts. The third grade objective includes the commutative property (turn around facts) and the distributive property (4 sevens is 2 sevens and 2 sevens; since 2 sevens is 14, then 4 sevens is 14 + 14 which equals 28) as well as anchor facts (facts "known" by the child) which is assessed by this item.

Suggested Scoring Guidelines: There are a variety of acceptable answers, but one point should be awarded for starting with the 4 x 9 = 36 fact and adding on to it. Another point is awarded for adding 18 and indicating why 18 was used. Acceptable explanations would include "add two more 9s" or "half of 4 x 9 is 18".

TEACHER NOTES: Item 15 (3 points)

Strand 2: Problem-Solving Strategies
Strand 3: Numbers and Number Relations

Objective 2.1 - select appropriate notation and methods for symbolizing the problem statement and the solution process
Objective 3.3 - from situations created in the classroom ... use ... conventional symbols to represent multiplication and division (extending from Grade two Objective 3.9- translate real-life situations involving addition and/or subtraction into conventional symbols of mathematics)

Students should be able to read word problems, determine what operation will lead to a solution, and translate the story into correct mathematical sentences using conventional symbols. Most computational drill should be in the form of word problems to provide practice in this problem solving skill, to provide the child context for determining whether the answer makes sense, and to provide the child practice in reading in mathematics.

Suggested Scoring Guidelines: One point for each correct number sentence.
Possible answers:

A. $36 \div 12 = 3$
$36 \div 3 = 12$
$12 \bullet 3 = 36$
$3 \bullet 12 = 36$
$36 - 12 - 12 - 12 = 0$
$12 + 12 + 12 = 36$

B. $36 = T + 10c$
$36 - 10 = 26$

C. $36 \div 4 = L$

TEACHER NOTES: Item 16 (2 points)

Strand 3: Numbers and Number Relations

Objective 3-4: translate real-life situations involving multiplication and division into conventional mathematical symbols

Suggested Scoring Guidelines: One point if the problem is truly a division problem and one additional point if the problem uses the numbers correctly.

TEACHER NOTES: Item 17 (4 points)

Strand 6: Measurement

Objective 6.2: count collections of coins and bills which include dollar, five, and ten dollar bills and compare values

Suggested Scoring Guidelines: One point for the correct answer ($32.76), one point for evidence of adding the savings, one point for correctly adding the savings ($27.24), and one point for subtracting the savings from 60 dollars.

TEACHER NOTES: Item 18 (2 points)

Strand 4: Geometry

Objective 4.4: describe a three-dimensional object from different perspectives (related to First Grade 4.3 identify two-dimensional shapes on three-dimensional objects)

The student should demonstrate an understanding of perspective by describing (orally, visually, or in writing) a three-dimensional object from different views, e.g., top, side, front, etc.

Suggested Scoring Guidelines: One point for saying that all sides will be rectangular. If students indicate that some faces are square that is acceptable. One additional point for saying either that there are six faces or discussing other shapes on a face.

TEACHER NOTES: Item 19 (3 points)

Strand 4: Geometry

Objective 4.4 - describe a three-dimensional object from different perspectives (related to First Grade 4.3 identify two-dimensional shapes on three-dimensional objects)

The student should demonstrate an understanding of perspective by describing (orally, visually, or in writing) a three-dimensional object from different views, e.g., top, side, front, etc. The enclosed patterns for the solid shapes should be cut out and assembled prior to the assessment or models of the solid shapes should be provided. The pyramid should be a triangular pyramid.

Suggested Scoring Guidelines: One point for using A and B to trace a circle, one point for using C and D to trace a triangle, and one point for using D to trace a rectangle.

TEACHER NOTES: Item 20 (3 points)

Strand 8: Data Analysis and Probability

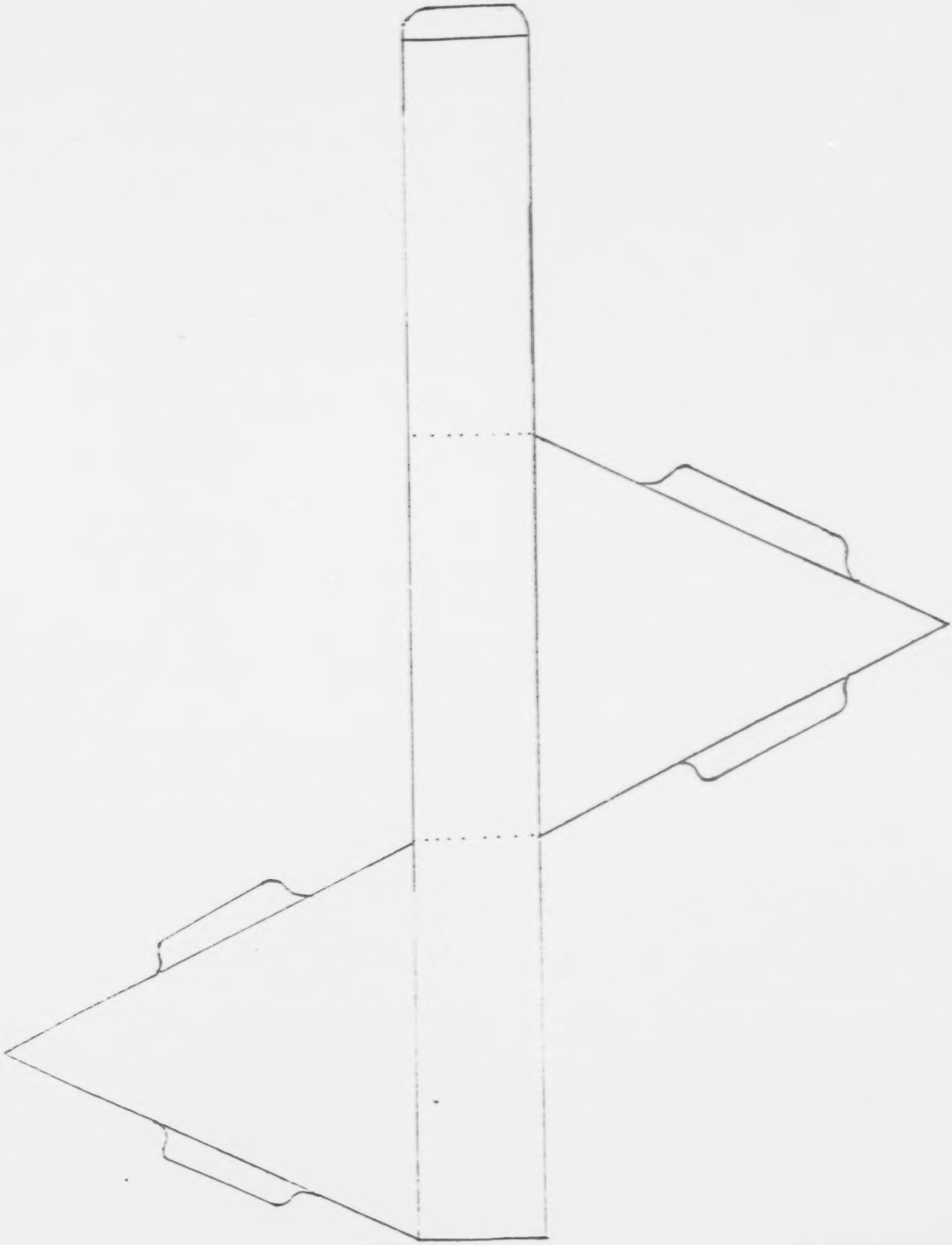
Objective 8.6 - investigate, display, and record all possible arrangements of a given set of objects.

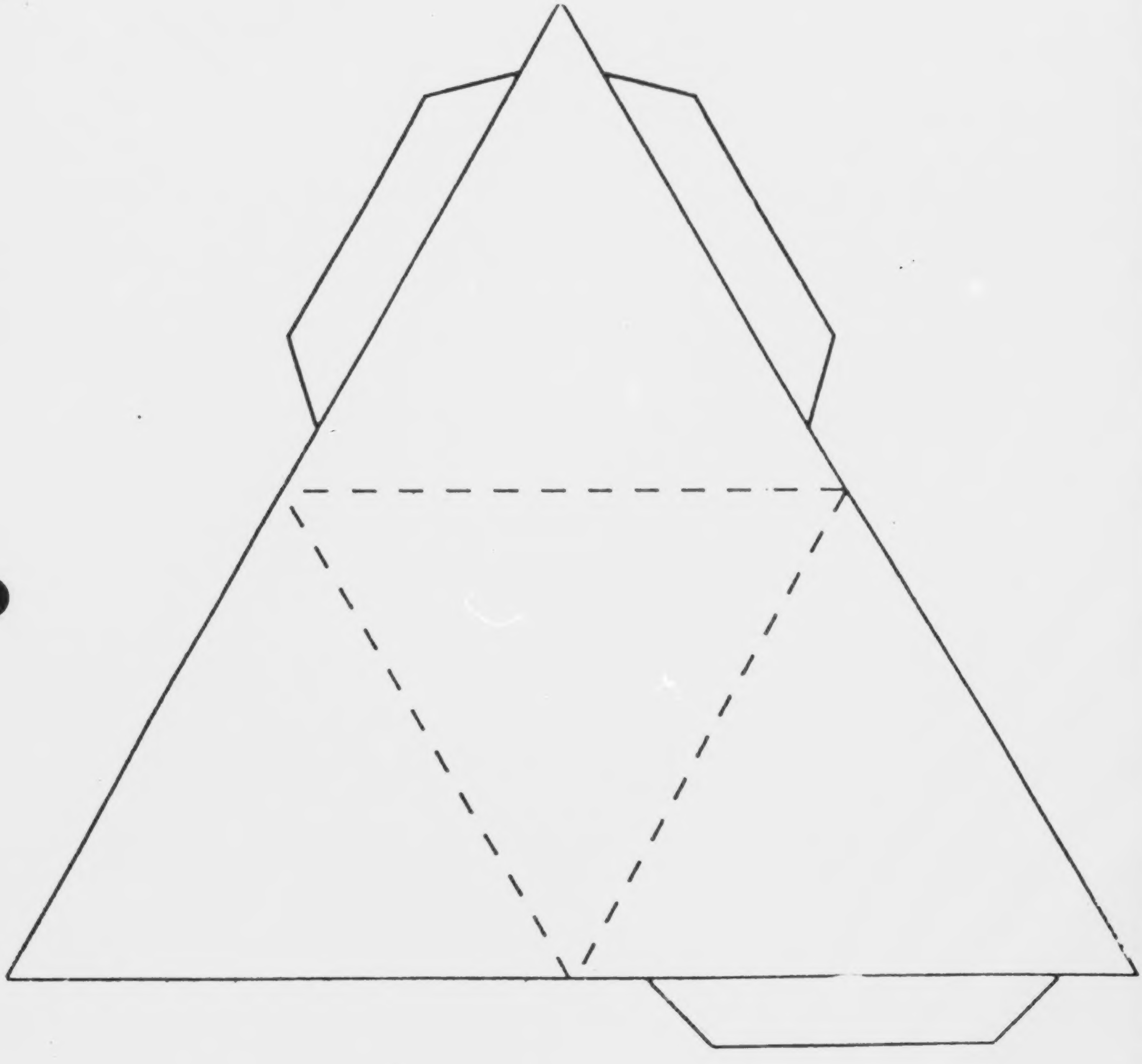
Suggested Scoring Guidelines: One point for correct answer (6), two points for complete explanation that includes a list or drawing. (Award one point for an incomplete — but not incorrect — explanation even if the answer is wrong.)

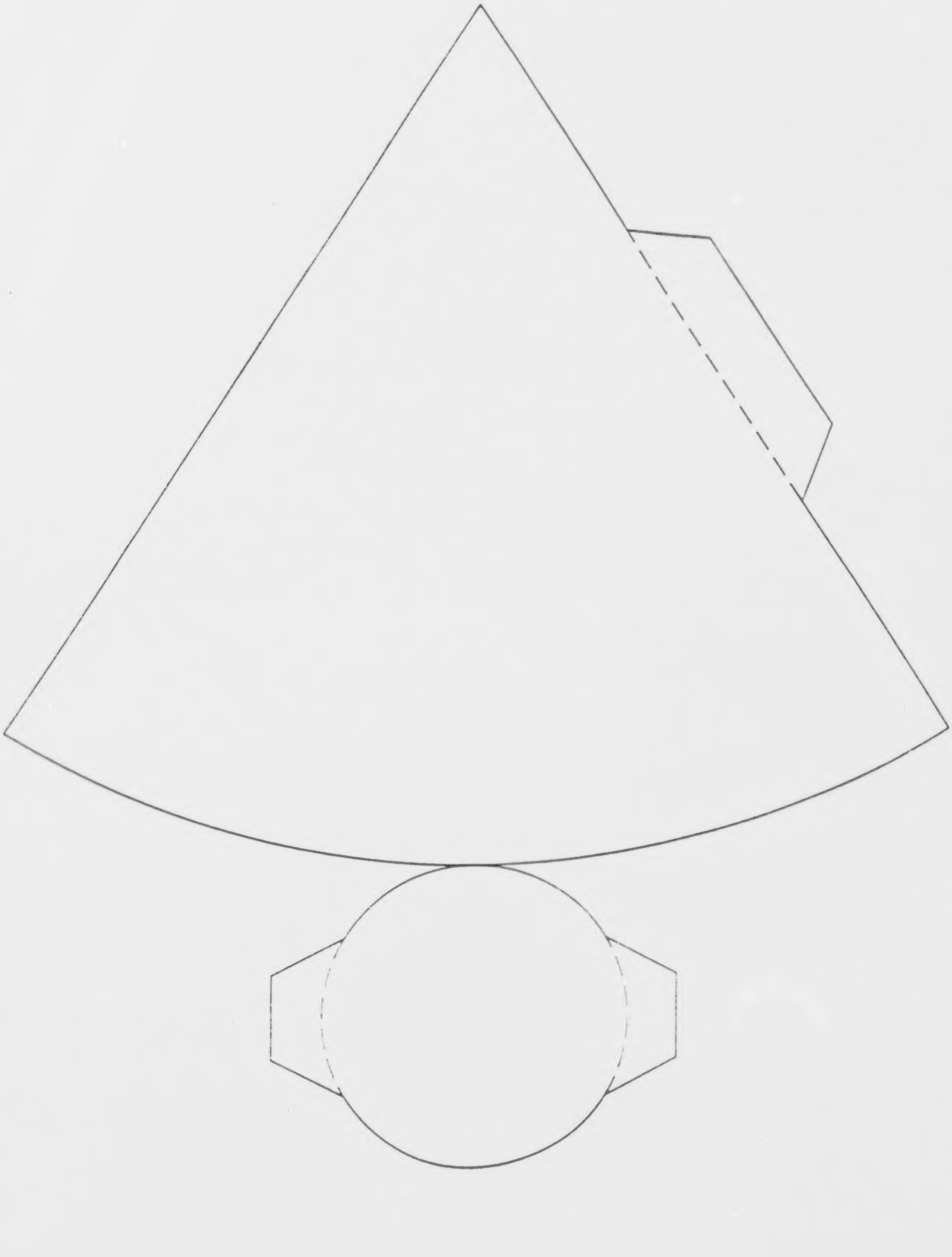
Information Sheet

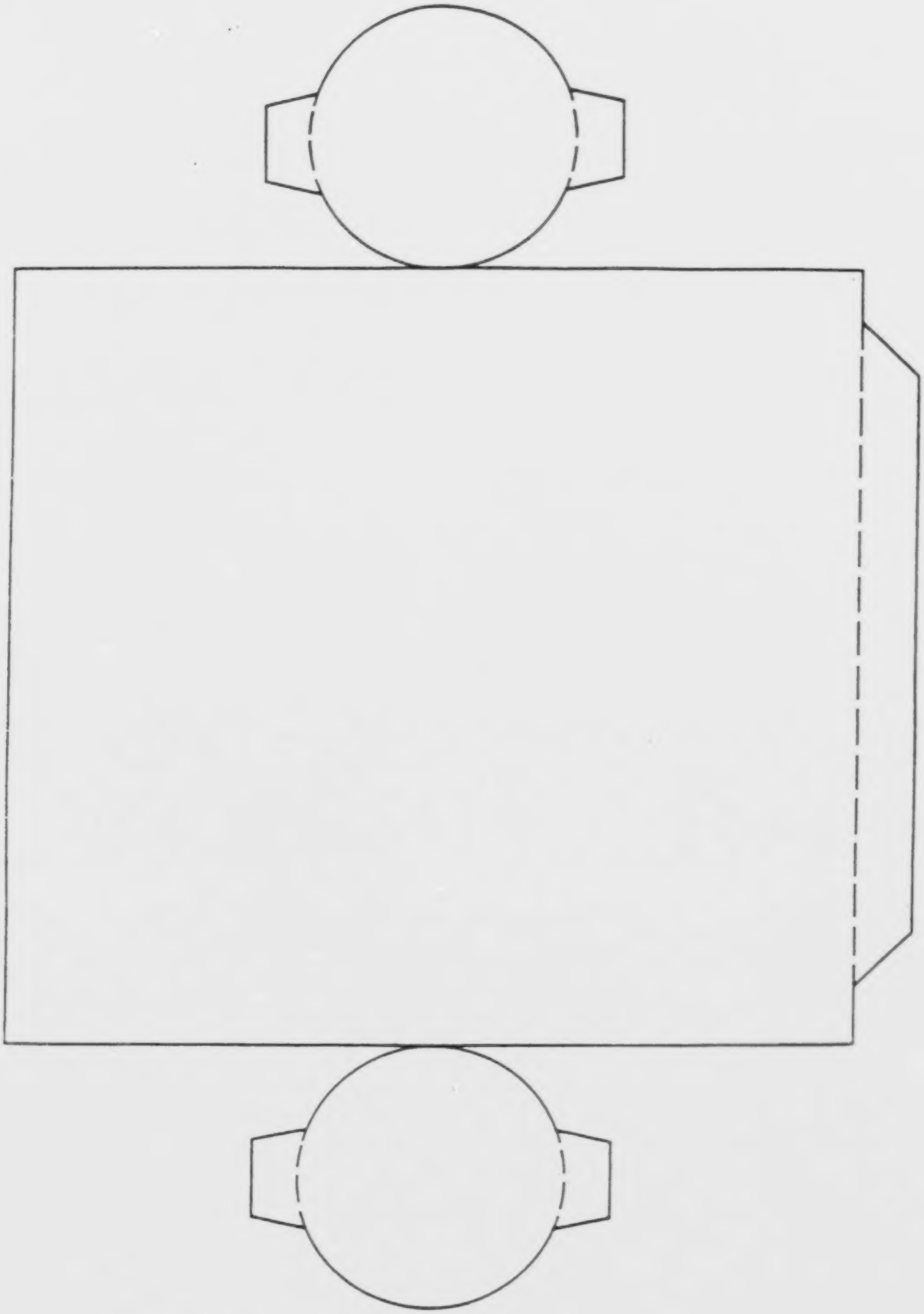
Item	Answer	Objective	Outcome Level*	Critical Objective
1.	diagram of 4 rows, 6 chairs See "Teachers Notes"	3-3-3	PS/A	Yes
2.	answers will vary See "Teachers Notes"	3-6-1	C	No
3.	mention combinations of 2 & 8 and 7 & 3	3-7-5	K/S	Yes
4.	Mickey will get a larger number	3-6-1	C	Yes
5.	See "Teachers Notes"	3-2-5	PS/A	Yes
6.	See "Teachers Notes"	3-6-1	C	Yes
7.	24	3-3-4	C	No
8.	C See "Teachers Notes"	3-2-1 & 3-3-3	C	Yes
9.	A See "Teachers Notes"	4-7-3	PS/A	No
10.	See "Teachers Notes"	3-8-7	PS/A	Yes
11.	12<16<27<52<94.	3-3-16	K/S	Yes
12.	13; subtract 2	3-1-3 & 3-1-4	K/S	Yes
13.	See "Teachers Notes"	3-3-16	C	Yes
14.	See "Teachers Notes"	3-3-8	C	Yes
15.	See "Teachers Notes"	3-2-1 & 3-3-3	C, K/S	No
16.	See "Teachers Notes"	3-3-4	C	Yes
17.	See "Teachers Notes"	3-6-2	PS/A	Yes
18.	See "Teachers Notes"	3-4-4	C	Yes
19.	See "Teachers Notes"	3-4-4	PS/A	Yes
20.	See "Teachers Notes"	3-8-6	C	No

* P S/A = Problem Solving/Application
C = Concept
K/S = Knowledge/Skills





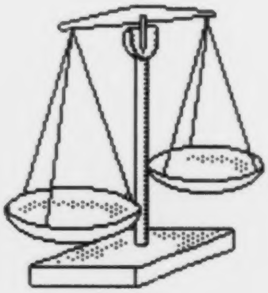




Directions: Answer the questions in the space provided.

1. There are 24 desks in the room. Draw a picture to show how to arrange the desks in four equal rows. Write a number sentence that goes with your picture.

2. Name something that weighs about a pound and something else that weighs about a gram.



______________________ about a pound

______________________ about a gram

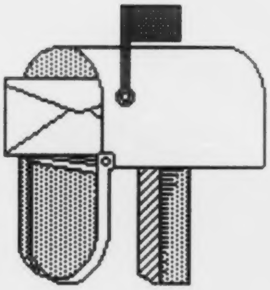
3. In the following exercise show how you could combine numbers to make ten so that adding them will be easier. Then give the sum.

$$7 + 5 + 3 + 6 + 8 + 2$$

4. Mickey weighs a big box using grams and Minnie weighs the same box using kilograms. Write below what you know about the numbers Mickey and Minnie will get.

5. It costs 22¢ to mail a post card and 32¢ for a letter. Billie told me he wrote five friends and spent $1.30 for postage. How many letters and how many postcards did he write?

   Use the guess and check strategy to solve this problem. Describe what you are thinking.

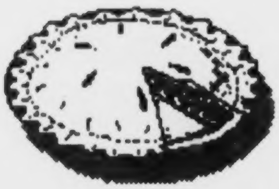


6. Evelyn wants to know the length of the playground fence. Name two units of measure that could be used to measure a fence. Tell which of the two units you would use to measure the fence and why you would use that unit.

The fence could be measured in ________________ or ________________.

I would choose ______________, because:

7.



Jimmy's mother baked three pies. If she cuts each pie into eight pieces, how many pieces of pie will she have? Show your work.

8. Tell which of the following problems is different from the rest and why you think it is different.

   A. Barbara has nine gum balls. She plans to give them equally to her three friends. How many gum balls will each friend get?

   B. Bill has two brothers. If the three made a total of 12 phone calls last week and they each made the same number of calls, how many calls did each brother make?

   C. Belle has four toy cars. How many different ways can she line them up?

   D. Betty bought three cassette tapes for $15.00. If the tapes cost the same, how much did each tape cost?

   Problem ____ is different from the others because:

9. If 124 and 15 are rounded to the nearest 10 and then subtracted, the difference will be:

   A. 100

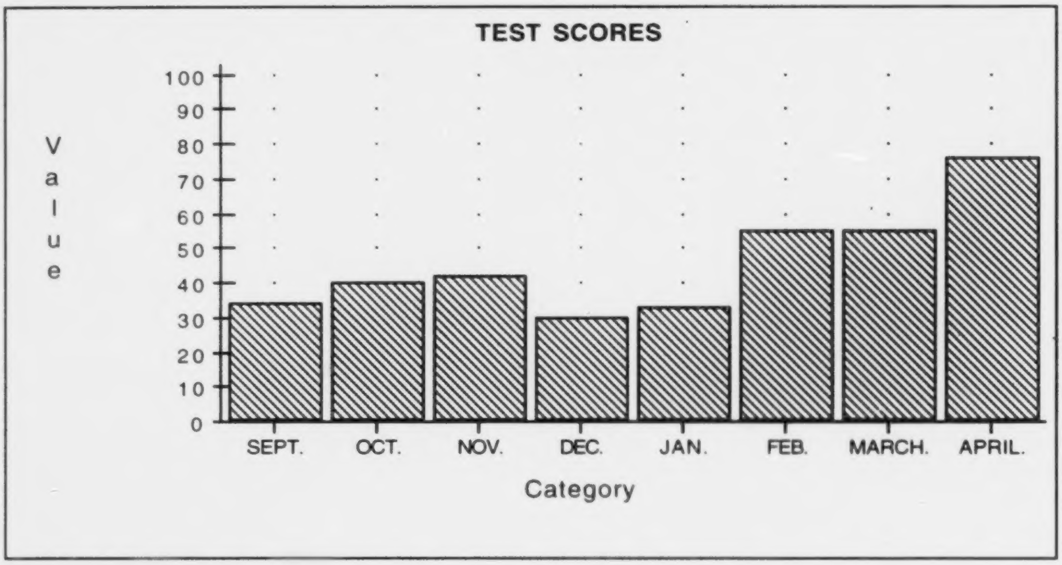
   B. 110

   C. 80

   Will this estimate be greater than, less than, or equal to the exact difference?

   Why?

10. Julie has been working very hard in third grade to improve her grades. She missed a lot of school in second grade because she was ill for several months. The bar graph below shows her scores on her spelling tests in third grade.



What do you think the MAY score will be ?

Why do you think that?

How will her May score compare to her score in September?

11. Place the following numbers in order from least to greatest

27, 52, 16, 94, 12

____ < ____ < ____ < ____ < ____

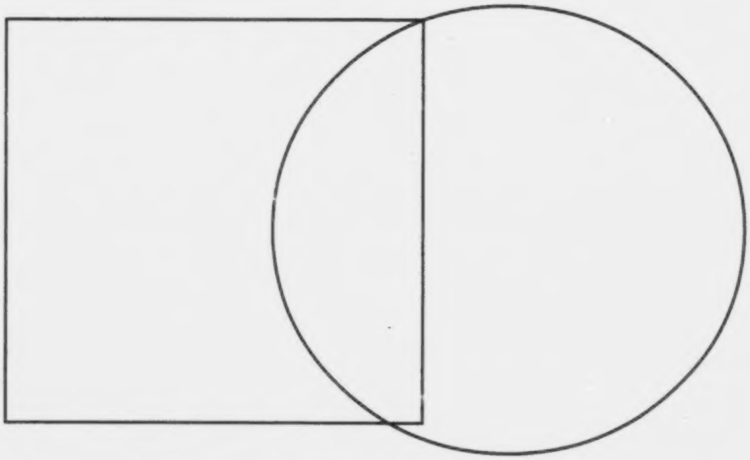
12. Identify the missing element in the table and tell the rule.

FIRST NUMBER	SECOND NUMBER
7	5
10	8
12	10
15	?

The missing number is _________

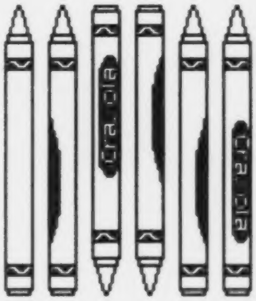
State the rule that connects the first number and second number.

13. In the drawing below, every number inside the circle is at least one, and every number inside the square is at most one. Write each of the numbers { 2 , $\frac{1}{2}$, 0 , 1 } where it should be.



14. If I know that 4 x 9 is 36, how can I find 6 x 9?

15. For each of the following, write a number sentence that you can use to answer the question.



Kathy had 36 crayons. She put 12 crayons in each box. How many boxes did she use?

Kathy had 36 crayons. She had 10 more crayons than Tom. How many crayons did Tom have?

Kathy had 36 crayons. After Lisa gave her four crayons, how many crayons did Kathy have?

16. Write a word problem that would be solved by the following division sentence.

$$36 \div 9 = 4$$

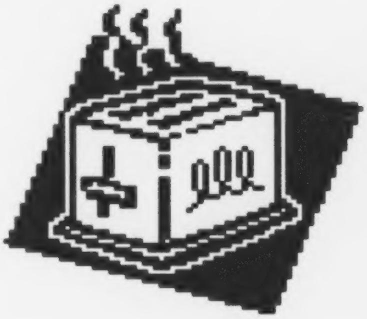
17. The table shows how much money Leslie put into her savings account each month. Leslie wants to buy a bicycle that costs $60.

LESLIE'S SAVINGS

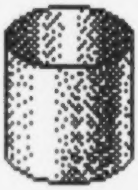
MONTH	**AMOUNT**								
April	$1.00	$1.00	25¢	25¢	25¢	25¢	25¢	5¢	5¢
May	$10.00	$1.00	$1.00	10¢	5¢	1¢	1¢	1¢	
June	$5.00	$5.00	$1.00	25¢	25¢	25¢	5¢	1¢	

How much more money does she need? Use words, pictures or numbers to explain your answer.

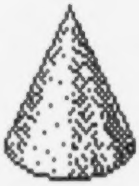
18. You are in the championship match of "Descriptionary". The game rules state that only basic geometric shapes - circle, square, oval, rectangle, parallelogram, triangle, trapezoid and rhombus may be used to describe an object. The item you are to describe is a "toaster". Using these terms describe what a toaster looks like from the top, front and side.



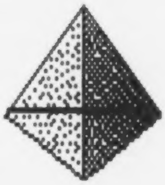
19. Look at the four solid figures below. You want to place the solid on a piece of poster board and trace around the solid to make shapes on a poster.



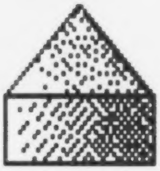
A



B



C



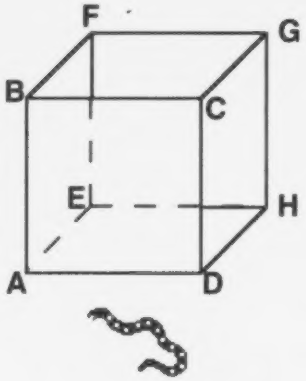
D

Name all of the solids (A, B, C or D) which could be used to trace a circle.

Name all of the solids (A, B, C or D) which could be used to trace a triangle.

Name all of the solids (A, B, C or D) which could be used to trace a rectangle.

20. How many ways can Wilhelm the Woolly Worm crawl from A to G along the wire frame if he takes a three-sided trip?

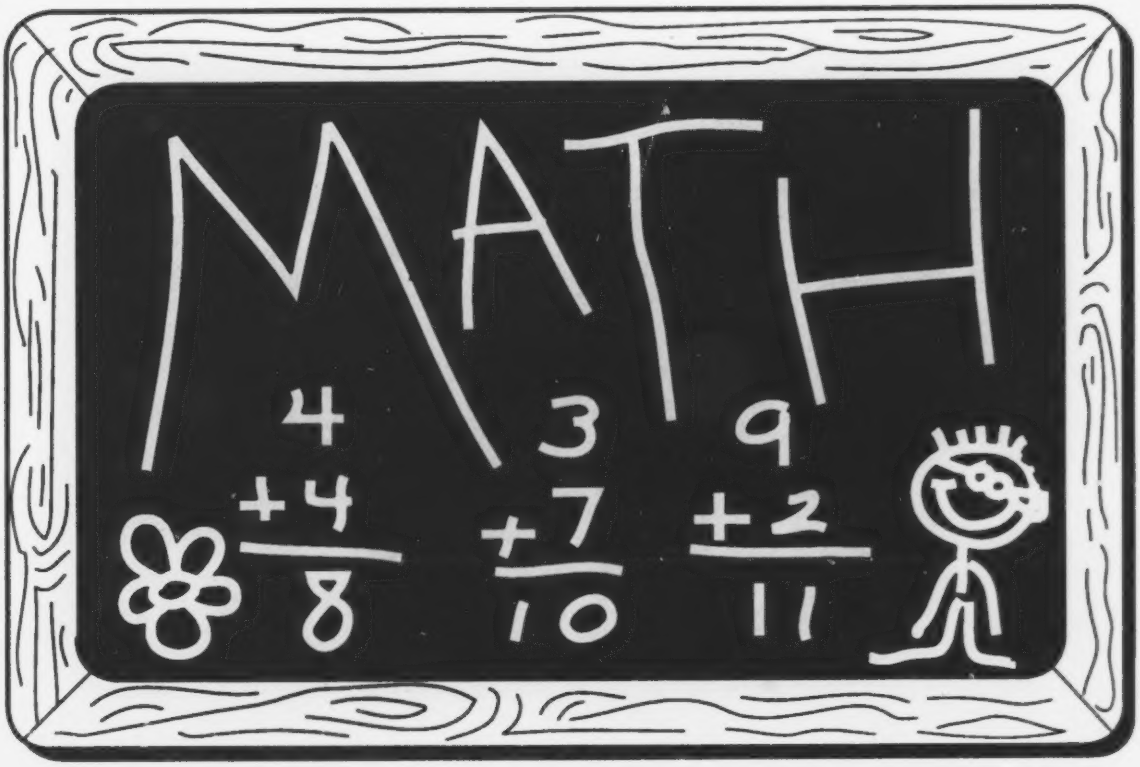


How do you know your answer is correct?

Competency-Based Education Assessment Series

Third Grade Mathematics

Type 3 Assessment



Developed by

Evelyn Altherr
Ethel Briggs
Margaret Comstock
Margaret Kasten
Anne Mikesell

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency assessment is comprised of tasks and activities keyed to the third grade objectives from Ohio's Model Competency-Based Mathematics Program. These assessments are designed to be used by teachers and districts committed to the use of authentic assessment tasks and activities. Use of non-traditional assessment requires extensive professional development for teachers, specialized instruction and preparation for students, and communication with parents and community. Each task or activity will require a substantial period of time to complete. These tasks or activities reflect ways in which mathematics can be applied (Finding Out About Pets, the Playground Problem, and Pencil Holders and Vases) or they relate to the intrinsic structure of mathematics (Finding Factors, Place Value and the Calculator, and A Function Machine). Multiple skills and understandings by students as well as a variety of tools and supplies may be required to complete each task. Rather than assessing single objectives the tasks may address several objectives from a strand or across strands.

The tasks and activities offer a good way for students to demonstrate progress in and understanding of mathematics. The activities suggested here are not intended to be a comprehensive, or even completely representative, set of tasks. Rather they should provide some guidance for those districts and teachers seeking to fully integrate assessment activities into the instructional program. Many times the best assessment activities arise from the daily curriculum and instruction program.

There are a variety of ways these tasks could be used to determine whether a student is prepared to learn successfully at the next grade level. It is not within the scope of this document to completely outline implementation options.

Activity 1

Finding Out About Pets

This activity specifically addresses the objectives

8-2 **create, read, and interpret tables and charts**
8-5 **collect and record data on the frequency of events**, and
8-7 **translate freely among pictographs, tables, charts, and bar graphs.**

Phase Two addresses the latter part of

6-2 **count collections of coins and bills which include one, five, and ten dollar bills and compare values.**

This activity should be preceded by other similar activities that may need to be conducted with a greater level of teacher assistance. For most students, Finding Out About Pets should be considered the culminating activity in a series. Obviously, those students who cannot complete this activity satisfactorily will need more experiences to achieve competency on these objectives. Suggested **preliminary** activities might include the following.

- Siblings - collect information from children about brothers and sisters, being careful to allow the children to define who their siblings are. Have the children, with help, determine how the information will be presented in table form. The beginning of the idea of average can be developed by having the children determine how many brothers and sisters each child would have if they shared all equally. Questions such as "What is the greatest number of sisters (or brothers, or siblings) any child has?" or "How many students have more than one sibling?", etc. can be asked and answered. Graphs should be developed by the whole class and used to find the answers to these questions.

- Crayons - have children count the number of crayons each has at school. Together develop a way to show the information in a table. Decide whether it is of interest to find out how many crayons are primary colors and how many are blends and put that information in the table. Develop a bar graph to show the information from the table. Again preview the idea of average by having the children look at "sharing" ideas.

- From Graph to Table - show the class a bar graph or a pictograph without a title. Have the children determine an appropriate title and develop a table of values from the graph. [You can get graphs for this from textbooks in other subjects and/or grades. You might also want to have the children write a story that fits the graph to determine their understanding of the data and to make a connection with language arts.]

Phase One

Procedure:

- Each child records the number and kind of family pets he/she has.
- The class, as a group, determines how to show the information in a table so that it is possible to answer questions such as "how many pets for each child?" and "how many pets of each kind?". [More than one table may be necessary.] This could get rather messy, if there are many kinds of pets. This would make the next step, determining categories, even more important. A sample table might be organized like this:

FAMILY PETS

NAME	DOG	CAT	RABBIT	HAMSTER	GUINEA PIG	WHITE RAT	FISH	TOTAL
Al	1	0	0	0	0	0	2	3
Becky	0	3	0	1	0	0	0	4
Craig	2	0	1	0	0	0	0	3
Dolores	0	2	4	0	3	0	0	9
Eddie	1	1	0	0	0	2	5	9

- The class determines, after looking at the information, what **categories** of pets are represented. This is a continuation of sorting activities introduced in first and second grades. Data is then displayed in table form to show the number of pets in each category. Categories might be *rodents, fish, birds, cats, dogs, or animals with four legs, pets that swim, etc.*
- Each child selects or is assigned one of the tables and develops a bar graph (or pictograph, if appropriate and if pictures are available). This may be done in class, or as a homework assignment. Since there will be multiple graphs for each table of values, each child will explain her graph to the class and the class will compare graphs.

Assessment:

Two modes of assessment are necessary, evaluation of an **artifact** and **teacher observation** of student behavior.

First, the **graph** -

> Does the graph have an appropriate title? Does it show the scale? Do the labels give enough information to the reader? Does it represent the data of the table with reasonable accuracy?

Second, the **presentation** of the graph -

> Does the presentation indicate that the student understands what has been done? (This is particularly important if the graph was done at home.) Can the student account for possible differences between her graph and another?

If the answers to these questions are "yes", then the student can be considered to have met the criterion.

Phase Two

If this extension is to be used as an assessment activity as well as a learning activity, there should be several preliminary activities of a similar nature that involve money. Examples are

- a restaurant price list and menu, a limited amount of money, and the task of ordering a meal for an individual or a family
- a catalog with toy prices and a list of people for whom gifts are to be bought as well as a limited amount of money

Procedure:

- Groups of two or three children work together to determine the cost of caring for a pet of their choice. The costs could be broken down into *Start-up Costs* (like collar, leash, water-bottle, cage, litter box, bed, etc.), *Periodic Costs* (like food, treats, bedding, litter, vitamins, flea collar, dog license, shots from the vet, etc.) and *Other* (like emergency visits to the vet, special grooming, etc.).

- Each team creates a report in a format suitable for display and posts it so that all children may see it. Class discussion about the accuracy and completeness of each report then takes place. Where a pet is chosen by more than one group comparisons are made. Reports are modified if necessary.
- Each student writes a paper on which pet she would choose to actually own and why she would choose it. The paper should include the cost of the pet and any advantages and disadvantages of having that particular pet.

Assessment:

Two modes of assessment are necessary, evaluation of an **artifact** and **teacher observation** of student behavior.

First the **teacher observation -**

> Does the child actively contribute to the group project? Does he/she use the resources of the written reports as well as personal experience?

Second the **evaluation of the paper** -

> Does the paper follow the general language arts rubrics for any written paper at the third grade level? Are the costs for a pet realistic and related to at least one of the reports? Are both advantages and disadvantages for having that pet listed?

If the answers to these questions are "yes", then the student has reached criterion. For students who do not reach criterion there should be more activities of this kind.

Activity 2

Playground Problem

This activity addresses the measurement objectives

6-1 **continue explorations of length, capacity, and weight, and extend familiarity of units to include kilometer, meter, mile, yard, foot, gallon, gram, ounce, and fractional parts of each**

6-5 **illustrate the approximate size of units (inch, centimeter, meter, and yard)**

as well as the geometry objective

4-5 **describe a three-dimensional object from different perspectives**

It should be preceded by a selection of similar activities that may require more assistance from the teacher such as

- measuring and recording parts of the body, such as arms, legs, neck, length of head, etc. and drawing to scale on grid paper

- measuring and recording dimensions of the classroom, including doors and windows, and drawing to scale on grid paper

Procedure:

Equipment needed - measuring tapes or measuring wheels in either US Standard or metric, with grid paper in centimeters or half inches. Non-standard units such as a tire that can be calibrated, string, adding machine tape. etc. are also valuable.

After a discussion of the project in the classroom students will proceed to the playground. Working in pairs, each piece of equipment and playing area (i.e. four squares, ball diamonds, hop scotch, etc.) will be measured - including the placement of posts and safety areas. Length and width as well as height when that can be safely done will be measured, even though height will not appear on the playground diagram. Depending on the number of pieces of equipment each team will measure each piece. If there is a great deal of equipment, the teacher may have to make assignments but each piece should be measured by several teams. Measurements should be entered on grid paper, one square per foot or meter (or other non-standard unit). This **measuring phase** may take more than one period. [The teacher should know the overall dimensions of the playground so that the grid paper provided will be large enough for the whole playground. If the playground is unusually large a part of it might be selected for this project.]

After the measuring phase is complete, teams will report results of their measurements and a large drawing of the playground and equipment will be cooperatively developed. Differences in measurements will be discussed and speculation as to why there are differences should be encouraged. Consensus on ballpark figures should be reached. This **reporting and comparing phase** should take one period.

Each student will make his own grid of the playground and equipment using the large classroom grid as a model. On this grid each student will add a piece of equipment using measurements provided by the teacher and will provide an explanation for the addition. [Principals usually have catalogs of playground equipment that include measurements. For example a map of the US takes about 45' x 35', hopscotch about 54" x 120". Slides and swing sets come in various sizes. Some children may wish to rearrange the present playground. In any case children should make a template of the equipment they wish to add or change so that they can try different positions without marking up their grid. This **planning for change phase** may take several days.

Assessment:

There is a single assessment of the playground diagram and accompanying explanation.

> Is the diagram reasonably accurate? Does the additional piece of equipment fit? Does the explanation of the addition to the playground follow the general language arts rubric for third grade? Is the reason given for the addition clear?

If the answers to these questions is "yes" then the student has reached criterion.

Activity 3

Finding Factors

This activity is designed to enable students to explore the representation of factors geometrically. It also gives a preliminary basis for the concept of area. Square numbers such as 9 and 25 are related to the geometric shape of a square. Objectives addressed are

3-3 **from situations created in the classroom**
- **a. develop models of multiplication and division (arrays)**
- **b. use invented and conventional symbols to represent multiplication and division**
- **c. describe multiplication and division in words,**

3-15 **relate even numbers to division by two,**

4-6 **use mathematically correct names for common geometric figures,** and

5-3 **understand the use of letters in statements such as ab=12 or 3c=d and find a when b is given, etc.**

It should be preceded by other activities such as

- illustrating such products as 2 x 3, 4 x 5 with counters placed in array form and skip counting to determine the total number of counters used for each example

- using counters and separating into groups to determine the missing number in number sentences such as $3 \bullet a = 15$ or $b \bullet 4 = 24$. [Use the dot to indicate multiplication rather than the x when you are using other letters. The form $3a = 15$ and $4b = 24$ should also be used so that the child can become familiar with this algebraic way of writing.] That is, for the sentence $3 \times a = 15$, the child takes 15 counters and partitions them into 3 equal sets. Since there are 5 counters in each set, the missing number is 5. Alternatively, the child may make as many three element sets as possible using the 15 counters. Five sets should be possible.

Procedure:

Equipment needed - square tiles or other square markers and grid paper for recording. It is important that the markers be square.

After a reminder of other activities with counters, the teacher leads the class in finding all possible ways of making rectangles with 12 squares. One way to ensure that all children can see is to place squares on an overhead. Be sure that the children include 1x12 as a rectangle! Record all possible arrangements on an overhead grid or on the chalkboard or other grid visible to all. Let the class decide whether a rectangle representing 3 x 4 will be considered different from a rectangle representing 4 x 3. When the teacher is sure that all students understand the process have each child work individually in the **investigation phase.** Each child should investigate the numbers from 1 through 25 and record the results. This may take more than one math period, but could also extend to a homework assignment.

After the investigation phase is complete, the teacher will ask students to report on the results for each number. As the children report, the teacher will make sure that the children understand the terms **prime, square,** and **even** numbers. Do not use formal definitions - just simple explanations such as, "if we can only make one long skinny rectangle for a number we call it a **prime** number" and "if one of the rectangles for a number has all sides the same we call it a **square** number" and "if one of the rectangles for a number has a side of 2 we call it an **even** number." This **reporting phase** should take one period.

In the **writing phase**, each child will choose or be assigned the task of finding all the **ways to make rectangles from** numbers from 26 to 36, identifying any prime, square, or even numbers and writhing a report on what was done. [or all the **square** numbers from 1 to 50 and writing a report that includes both a list of such numbers, an explanation of why each of the numbers is included in the list, and a description of how the child discovered the numbers.] This writing phase may take more than one math period.

Assessment:

There is a single assessment of the diagrams or list and the accompanying explanation.

> Are the diagrams reasonably complete and accurate? (Is the list reasonably complete and accurate?) Does the explanation follow the general language arts rubric for third grade? Does the explanation show an understanding of the processes and terms used?

The answers to all questions should be "yes". If not, the child needs more experiences like this.

Activity 4

Place Value and the Calculator

This activity addresses the concept of multiples of ten and their relation to place value. It is a variation of the calculator game "Wipe Out". Objectives addressed are

2-10 **develop a convincing written argument for the correctness of a solution**,
3-1 **add and subtract numbers fluently using any strategy**, and
7-6 **subtract mentally using multiples of ten.**

Familiarity with the use of a calculator is assumed as well as knowledge of the names and positions of units, tens, hundreds, and thousands.

Procedure:

Materials needed - a calculator for each student and paper on which to record calculator keystrokes.

The class, as a group under the leadership of the teacher, investigates the following situations.

- The number 12285 is mistakenly entered into a calculator. The intended number was 12085. What one number can be subtracted from 12285 so that the calculator correctly reads 12085?
- Somehow the number 21367 is in the display of the calculator. It should be 21337. What one number can be subtracted from 21367 so that 21337 will appear?
- The number 32667 is in the display of the calculator. The number should be 33667. What can you add to get the correct number?
- Your calculator has something wrong with it. For some strange reason you always have to add before you can subtract, and you can't just add zero. How can you change the display from 54367 to 54167?

The teacher should make up additional examples of each case until the class seems to understand them. You may want to begin with three digit numbers and progress to additional digits. This should take one math class period.

The task for each student is to make up three examples: one that requires only addition, one only subtraction, and another that requires a combination of both for a classmate to do and should provide, on a separate sheet of paper, clear instructions of how to solve the problems. The instructions should include correct place value names as well as a clear explanation of why the solutions are correct. This should take one class period.

Assessment:

> Does the paper follow the general language arts rubric for any written paper at the third grade level? Do the examples meet the given requirements? Are correct place value names used? Are the given solutions correct and is the explanation clear?

If the answers to these questions are "yes", then the student has reached criterion for this activity. Students having difficulty with this should have additional activities of this kind.

Activity 5

Pencil Holders and Jewelry Boxes

This activity specifically addresses the objectives

4-1 **explore properties of geometric figures and relationships by measuring, coloring, folding, cutting, making models, and using tiles and geoboards,**
4-5 **investigate area by covering regions with standard and non-standard units,** and
4-6 **use mathematically correct names for common geometric figures.**

A suggested preliminary activity might be the making and taking apart of cardboard models of three dimensional figures.

Students will make a pattern on grid paper to cover a can, a small box, or other three dimensional container and then will transfer the pattern to any of the covering materials provided to transform the small containers to pencil holders, vases, and/or jewelry boxes. The teacher can use this activity as a means to begin to develop the idea of surface area by having the children count the grid squares on their patterns.

Procedure:

Materials - soup or other cans, small boxes, other containers, wallpaper samples, wrapping paper, material scraps, glue, grid paper

Students will use the grid paper to draw a pattern for the box and/or can to be covered. Once the pattern is made, the student will draw in tabs for extra material to cover the edges of the container. Students will then transfer the pattern to the covering material they select, cut out the material, and glue it to the chosen container to transform it into the desired useful and decorative object.

Students will also make a list of the geometric shapes they see on the patterns they made.

Assessment:

The teacher will determine if the finished product and pattern are reasonably correct and neat and if the list of geometric shapes is complete and correct. If they are the student has reached criterion for this activity.

Activity 6

A Function Machine

This activity addresses the objective

1-4 **make a table of values to record the pairing of members of two sets, determine the relationship (rule) between each pair, and use the rule to generate additional pairs.**

This activity can be **introduced** in a game format or a simple small group activity. In one version ,

- one child builds a simple function machine using pieces from the attached blackline master, and
- another child enters a number [input], records the changes as the number passes through the machine, and determines what number the function machine will produce [output].

For example, the first child selects the pieces x 3 and +2 , in that order. The second child says and writes "5, 15, 17 " and then justifies the result of 17.

In another version,

- the first child gives an input number and the output number, and
- the second child selects pieces for the machine that give the correct result, and justifies her selection.

For example, the first child selects the initial number 5 and the final number 15. The second child says and writes "times 2 plus 5" and then justifies her answer by saying "5 times 2 is 10 and 5 more makes 15." Note that there might be more than one correct answer .

[At this grade level it is probably not appropriate to insist on the correct order of operations. If addition precedes multiplication just apply the operations in sequence *unless you are using the Texas Instrument Mathmate calculator or any other calculator which uses the algebraic hierarchy*. That is, consider that + 3 followed by x 2 applied to the number 5 would give 5, 8, 16 rather than the mathematically correct order of operations which would give an output of 11. Try this on different calculators.]

Procedure:

Each child will build a function machine and will construct a table of values with at least 6 input numbers and the associated outputs. The last input in the table will be the letter **d**. The output for **d** will be an algebraic expression. For each input number the student will also give the calculator keystrokes that represent the action of the machine. In addition to the table of values the child will write a justification for each output.

Assessment:

A correct table of values plus a justification that follows the general language arts rubric for third grade and is reasonably complete indicates that the student has reached criterion.

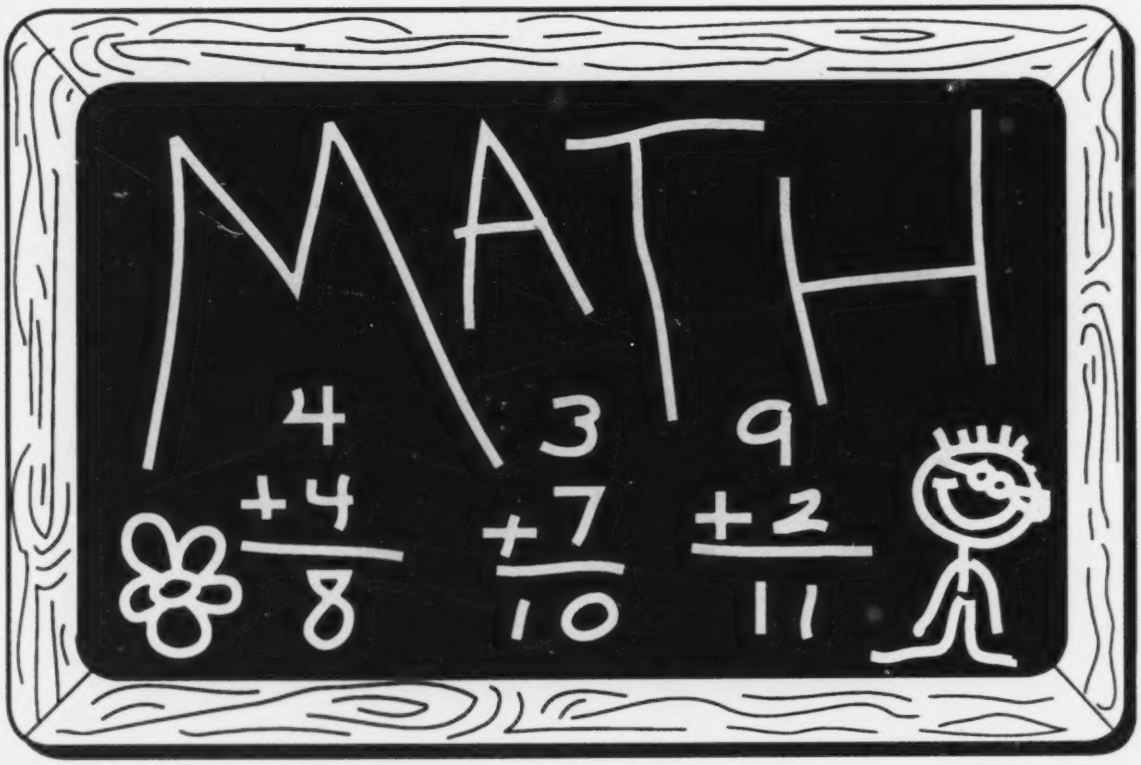
FUNCTION MACHINE PARTS

+ 1	X 1	- 1	÷ 1
+ 2	X 2	- 2	÷ 2
+ 3	X 3	- 3	÷ 3
+ 4	X 4	- 4	÷ 4
+ 5	X 5	- 5	÷ 5
+ 6	X 6	- 6	÷ 6
+ 7	X 7	- 7	÷ 7
+ 8	X 8	- 8	÷ 8
+ 9	X 9	- 9	÷ 9
+ 10	X 10	- 10	÷ 10

Competency-Based Education Assessment Series

Fourth Grade Mathematics

Type 2 Assessment



Reviewed by

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Vickie Hesler
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Kelly Polling
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Terry Rounds

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency assessment is intended to go beyond the traditional multiple choice assessment. Unlike other grade levels, nearly all of the items and tasks are adapted from other sources. The fourth grade, perhaps more than any other grade level, has been the focus of a variety of assessments. There exists a myriad of released items and tasks at the fourth grade level. The National Assessment of Educational Progress (1990, 1992, and 1994), Measuring Up: Prototypes for Mathematics Assessment (1993) developed by the Mathematical Sciences Education Board of the National Research Council, Great Tasks and More!! (1996) compiled by the National Council of Supervisors of Mathematics, and various state assessments (i.e., California, Kentucky, and Oregon) have focused on the fourth grade. These items provided the basis for the items used in this assessment. When statistical results were available and appropriate that information is included in the Teacher Notes below.

The length of time a student will need to complete the instrument may extend beyond the typical 40 minute class period. It is not intended to be a timed test and district or teacher discretion should be used when determining the schedule for assessment. The instrument is a mixture of open form items and items requiring extended responses. Some items are direct, straightforward measures of particular objectives. Some items assess combinations of more than one objective.

Scoring of this instrument will require the use of rubrics or other holistic scoring procedures. The entire instrument has a suggested point value of 58 points. Please note that districts are encouraged to consider adapting any or all parts of the test, including scoring suggestions.

Because these are not original items, there are no "developers" listed. Rather a group of Ohio educators reviewed items for clarity and appropriateness. The names of those individuals are listed on the cover. The minor editorial changes nevertheless render a direct comparison of student achievement, using the statistics provided, inappropriate.

The Teacher Notes that follow include discussion about each item. The items have been chosen to measure progress on the fourth grade objectives and to measure achievement on learning outcomes from the Fourth Grade Proficiency Test. Information related to proficiency test objective most closely related to the item is also included.

Teacher Notes: Item 1 (3 points)

> This item has been substantially changed from the National Assessment item. It is intended that students should see that the first step corresponds to a wall that is one wide and two high (1 X 2 = 2 blocks), the second step corresponds to a wall that is two wide and three high (2 X 3 = 6 blocks), and so on until the seventh step would be seven wide and 8 high (7 X 8 = 56 blocks). The "56"

could be awarded one point, and an additional two points could be awarded for an explanation that includes seeing the relationship between the "step" and the width and that the height is one more than the width.

Teacher Notes: Item 2 (1 point)

This item was also drawn from the NAEP released item bank. Teachers may or may not find a difference in students ability to solve the exercise if it is "set up" that is, $9\overline{)108}$. There may be a difference in results if the question were asked as 108 + 9. If students perform differently, depending on the form of the question, perhaps the concept is not understood or the notation is not understood. The NAEP results indicated that 89% of the students could preform this computation correctly.

Teacher Notes: Item 3 (1 point)

This is a standard NAEP item that asks students to "change from words to numbers." It is a decimal item, but includes only "tenths." The "wrong" answer chosen by a student may give a clue about what the student is thinking. For example if the student chose "C," it may indicate that the words "four," "five," and "three" were focused on outside the context of the rest of the statement. Sixty-nine percent of the four-graders answered this correctly on NAEP.

Teacher Notes: Item 4 (1 point)

Item 4 requires children to read a word problem and perform a relatively simple computation. Sixty-seven percent of the fourth graders taking the NAEP were able to do this successfully.

Teacher Notes: Item 5 (1 point)

This item is virtually unchanged from the NAEP item. In the 1992 report, 91% of the fourth graders answered this correctly. Any student difficulty with this item merits further discussion with the student about patterns.

Teacher Notes: Item 6 (2 points)

This item was also included in 1992 released items from NAEP. At that time 45% of the fourth graders responded correctly by identifying both items. One point can be awarded for each answer. This item assesses estimation, but is not a direct application of front-end estimation.

Teacher Notes: Item 7 (2 points)

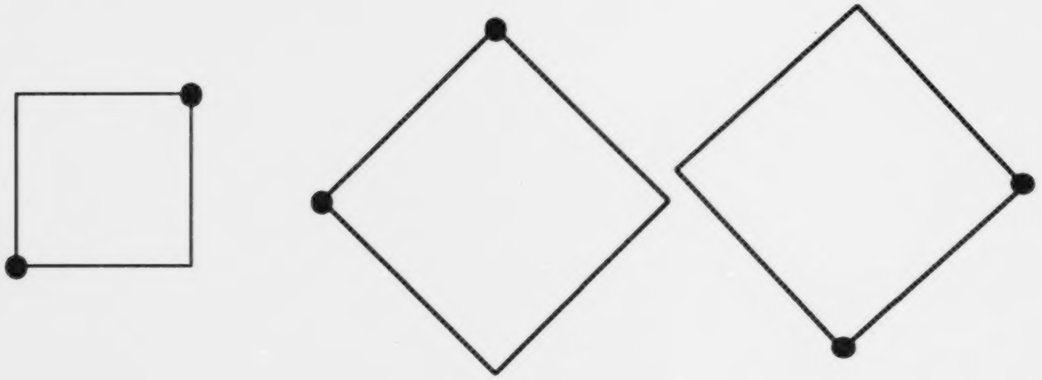
National results from the NAEP 1992 released items indicated 52% of the fourth graders were able to find the total number of points earned from the mathathon. The students did slightly worse on part 2 of the question--49% gave the correct answer. It is suggested that the students be awarded one point for each answer.

Teacher Notes: Item 8 (1 point)

The scale reading problem was a NAEP item released in 1992. At that time, 44% of the fourth graders in the study answered the question correctly.

Teacher Notes: Item 9 (1 point)

This item was drawn from the same set of NAEP released item. There are several ways to correctly complete the square. Only 40% of the fourth graders correctly answered this question. Most fourth grade students have spent a great deal of time identifying shapes, but they may not spent time creating shapes or analyzing characteristics of shapes.



Teacher Notes: Item 10 (2 points)

This item requires students to perform two measurements. The National Assessment data indicated that 52% of the fourth graders answered part "a" correctly and 60% answered part "b" correctly. This may be surprising because part "b" requires the measurement of a "distance" as opposed to a length. It also indicates that the term "diagonal" did not appear to be a problem for fourth graders.

Teacher Notes: Item 11 (2 points)

By late in the fourth grade most students have spent a lot of time working on multiplication. However they may not have much experience with exercises of this type. National Assessment results indicated that 58% of the fourth graders answered this item correctly.

Teacher Notes: Item 12 (1 point)

This item addresses the idea of variable in a most basic fashion. Forty-eight percent of the students answered this item correctly on the National Assessment.

Teacher Notes: Item 13 (2 points)

This was one of the items that was adapted by Ohio educators. In the NAEP version, there was only one question: "How long will it take her to earn $45.00?" and only 22% of the fourth graders answered correctly. The insertion of an intermediate question, "How much does Jill earn in one week?" was an attempt to help students identify a prerequisite step before getting the final answer.

Teacher Notes: Item 14 (1 point)

Only 22% of the students taking the NAEP assessment were able to answer this correctly. This would require the students to do some three-dimensional thinking. Prior hands-on experiences with solids, especially cubes, would be helpful for students.

Teacher Notes: Item 15 (1 point)

Only 25% of the students taking the NAEP assessment answered this correctly. Since the item clearly touches upon both conceptual fraction understanding and probability, it is difficult to pinpoint the cause of such low performance. One might ask a similar question: "In a bag of marbles, 6 are red, 3 are blue, 2 are green, and 1 is yellow. If a marble is taken from the bag without looking it, is it most likely to be red, blue, yellow or green?" If students respond correctly in this case that it is most likely to be red then any difficulty with item 15 may be with fractional concepts instead of probability.

Teacher Notes: Item 16 (1 point)

This item was changed slightly from the one used on national assessment. It now includes a unit designation. Twenty-five percent of the students answered the item correctly on the National Assessment.

Teacher Notes: Item 17 (1 point)

This item is designed to assess student understanding of place value. Thirty-six percent of 4th grade students answered this National Assessment item correctly.

Teacher Notes: Item 18 (1point)

This item is a very direct assessment of learning outcome 2 on the 4th grade proficiency test. Only 42% of the students taking the National Assessment examination answered the item correctly.

Teacher Notes: Item 19 (2 points)

This item provides an opportunity for students to demonstrate an understanding of the concept of area without having to know any formula. Any rectangle with an area of 12 is a correct answer. The dimensions could be 1 X 12, 2 X 6, or 3 X4. Forty-two percent of the students answered this correctly on the NAEP test.

Teacher Notes: Item 20 (5 points)

This item was adapted from an assessment divised by the California Department of Education. There are no student performance data available. Teachers should carefully consider a scoring rubric before using the item. One point might be awarded for each correct sketch, and 2 points for a correct explanation.

Teacher Notes: Item 21 (3 points)

This is a difficult item. It requires students to understand "perimeter" as well as have a good conceptual understanding of "square" that can be applied in a problem solving situation. One point might be awarded for words or picture that reflect an understanding of perimeter as distance all the way around. Another point could be given for an understanding that the original rectangle is two squares. And finally one point given for the correct answer.

Teacher Notes: Item 22 (1 point)

This item was adapted from an extended response NAEP item. The decision to change the item to a multiple choice item means data will not be comparable. It addresses proficiency outcome 15.

Teacher Notes: Item 23 (1 point)

This item may not be appropriate for students who have never used Venn diagrams. It is one way to assess a student understanding of "sorting" on multiple attributes.

Teacher Notes: Item 24 (3 points)

This item is designed to measure a student's conceptual understanding of the relative size of fractions. One point could be awarded for correctly identifying a fractions between one-fourth and one-half, and two additional points for an adequate explanation.

Teacher Notes: Item 25 (3 points)

One point could be awarded for correctly telling how many cookies in each package. Two points could be given for an adequate explanation. Notice the relationship between this item and item 2.

Teacher Notes: Item 26 (3 points)

One point could be awarded for selecting the correct spinner and two points for the explanation. It is not necessary that students use technical language in their explanation. For example a student might say that the answer is B. because "the 2s take up more space on the spinner."

Teacher Notes: Item 27 (3 points)

One point should be awarded for a correct graph, one point for a title, and one point for a correct label. This is a relatively straightforward item, but it does require students to find some method of "organizing" data.

Teacher Notes: Item 28 (1 point)

There is some reason to believe that fourth graders have more difficulty with the concept of "perpendicular" than with the concept of "parallel."

Teacher Notes: Item 29 (1 point)

This item may prove difficult for fourth graders because of the word "NOT."

Teacher Notes: Item 30 (1 point)

On the NAEP assessment 51% of fourth graders selected the correct option. This item assesses division and also an understanding of the term "sum."

Teacher Notes: Item 31 (1 point)

This item also requires the student to apply some basic understanding related to operations with whole numbers. Fifty-two percent of the fourth-graders answered this correctly on the National Assessment.

Teacher Notes: Item 32 (1 point)

Sixty percent of the students answered this item correctly on the National Assessment. This item attempts to assess directly a student's ability to understand a word problem.

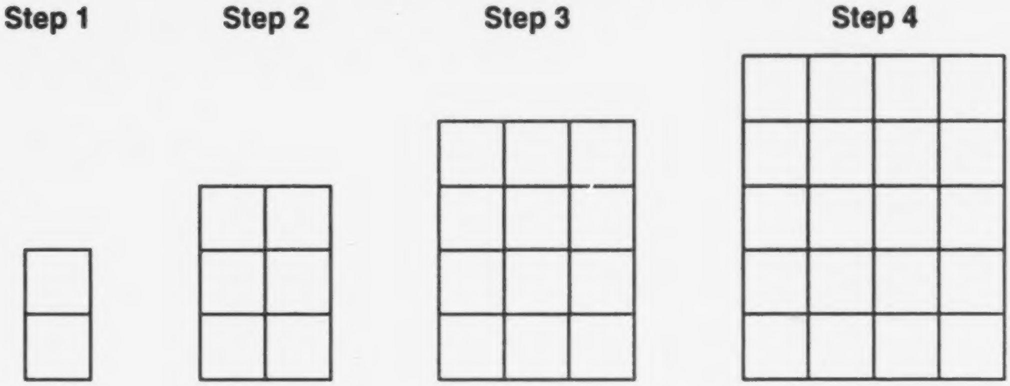
Teacher Notes: Item 33 (4 points)

This item requires a student to do a variety of things. The student might be awarded one point for correctly calculating how much money Jason has ($55.00), one point for correctly calculating how much money he will spend if he buys the items listed ($49.96), one point for a correct subtraction, and one point for an appropriate explanation.

Information Sheet

Item	Answer	OBJECTIVE	OUTCOME LEVEL	CRITICAL OBJECTIVE
1.	56	4-4-2	PS/A	no
		4-4-1	PS/A	yes
2.	12	4-3-11	K	yes
3.	B.	4-3-9	K	no
4.	B.	4-2-5	PS/A	no
5.	B.	4-1-1	C	no
6.	Cheeseburger; Yogurt	4-7-3	C	yes
7.	1200; 128	4-2-3	PS/A	yes
8.	D.	4-6-1	C	no
9.	See Teacher Notes	4-4-2	C	yes
10.	7 cm, 8 cm	4-6-2	K, C	yes
11.	7	4-3-11	C	yes
12.	B.	4-2-4	C	no
13.	$15; 3 weeks	4-2-5	K, PS/A	no
		4-3-11		yes
14.	A.	4-4-3	C	no
15.	red	4-3-6	C, PS	yes
		4-8-6		yes
16.	C.	4-7-2	K	no
17.	40	4-3-13	K,C	no
18.	A.	4-1-2	K,C	yes
19	See Teacher Notes	4-6-2	C	yes
20.	See Teacher Notes	4-3-7	C,PS/A	no
21.	8 by 4	4-4-4	C,PS/A	yes
22.	C	4-5-3	C	yes
23.	D.	3-2-4	C	no
24.	See Teacher Notes	4-3-6	C	yes
25.	12; See Teacher Notes	4-3-6	PS/A	no
		4-3-11		yes
26.	B.	4-8-6	C	yes
27.	See Teacher Notes	4-8-1	PS/A	no
28.	B.	4-4-2	K	yes
29.	D.	4-4-4	K	yes
30.	A.	4-3-11	K	yes
31.	D.	4-7-3	C	yes
32	C.	2-2-1	PS/A	no
33.	$5.04 left over	4-6-6	K	yes

1. Sarah is building a wall with blocks. She is building the wall using a pattern. The first four steps of the pattern are shown below.
If the pattern continues, how many blocks will be in Step 7?________________



How did you find your answer?

2. Divide 108 by 9. Answer________________

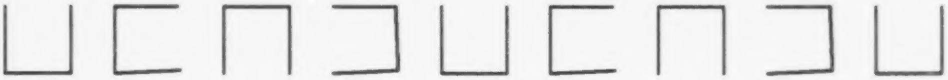
3. What number is four hundred five and three-tenths?

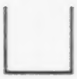
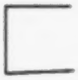
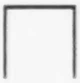
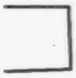
A. 45.3
B. 405.3
C. 453
D. 4,005.3

4. There are 50 hamburgers to serve 38 children. Each child is to have at least one hamburger. How many can have a second hamburger?

A. 6
B. 12
C. 26
D. 38

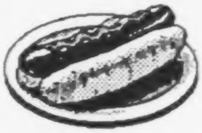
5. If the pattern were continued, which figure would be next?



A.  B.  C.  D. 

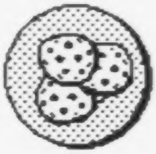
6. Which two different items below would provide a total of about 600 calories?

Cheeseburger
393 Calories



Hot Dog
298 Calories

Yogurt
214 Calories



Cookie
119 Calories

__________________ and __________________

7.

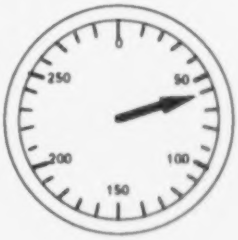
POINTS EARNED FROM SCHOOL EVENTS		
CLASS	MATHATHON	READATHON
Mr. Lopez	425	411
Ms. Chen	328	456
Mrs. Green	447	342

What was the total number of points earned from the Mathathon?______

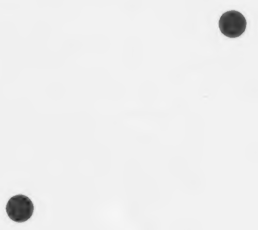
How many more points did Ms. Chen's class earn from the readathon than from the mathathon?_____________

8. What is the weight shown on the scale?

- A. 6 pounds
- B. 7 pounds
- C. 51 pounds
- D. 60 pounds



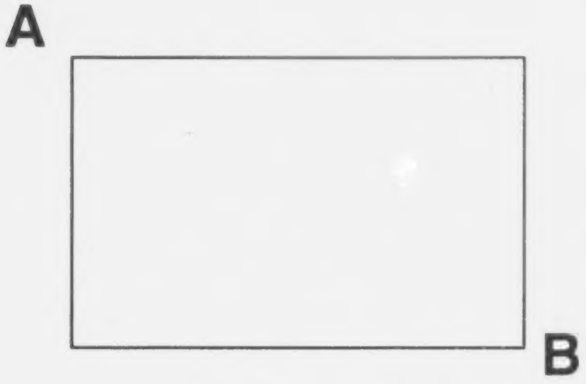
9. In the space below, use your ruler to draw a square with two of its vertices at the points shown.



10. Use your centimeter ruler to make the following measurements to the nearest centimeter.

a. What is the length in centimeters of one of the longer sides of the rectangle?____________

b. What is the length in centimeters of the diagonal from vertex A to vertex B?____________



11. In the multiplication problem below, write the missing number in the box

$$\begin{array}{r} 23\square \\ \underline{\times 8} \\ 1{,}896 \end{array}$$

12. If Δ represents the number of newspapers that Lee delivers each day, which of the following represents the total number of newspapers that Lee delivers in 5 days?

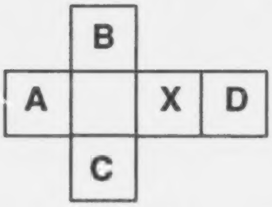
A. 5 + Δ
B. 5 X Δ
C. Δ + 5
D. (Δ + Δ) X 5

13. Jill needs to earn $45.00 for a class trip. She earns $2.00 each day on Mondays, Tuesdays, and Wednesdays, and $3.00 each day on Thursdays, Fridays, and Saturdays.

How much money does Jill earn in one week?____________________

How long will it take her to earn $45.00?________________________

14.



The squares in the picture above represent the faces of a cube which has been cut along some edges and flattened. When the original cube was resting on face X, which face was on top.

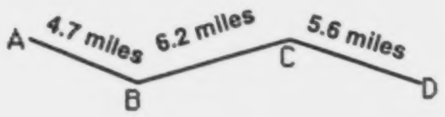
A. A
B. B
C. C
D. D

15. In a bag of marbles, $\frac{1}{2}$ are red, $\frac{1}{4}$ are blue, $\frac{1}{6}$ are green, and $\frac{1}{12}$ are yellow. If a marble is taken from the bag without looking it is most likely to be

A. red
B. blue
C. green
D. yellow

16. Carol wanted to estimate the distance from A to D along the path show on the map . She correctly rounded each of the given distances to the nearest mile and then added them. Which of the following equations represents her work?

A. 4 + 6 + 5 = 15
B. 5 + 6 + 5 = 16
C. 5 + 6 + 6 = 17
D. 5 + 7 + 6 = 15



17. By how much would 217 be increased if the digit 1 were replaced by a digit 5?

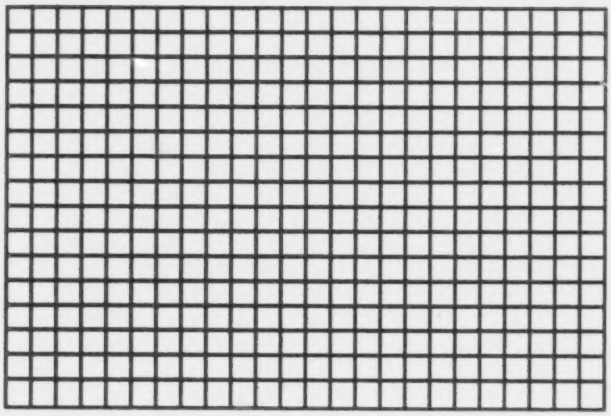
A. 4
B. 40
C. 44
D. 400

18. What rule is used in the table to get the numbers in column B from the numbers in column A?

Column A	Column B
12 →	3
16 →	4
24 →	6
40 →	10

A. Divide the number in column A by 4
B. Multiply the number in column A by 4
C. Subtract 9 from the number in column A.
D. Add 9 to the number in column A.

19. On the grid below, draw a rectangle with an area of 12 square units.



20. Kim's teacher asked her to use four colors to divide a square into parts, and to color the parts as follows:

$\frac{1}{2}$ is colored red $\frac{1}{8}$ is colored green $\frac{1}{4}$ is colored blue

Any other part is to be left white

- On the squares below, draw three different designs to help Kim. Each square must contain all four colors.
- On your sketches, write R for red, B for Blue, G for green, and W for white.
- Circle one of your designs and explain to Kim how you know that the parts you colored are the correct fractions.

21. Wayne cut a rectangle with a perimeter of 24 inches into two squares. What were the dimensions of the rectangle? Explain your thinking at each step and your answer(s) using pictures, numbers or words.

22. Laura wanted to enter the number 8375 into her calculator. By mistake, she entered the number 8275. She wants to correct the mistake without clearing her calculator. Which of the following keying sequences would accomplish that goal?

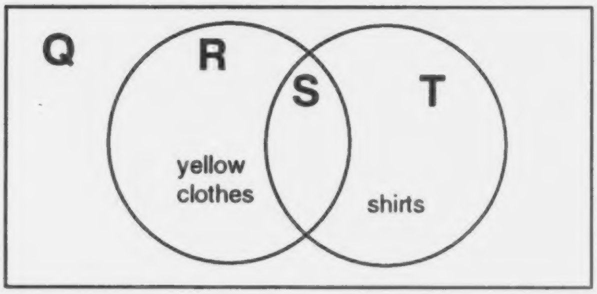
A. $\boxed{+}\ \boxed{3}\ \boxed{0}\ \boxed{0}\ \boxed{=}$

B. $\boxed{1}\ \boxed{0}\ \boxed{0}\ \boxed{=}$

C. $\boxed{+}\ \boxed{5}\ \boxed{0}\ \boxed{=}\ \boxed{+}\ \boxed{5}\ \boxed{0}\ \boxed{=}$

D. $\boxed{+}\ \boxed{1}\ \boxed{+}\ \boxed{0}\ \boxed{+}\ \boxed{0}\ \boxed{=}$

23. In the diagram below, one circle represents yellow clothing, and the other circle represents shirts. Which letter shows the part of the diagram where you would put a green shirt?

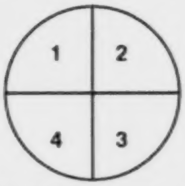


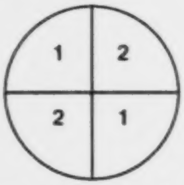
A. Q
B. R
C. S
D. T

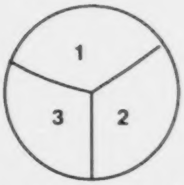
24. Find a number between $\frac{1}{4}$ and $\frac{1}{2}$. What number did you find?__________
Explain how you know for sure it is between $\frac{1}{4}$ and $\frac{1}{2}$.

25. Mrs. Baker's fourth grade class is packaging cookies for the school carnival. They have 108 cookies. Mrs. Smith says they should make 9 packages. How many cookies will be in each package?________
Explain how you found your answer using pictures, numbers or words.

26. You want to spin a 2. Which spinner would give you the best chance of spinning a 2 on the first spin?_______

A.


B.


C.


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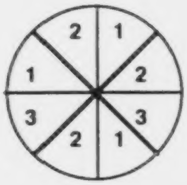


Explain why the spinner you picked gives you the best chance.

27. You have taken a survey of some second graders, asking them "What is your favorite color?" Here are the answers you received

Red
Pink
Orange
Pink

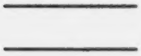
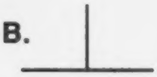
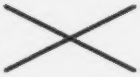
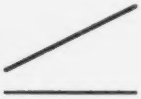
Blue
Red
Blue
Black

Blue
Green
White
Blue

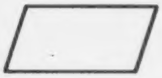
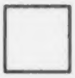
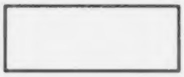
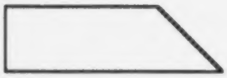
Green
Yellow
Black
Red

On the grid below, make a bar graph that shows all the information about the second graders favorite colors. Be sure to title and label all parts of your graph.

28. Which of the drawings below shows perpendicular lines?_______

A. 
B. 
C. 
D. 

29. Which figure is NOT a PARALLELOGRAM?

A. 
B. 
C. 
D. 

30. If the sum of 39 and 66 is divided by 3, the result is

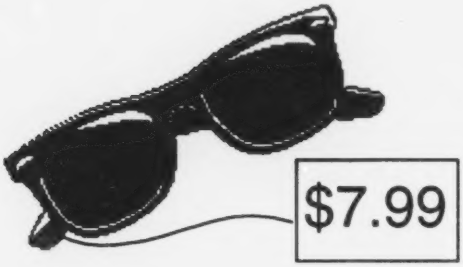
A. 35
B. 61
C. 79
D. 315

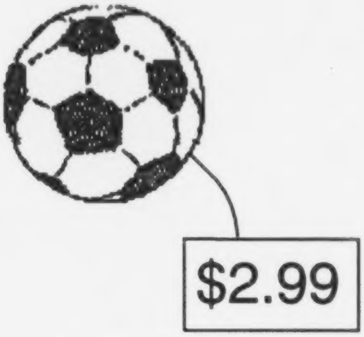
31. When you subtract one of these numbers from 900, the answer is greater than 400. Which number is it?

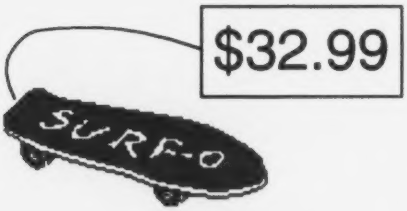
A. 712
B. 667
C. 579
D. 459

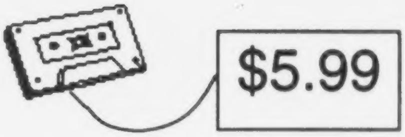
32. On a flight from Los Angeles to New York, the cost of the fare was $400. Every seat was sold. What additional information do you need to find the total for all fares?

A. None
B. The number of employees on the plane.
C. The number of passenger seats on the plane.
D. The distance from Los Angeles to New York.







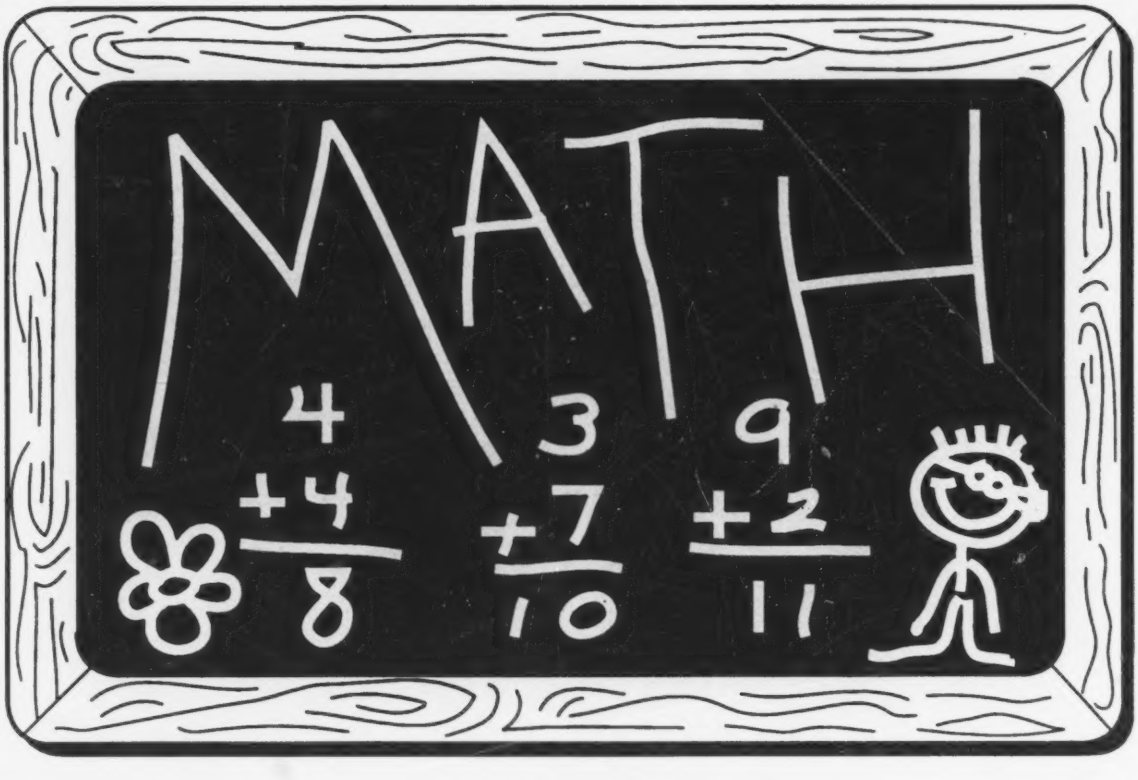


33. Justin just had a birthday. He got money from his grandparents (two twenty dollar bills), his Aunt Millie (a ten dollar bill), and his cousin Butchie (a five dollar bill). His dad told him he would pay the tax on whatever he decides to buy. Can he buy a skate board, a pair of sunglasses, a soccer ball and a cassette tape? If he can, how much money will he get back? If he can't, how much more money does he need? Explain how you are sure you are right.

Competency-Based Education Assessment Series

Fourth Grade Mathematics

Type 3 Assessment



Reviewed by

Karen Ellerson
Vickie Hesler
Margaret Kasten
Kelly Polling
Pushpa Ram
Terry Rounds

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency assessment consists of performance tasks, each requiring a substantial period of time to complete. The tasks involve more authentic-type situations that reflect the ways in which mathematics is applied. Multiple skills, understanding, and thinking are required to complete each task. Rather than assessing single objectives, many of the tasks address multiple objectives drawn from a variety of strands. Several objectives may be assessed in a single task. The manner in which the assessment is carried out may also reflect the outcomes level (knowledge/skills, concept, or problem solving/application) measured relative to those objectives.

There are four separate "activities" at the fourth grade level, each with several "tasks." Taken in total the activities provides multiple ways to assess a variety of outcomes. The results of the assessment may also be used in several ways, to make instructional decisions, to make programmatic decisions, or even to diagnose student misconceptions.

Activity 1 (A Birthday Party for the Twins) is actually a series of instructional activities/assessments, some of which are performance tasks and some of which are not, done in the "context" of a birthday party. A series of assessments done in a single context will help students relate to and "visualize" the situations being described. Children are often able to "read" the words in a story problem, but unable to "interpret" or understand what the problem is asking. The contextualization of problems can help some children "see" what is being asked. The first activity contains a variety of tasks which include both standard story problems and activities that require student production of graphs or budgets.

Activity 2 (Patterns, Shapes and Transformations) is based on an instructional activity in a publication from the National Council of Teachers of Mathematics and illustrates the integration of assessment and instruction. Students have opportunities to learn "new" things and to apply old learnings in new ways. The Teacher Reviewers felt strongly that this set of activities would require specific teacher leadership and direction. Materials originally designed as student "worksheets" have been converted to teacher directions. There are six related tasks in this activity.

Activity 3 (Bears) is a relatively brief task, which nevertheless assesses a variety of the proficiency outcomes. This task is found in a publication of the Mathematical Sciences Education Board, *Measuring Up: Prototypes for Mathematics Assessment.* The book contains a variety of performance tasks and activities designed for use at the fourth grade level.

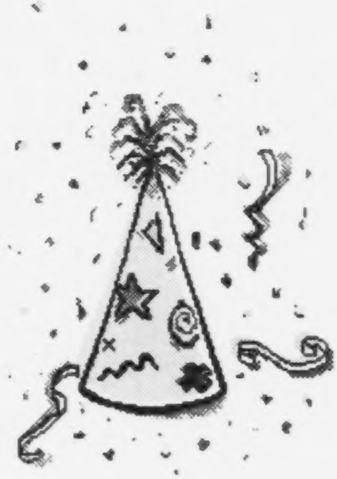
Activity 4 (Graphs) is also found in the same publication. However, there are a variety of suggested "Extensions" that are not in included in *Measuring Up: Prototypes for Mathematics Assessment*. This is intended to provide an illustration of ways in which teachers might adapt commercial or other materials as assessment tasks appropriate for their specific classrooms.

Additional discussion of the activities is provided in the introduction to each activity.

These tasks, to be used at local discretion, are intended to be adopted or adapted by teachers and districts in ways that support local programs and goals. The activities and tasks can be used in a variety of ways. In addition to serving as examples which can be used directly with students, they are also examples of ways that teachers might design assessment activities.

Activity 1-A Birthday Party for Twins

Teacher Notes:



What follows provides an example of a set of related activities and tasks designed to assess understanding of a variety of fourth grade objectives. These activities and tasks are related to the planning of a birthday party for the Hatch twins. This is a good opportunity for the children to discuss a hypothetical situation. Some of the tasks are little more than review sheets; others represent a true integration of assessment and instruction. Student worksheets are provided for many tasks. In one case, a "Teacher Sheet" provides specific suggestions. As is the case with all of these materials, teachers should adopt or adapt as they see fit.

Discussion of Tasks:

Task 1: A Birthday Party for the Twins

This introductory activity helps set a "context" for the rest of the activities. One of the principal difficulties that fourth graders experience in mathematics is with word problems. Teachers often worry that "the problem is not with mathematics it is with reading." But when asked to "read" the problem aloud most of the students can. In fact the problem is not with "reading" it is with "understanding" what is read, "visualing" the problem situation, and "interpreting" the mathematical meaning of the words, diagrams, situations. Structuring a series of problems or activities around a common theme should help students create "mental pictures" that will help them become better at solving mathematical problems. The first activity should help begin this process. Students are asked what decisions have to be made. Hopefully, they will come up with a list that will include: when and where to have the party, who to invite, what to do at the party, and what to eat at the party. Help students think through which of these involve mathematics (they all do). "When" involves both date and time of day. "Where" should cause students to think about space and numbers of guests. "What to eat" can give rise to discussions of recipes and serving size. Of course, any kind of party costs money--an important fourth grade concept. This first activity is teacher led and relies on an informal assessment that only a teacher can do.

Task 2: Calendar Counting

This activity requires students to collect, organize and display data. There is a great deal of flexibility in this activity. It can be done as a whole group exercise. It can be done in groups or with students working individually. The teacher may just pass the "Any Month" calendar around and have each student sign in the correct "day", or the calendar may be posted, or the data collection can be a teacher led activity. Likewise, each student might graph the data independently or the graphing can be done as a group project. As an extention, you might ask students if there are other ways to display the data. The students might discuss how they would use a circle graph or a line graph to show the same data.

Task 3: Time at the Birthday Party

This activity requires students to work with elapsed time as well as interpret times. There are two "student sheets" associated with this task. The first asks students to draw hands on a clock face. There is an event associated with each of the times they are asked to represent. The second student sheet requires the students to use the information on the first sheet to answer some questions. They are required to calculate elapsed time as well as answer other "time-related" questions.

Task 4: Shopping for the Birthday Party

For this task there are two alternatives. Both alternatives require students to use two "advertisements" to decide where to buy their party supplies. The first alternative requires class consensus on what to buy and how many to get of the different items. The second alternative supplies that information. Teachers should decide which would be best for their individual classes

Task 5: Creating Number Sentences

This assessment asks students to write number sentences or equations that would help solve the problem. It also tells students not to solve the problem. This is so students will focus on the mathematical processes instead of getting an answer. There are a variety of ways students can answer correctly. The following are examples of correct responses:

1. 24 ÷ 3 = the number of students who like chocolate cake
2. 24 X 3 = the number of jelly beans
3. 120 ÷ 24 = the number of balloons
4. $10.00 - $5.95 = Barbara's changes

Task 6: Word Problems

For this set of problems, students should provide solutions and also show their work. The first problem is a routine subtraction problem, but it does require students to know that two twenty-dollar bills is forty dollars. The subtraction problem is relatively difficult. The second problem, which had two parts, requires students to think about "change" in two different forms. They should know from the first problem that the change is $15.05. The second problem requires them to find two different ways that the same amount of money could be given as change. The third problem is a calculator sequence problem. The mixing of "problem types" is a good strategy to help students get in the habit of "thinking" about each problem. Answers for the third problem will vary, one acceptable keying sequence would be:

1	0	0	-	(	2	X	2	2	)	=		

An alternative correct keying sequence is:

1	0	0	-	2	2	=	-	2	2	=		

This problem provides an excellent opportunity to discuss the differences in algebraic and arithmetic logic found in calculators. The sequence:

1	0	0	-	2	X	2	2	=				

would yield a correct answer using an algebraic logic calculator and an incorrect answer using an arithmetic logic calculator.

Task 7: Birthday Cakes

This task provides an opportunity to assess several of the geometry objectives found in the Model Competency-Based Mathematics Program as well as 4th Grade Proficiency Learning Outcomes. Since Mary and Tom both have first names that begin with "symmetric" letters, this is an opportunity to find symmetry in letters. You may want to ask your students, "How many of the letters in 'HAPPY BIRTHDAY' are symmetric?" Students might check one anothers' answers to verity the presence of parallel, intersecting and perpendicular lines and right angles in the letters. Obviously, the assessment could be extended by having students decorate boxes or cylinders with paper and crayons or markers.

Task 8: Presents!

This set of exercises is designed to target both measurement and geometry objectives and to assure that students have experience with a three dimensional figure which they "construct." In addition to having to make simple measurements on a line in both the metric and the standard system, students are asked to determine which box is Karen's by measuring the boxes using a centimeter ruler. Have them decorate the box before cutting it out and taping it together.

Task 9: Review

This final task asks students to take information from the drawing and use it in the creation of number sentences, solve two conceptual fraction problems, and create a story problem that in their judgement a 1st grader could solve. The picture is rich with mathematics: the cake has two layers and is shaped like a cylinder, and there are 11 candles and 9 stars. This activity can help students see "quantitative and geometric" things in the environment.

A Birthday Party for Twins

Mary Hatch and Tom Hatch are twins. They are going to be 10 years old in June. They want to have a birthday party. Their mother, Mrs. Hatch, says she will help them, but wants them to do most of the planning AND she wants them to agree BEFORE they come to her with their decisions. They will have to decide all kinds of things, but first Mary and Tom must think about all the decisions they have to make.

List all of the things you can think of that Tom and Mary will have to decide.

Calendar Counting

Teacher Notes:

You will need to have at least one June calendar that can be seen by all the children. You might say something like:

Tom and Mary were born in June, and they want to have the party on a Saturday. Look at the calendar for this year. If they want to have the party on a Saturday, what are the dates they might choose?

This is a very easy activity and should allow all the students to have the feeling that they are starting from the same point. The first question can be followed by questions such as:

If their birthday is June 14, which Saturday is closest to their birthday? Which Saturday is the best day for the party?

All of this serves as an introduction to have students begin thinking about their own birth date. You might say something like:

When you asked someone to tell you his or her birthday, he or she will tell you a month and a day. For this next thinking exercise everyone in the class will need to think about their birthday (not the month, just the number.) We are going to collect data about everyone's birth date.

Use the following "Any Month" calendar to collect the data. (See Discussion)

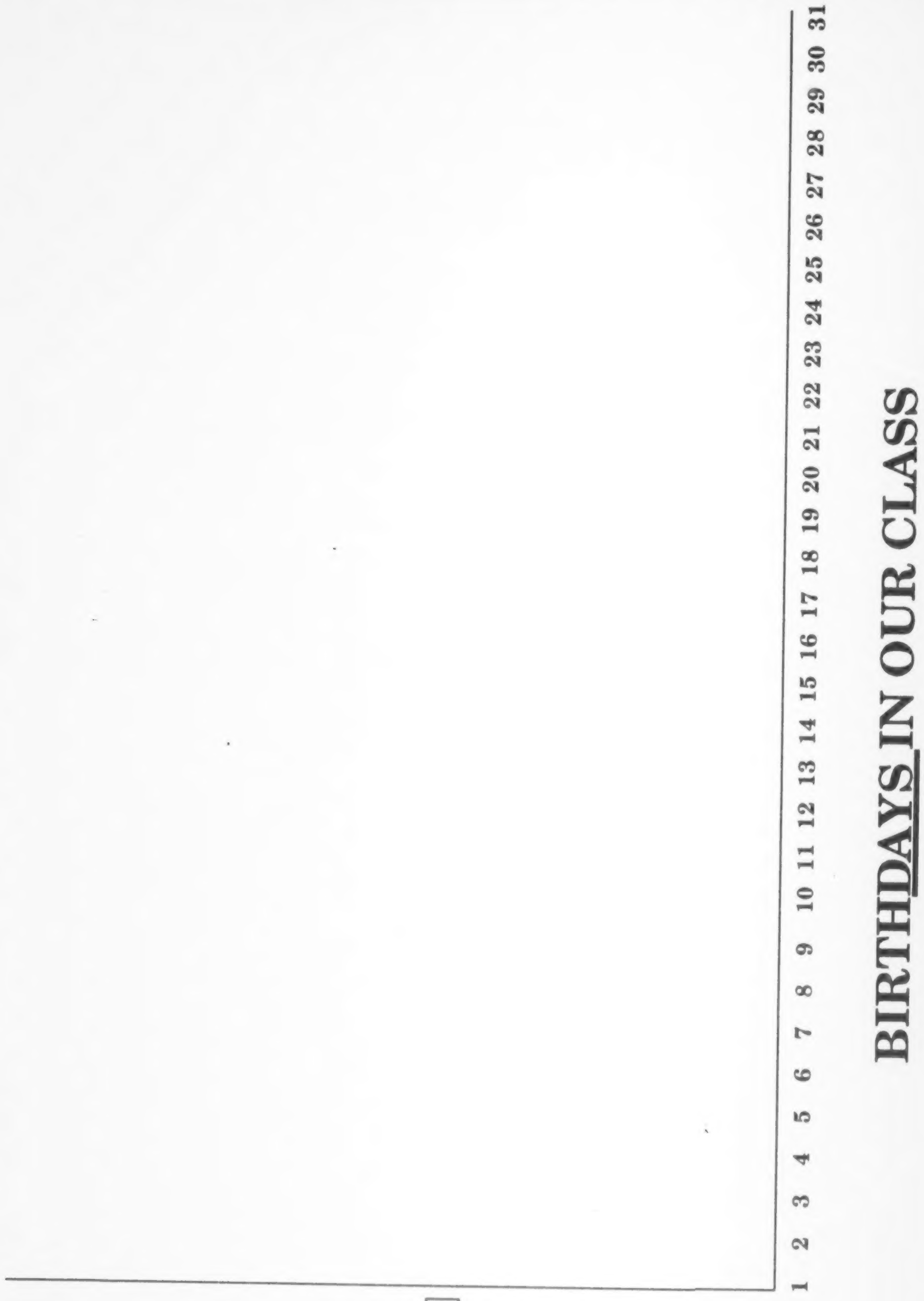
Follow-up question:

Do you think there is the same chance of being born on the first day of the month as there is of being born on the thirty-first day of the month?

Calendars

Any Month

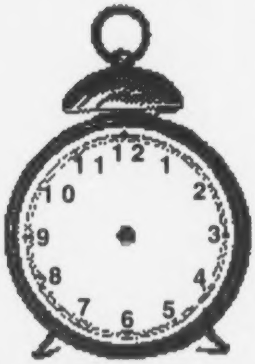
1	2	3	4	5	6	7
8	9	10	11	12	13	8
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				


BIRTHDAYS IN OUR CLASS
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
21
22
23
24
25
26
27
28
29
30
31

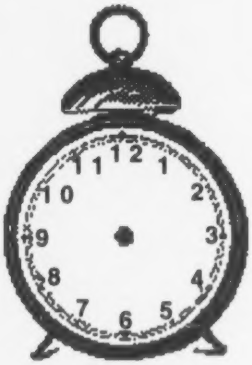
Time at the Birthday Party

Activity 1
Task 2

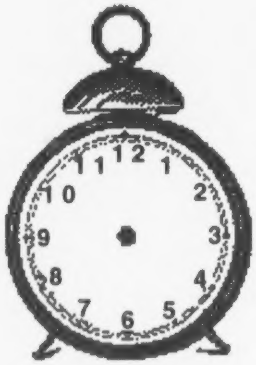
Draw hands on each clock to show the time given. You may use the drawings to help answer the questions on the next page.



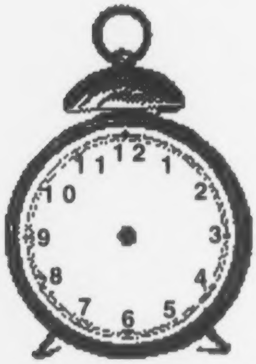
The party started at 1:00 o'clock.



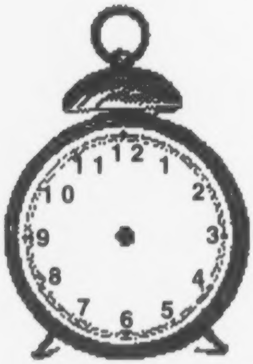
The party was over at 3:30 o'clock.



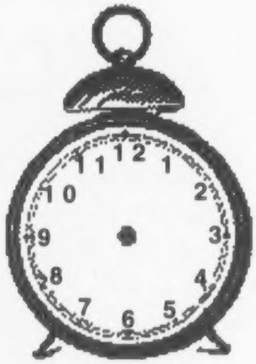
Mary and Tom opened their presents at 2:15 o'clock.



Mary and Tom 's mother cut the cake at 2:45 o'clock.



Susan's mother picked her up 8 minutes before the party was over.



It took Mary and Tom 1 hour and 40 minutes to clean up after the party was over.

How long did the birthday party last?____________

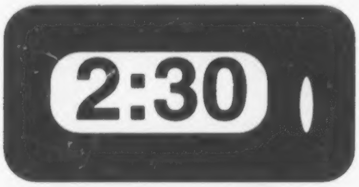
If Susan got to the party when it started, how long was she at the party?____________________

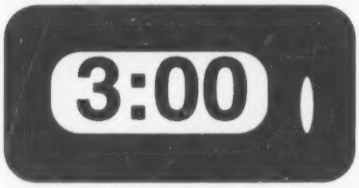
What time did Mary and Tom finish cleaning up?________________

If the cake was cut as soon as Tom and Mary finished opening their presents, how long did it take them to open their presents?____________

After the party started, how long did Tom and Mary wait to open their presents?________________

Look at the digital clocks below. In the box write one thing that might have been happening at the party at the time shown.





In the box below write another problem about time at Tom and Mary's Birthday Party.

Shopping for the Birthday Party

Activity 1
Task 4 Alternative 1

Their mother told Tom and Mary that she will have time to go to only one store to pick up the things for the party. She gave Tom and Mary the advertisements from the newspaper for the *Party Discount Store* and the *Birthday Store*. She asked them to figure out how many things they needed and then decide which store would be the least expensive.

Tom and Mary want to invite all the children in their class (there are 24 students in the class--counting Tom and Mary).

List what Tom and Mary need to buy.

Which store should they use? Why?

Their mother has allowed them $100.00 to spend on the party. How much money will they have left?

Shopping for the Birthday Party

Activity 1
Task 4 Alternative 2

Tom and Mary's mother told them that she will have time to go to only one store to pick up the things for the party. She gave Tom and Mary the advertisements from the newspaper for the *Party Discount Store* and the *Birthday Store.* She asked them to figure out how many things they need and which store is the least expensive.

Tom and Mary decided that they would need:

1 invitation for each person they invite
1 plate and 1 cup for each person at the party
2 napkins for each person at the party
1 table cloth
2 balloons for each person at the party
300 feet of streamer to decorate

If Tom and Mary want to invite all the children in their class (there are 24 students in the class--counting Tom and Mary), which store is least expensive?

Their mother has allowed them $100.00 to spend on the party. How much money will they have left?

Party Discount Store
BIG SALE

plates	10 for $2.00
cups	10 for $3.00
napkins	20 for $1.50
table cloths	$2.00
birthday candles	24 for $2.00
balloons	12 for $3.00
invitations	10 for $3.00
streamers	150 feet for $2.00

Big Sale at the Birthday Store!

Big Sale at the Birthday Store!

Big Sale at the Birthday Store!

Paper Products

Napkins --25 for$2.00
Table Cloths--$3.00 each
Paper Plates--50 for $5.00
Paper Cups--50 for $6.00

Special Price on our deluxe invitations--25 for $6.00

Stock-up Sale Saturday Only

Decorations

- ✓**Balloons 8 for $2.00**
- ✓**Streamers 100 feet for $1.00**

Creating Number Sentences

Activity 1
Task 5

All of the students in Tom and Mary's class are excited about being invited to the birthday party. Write a number sentence or equation that would help solve each problem below. DO NOT solve the problem.

1. There are 24 students in Mary and Tom's class. One third of the students like chocolate cake. The rest of the students like white cake. How many of the students like chocolate cake?

2. Tom and Mary ordered a sheet cake for their birthday cake. Their mother wants to cut it into 24 pieces. If she wants to put 3 jelly beans on each piece, how many jelly beans will she need?

3. A clown is coming to the birthday party and will bring 120 balloons. If there are 24 children at the party and each child will get the same number of balloons, how many balloons will each child get?

4. Barbara wants to get Mary a Beanie Baby. They cost $5.95. If she gives the clerk a ten dollar bill how much change will she get?

Word Problems

Activity 1
Task 6

Find the answer to each story problem. Show your work.

1. The birthday cake for the party came from the grocery store. It cost $24.95. If Mrs. Hatch paid for the cake with two twenty dollar bills, how much change did she get?

2. When Mrs. Hatch looked at the money she got back from the clerk, she had a nickel and some bills.

   a. If she had a five dollar bill and one other bill, what denomination was the other bill?

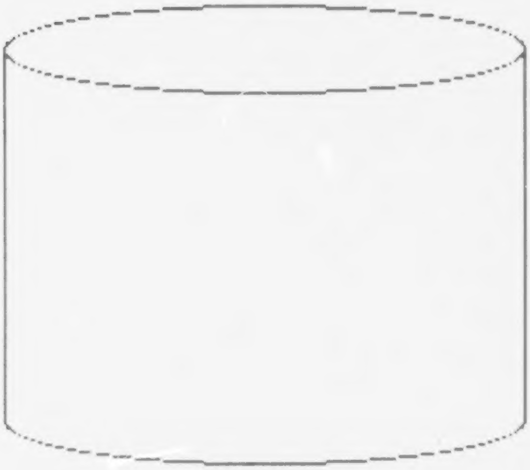
   b. If she had five one dollar bills and two other bills that were the same, what were they?

3. Show what buttons you would push on your calculator (the keying sequence) to figure out how many balloons Tom and Mary would have left if they started out with 100 balloons and each one of their 22 friends takes two balloons home. (Just draw a line through any squares you don't use, you may add squares if you need to).

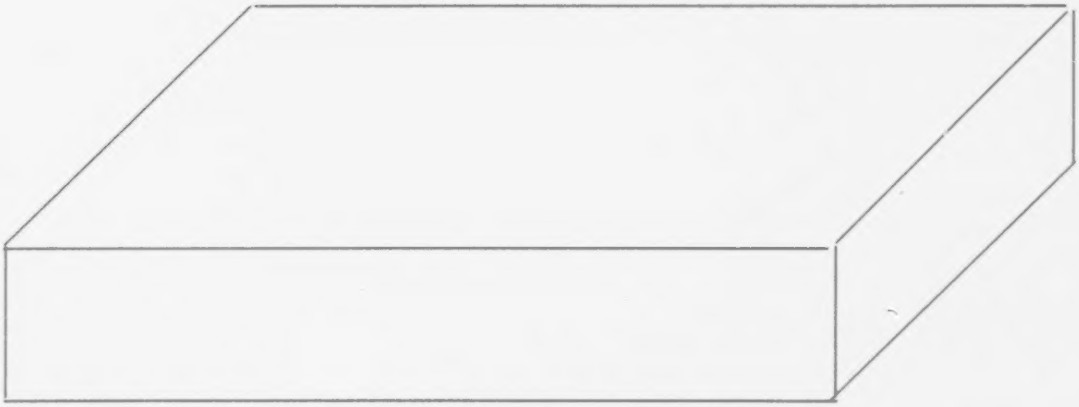
☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐ ☐

Thinking about Birthday Cakes

Activity 1
Task 7



Mary's Birthday Cake



Tom's Birthday Cake

Tom and Mary need to decide how they want their birthday cakes decorated. Choose either Mary's cake or Tom's cake and show how you would decorate the top and sides of the cake. Use crayons or markers. Each design should contain parallel, intersecting, and perpendicular lines and right angles. Each design should also contain at least one example of symmetry and the shapes "square," "circle," and "triangle."

Activity 1
Task 7

Show how you would decorate the top of Mary's cake:

Show how you would decorate the side of Mary's cake:

Activity 1
Task 7

Show how you would decorate the top of Tom's cake:

Show how you would decorate the sides of Tom's cake:

Presents!!!

Activity 1
Task 8

Anna and Karen both bought Mary bracelets. Anna put her bracelet in a box that was 1 inch high, 2 inches wide, and 3 inches long. On the line below use your ruler to mark off a 2 inch segment.

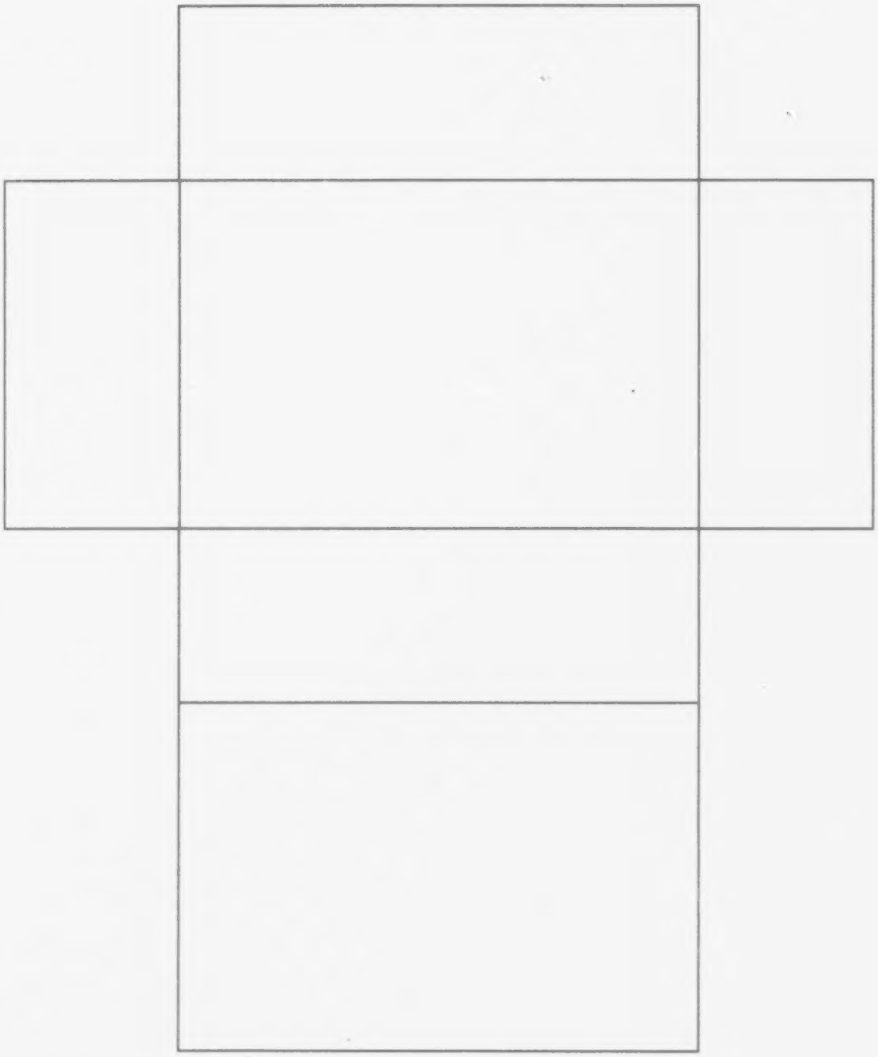
Karen put her bracelet in a box that was 3 centimeters high, 6 centimeters wide, and 8 centimeters long. On the line below use your ruler to mark off a 6 centimeter segment.

On the next two sheets are "nets" that can be cut out and folded up to make boxes.

Which box, A or B, is the same size as the box Karen used?___________

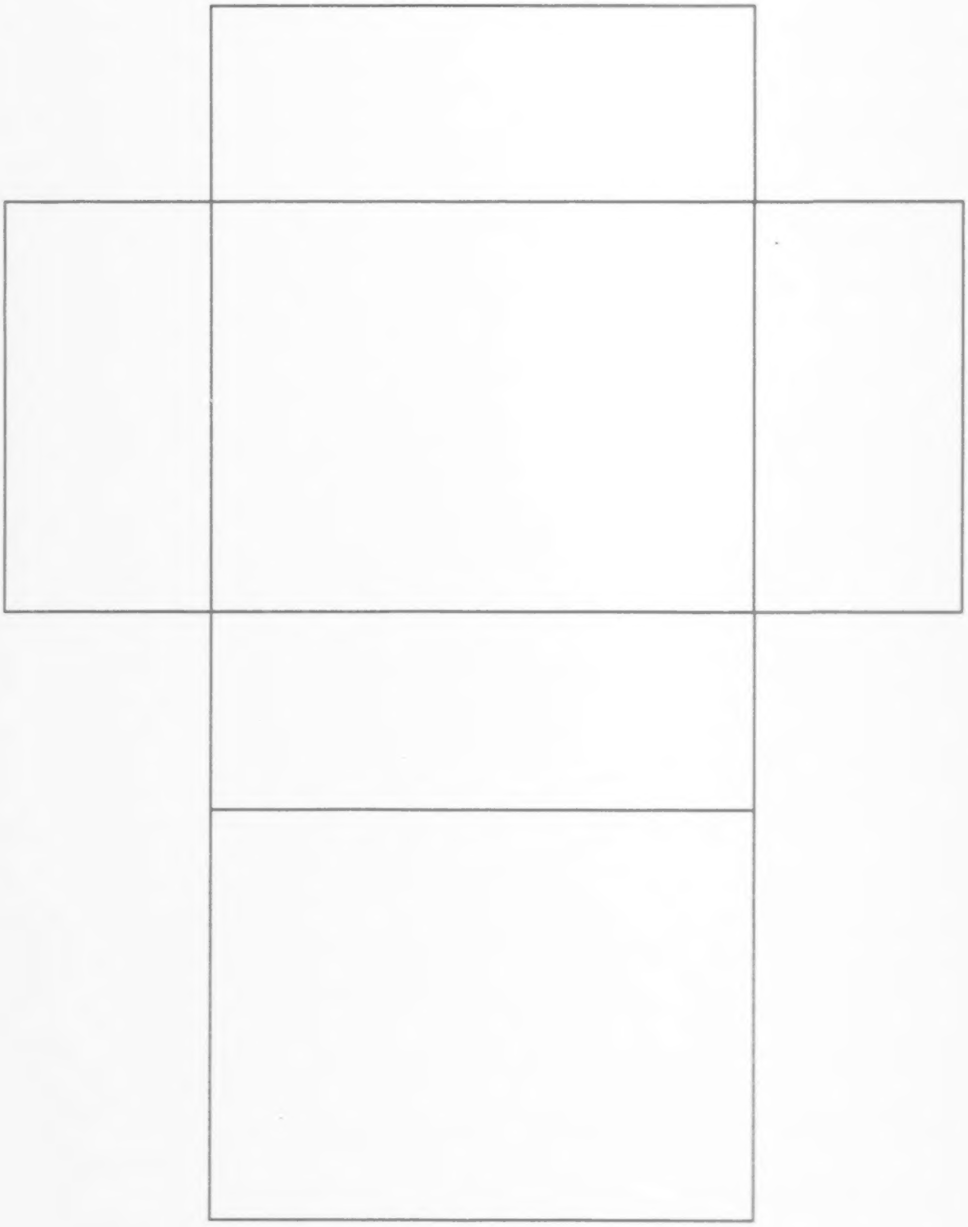
Choose one of the boxes. Decorate the box. Find a partner and see if you can find examples of symmetry, congruence, simple closed curves, circles, rectangles, parallel or perpendicular lines on your boxes.

Activity 1
Task 8



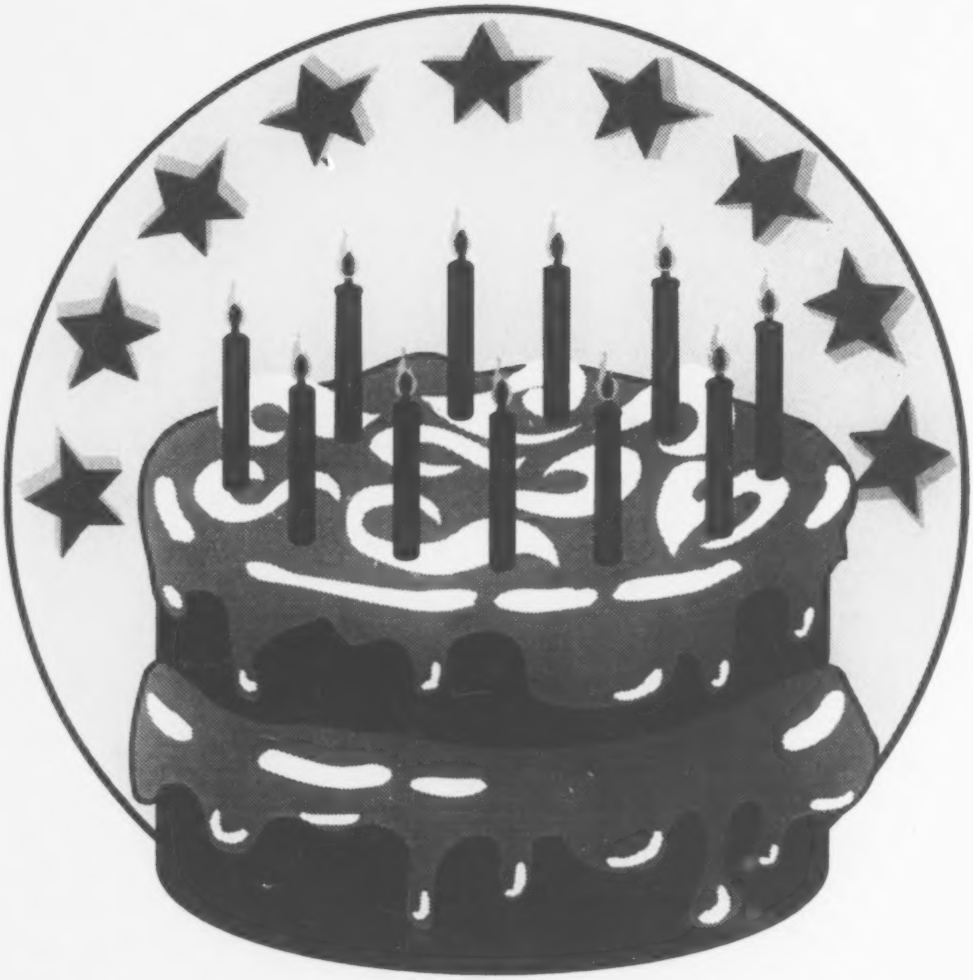
Box A

Activity 1
Task 8



Box B

Activity 1
Task 9



If S stands for the number of stars and C stands for the number of candles, write an equation in the space provided that uses both S and C. Equation:_________________________________

Circle two-thirds of the Stars.

Add candles to the cake so that one-fourth the number of candles is 4.

Write a problem, on the back of this page, about the cake that a 1st grader could do.

Activity 2-Patterns, Shapes, and Transformations

This set of related activities is based on the article "Making Patterns with a Square" from Geometry for Grades K-6: Readings from the Arithmetic Teacher, a publication of the National Council of Teachers of Mathematics.

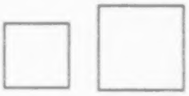
This assessment activity relies heavily on teacher direction and questioning. The suggestions that follow are just that--suggestions. Teachers should use the materials in ways that support their teaching objectives and style. This set of tasks provides a concrete example of the integration of assessment and instruction.

Task 1: Drawing the Lines of Symmetry

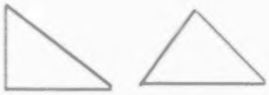
Teachers might introduce this series of instructional and assessment activities in a 10 or 15 minute time period at the end of a class period. Begin by distributing a copy of handout 1 and asking all students to draw in all the lines of symmetry in the square. After all the students have completed the task and there has been some discussion give Task 2 as a homework assignment.

Task 2: Thinking About Congruent Triangles

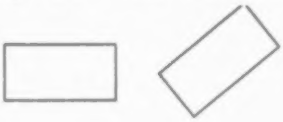
It is suggested that students might do this activity as a homework assignment. It is a writing assignment that will help them clarify their own understanding about congruence. Students should understand that congruence means "same size and same shape" but not necessarily same orientation. They should understand that the two squares are the same shape, but not the same size and therefore not congruent.



Shapes like the two triangles appear to be close to the same "size" but they are not the same shape and therefore not congruent.



On the other hand "orientation" does not make a difference so shapes that are the same shape and the same size are congruent whether or not they are "in the same position." So these shapes are congruent:



Task 3: Coloring Half

This cutting and coloring task is the heart of this activity. (see page 9). In some ways having the students cut out the squares before coloring them makes this a "messier" project. But if that is not done students have a much more difficult time understanding "different." Certainly, if the shapes are immobile on a piece of paper it is more difficult to understand why Figures A through D are the same.

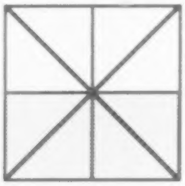


Figure A

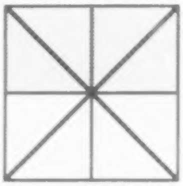


Figure B

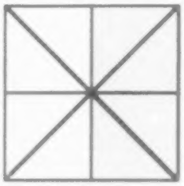


Figure C

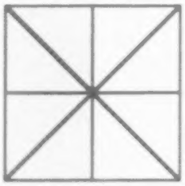


Figure D

But if students cut out the squares and rotate them, it is easy to see that this is not four different ways to show one-half, but the same way, turned in four different positions. There is a black line master provided for use if desired (page 8). Alternatively the sheet could be copied and given to students. There are 13 different ways to color the square (see page 10). This is a challenging activity for fourth graders. It is important to allow plenty of time for students to experiment, to discuss and to challenge one another.

Task 4: Student Questions

The four questions asked of students on this page touch a variety of objectives. They require students to "explain" how they know that they have all of the different ways to color one-half of the squares. This requires students to do some fairly sophisticated thinking. Answers should give some indication of a systematic approach to the problem as opposed to a random or haphazard approach.

Task 5: Perimeters

This task is important for several reasons--it reviews the concept of perimeter, it requires students to look at the perimeter in a different way, it requires them to communicate about mathematics, and finally it requires verification of an answer by a nontrivial measurement exercise. When students look at the figures they should realize that perimeter of the shaded region

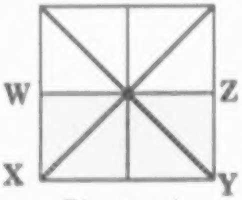


Figure A

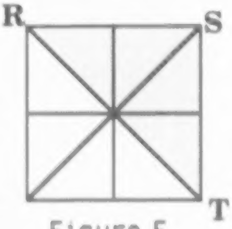


Figure E

in Figure A is the distance from X to Y, plus the distance from Y to Z, plus the distance from Z to W, plus the distance from W to X; and the perimeter of the shaded region in Figure E is the distance from R to S, plus the distance from S to T, plus the distance from T to R. Since the distance from X to Y is the same as the distance from R to S and the distance from W to X plus the distance from Y to Z is the same as the distance from S to T, what really matters is how the distance from R to T compares to the distance from W to Z. Since the distance from R to T is greater than the distance from W to Z, the perimeter of the shaded region in Figure E is greater than the distance in Figure A. This is one of those tasks that is easier to "see" or "understand" than it is to "explain in words."

Task 6: Lines

This task is one of the simplest in this packet. It requires students to identify particular kinds of line relationships. It may be different from other exercises in that they are asked to actually draw the lines instead of just "identifying" lines.

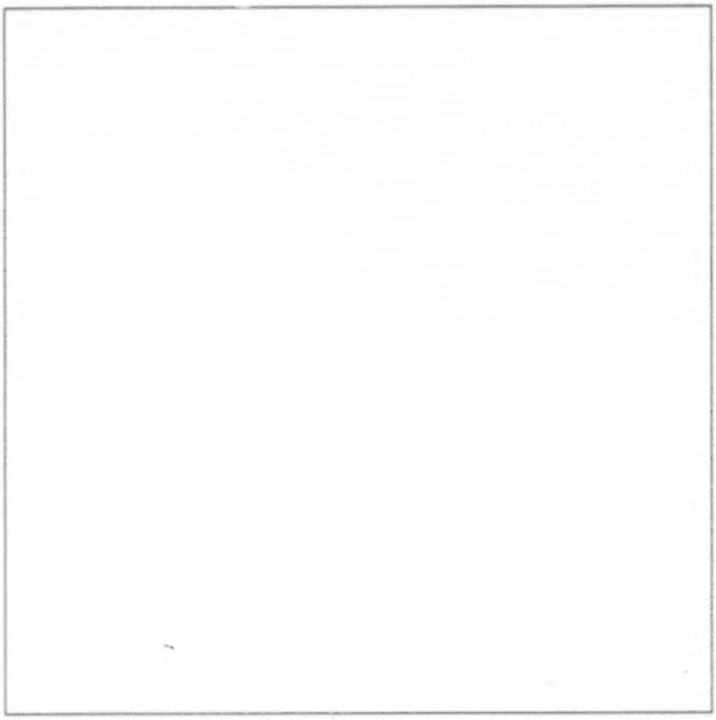
In total this activity addresses several of the fourth grade proficiency outcomes. Those most directly addressed are in the geometry area, but several of the problem solving objectives are also addressed.

There are several important aspects of this, one of the most important involves physical manipulation by students of their patterns, so that they really understand that when "orientation" does and when it does not "matter." A second important idea is "how do you know" when you have them all. The "how do you know" question is important on several levels. It helps students communicate their own thinking, but it also begins to help them understand the importance of examining mathematical ideas.

Drawing the Lines of Symmetry

Activity 2
Task 1

Draw all lines of symmetry

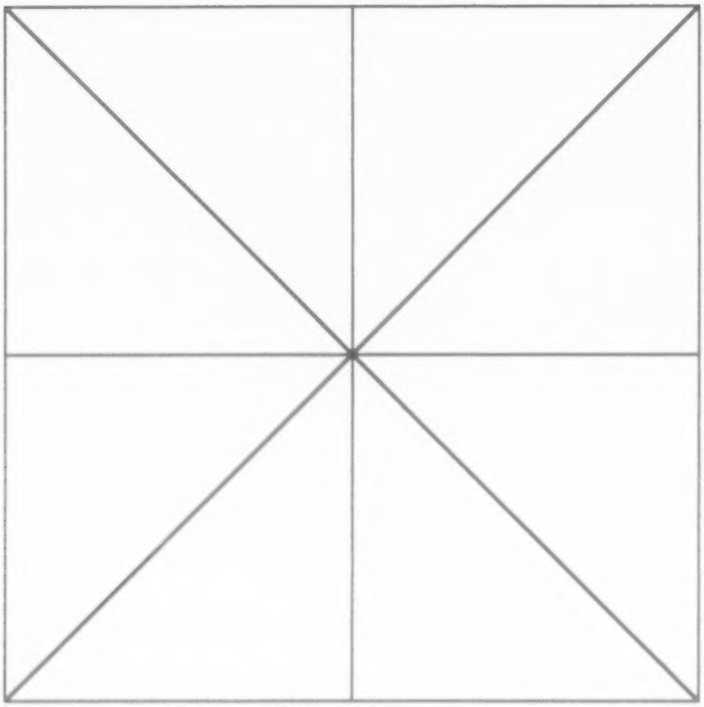


How do you know you have them all?

What shapes can be found in the interior of the square?

Thinking About Congruent Triangles

Activity 2
Task 2



Your teacher told you in class today that the square above is divided into 8 congruent triangles. After you got home your best friend called and said he did not know what that meant. What would you say to your friend to help him understand?

Teacher Directions: Working on the Problem -Task 3

Give each student one and a half " Divided Squares" worksheets. Ask students to cut out the squares that are divided into eight congruent triangles. Give the following directions.

1. Color half of the triangles in the first square.

2. Color half of the triangles in the second square in a different way. How can you tell if it is really "different?" (It is not "different" if I can just move the square to get it to "look" the same. Figure A, B, C, and D are all the "same" because you can get them all by turning Figure A.) Note: Blackline masters for transparency are provided.

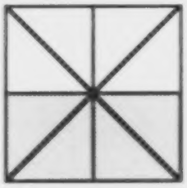


Figure A

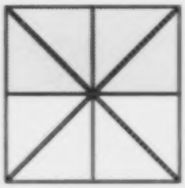


Figure B

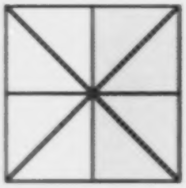


Figure C

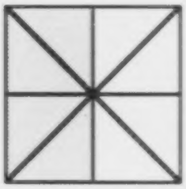


Figure D

Figure A below is "different" than Figure E, because you cannot turn, flip, or slide Figure A to make it look like Figure E.

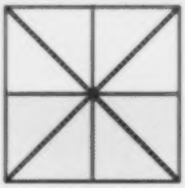


Figure A

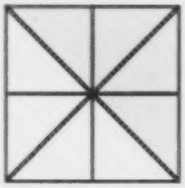


Figure E

3. Tell students they are going to answer the question, "How many different ways can you color half of the triangles in a square? "

These patterns are the SAME

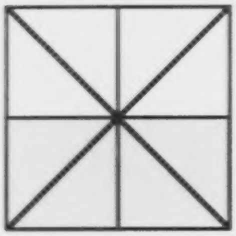


Figure A

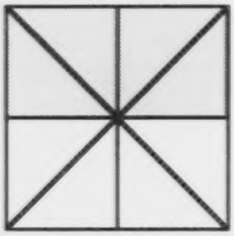


Figure B

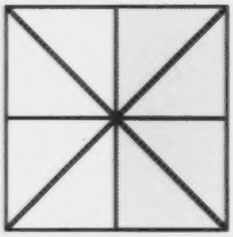


Figure C

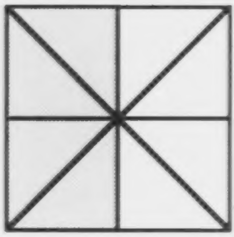


Figure D

These patterns are the DIFFERENT

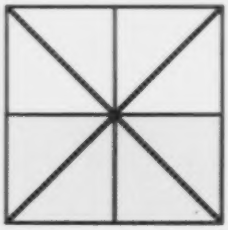


Figure A

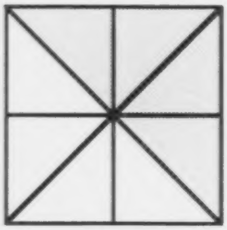
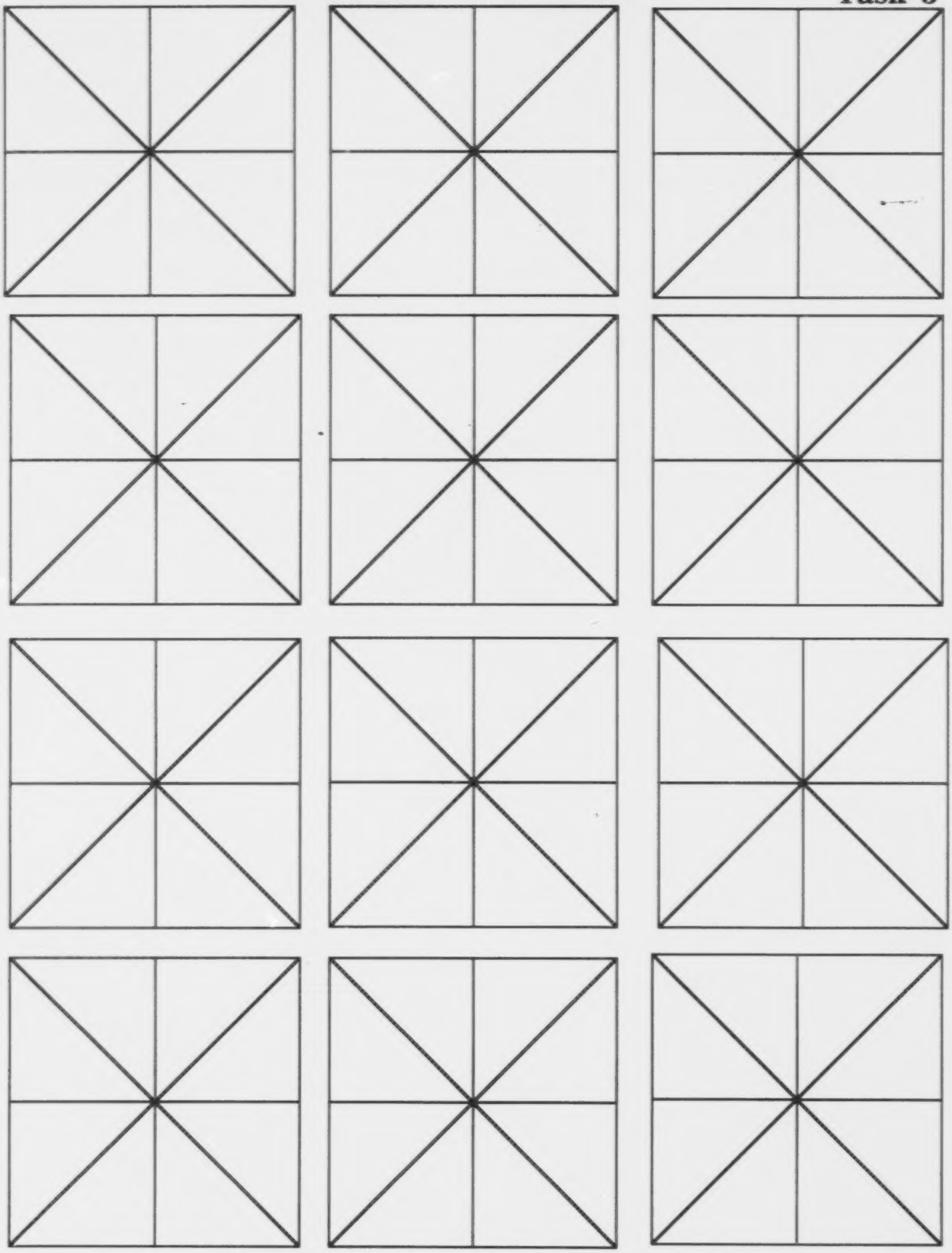


Figure E

Working on the Problem

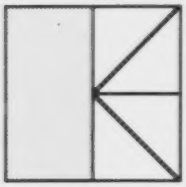
Activity 2
Task 3



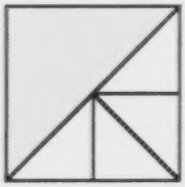
Answer Sheet

Activity 2
Task 4

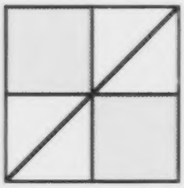
The 13 basic patterns



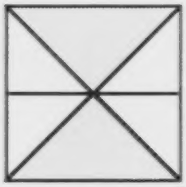
Pattern 1



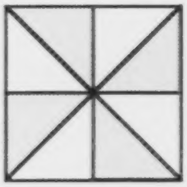
Pattern 2



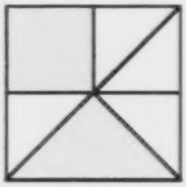
Pattern 3



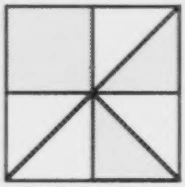
Pattern 4



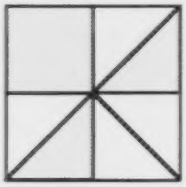
Pattern 5



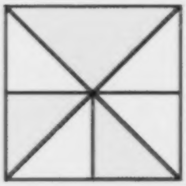
Pattern 6



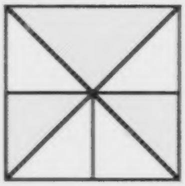
Pattern 7



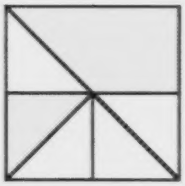
Pattern 8



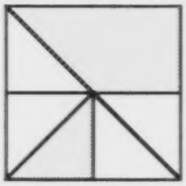
Pattern 9



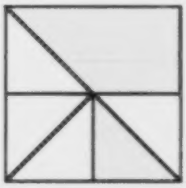
Pattern 10



Pattern 11



Pattern 12



Pattern 13

Student Questions

Activity 2
Task 4

1. How can you be sure when you have found all the ways to color half the triangles in a square?

When you colored half the triangles in a square you created patterns.

2. How many of the patterns are symmetric?_____ Sketch all the symmetric patterns in the space below.

3. Put two of your squares together to make a rectangle, what fractional part of the rectangle is colored?_____ Is the answer the same no matter which two squares you choose?_____Explain why this is true.

4. On the back of this page, list all of the different shapes that you see in the patterns you created.

Perimeters

Activity 2
Task 5

Look at the perimeters of the shaded regions in the figures below. Is the perimeter of the shaded region in Figure A greater than, less than, or the same as the perimeter of the shaded region in Figure E. How do you know? After you have explained your reasoning, verify the result by measuring the perimeters.

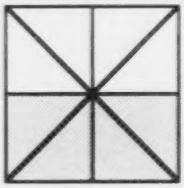


Figure A

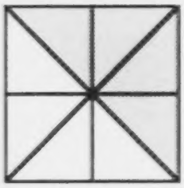
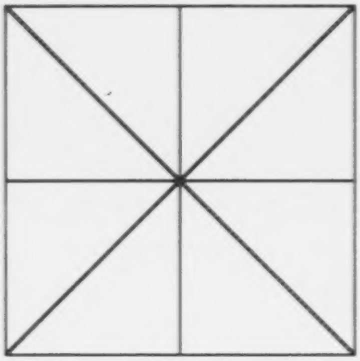


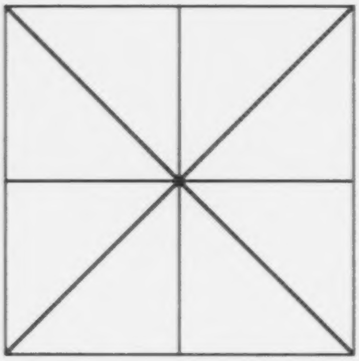
Figure E

Lines

Activity 2
Task 6

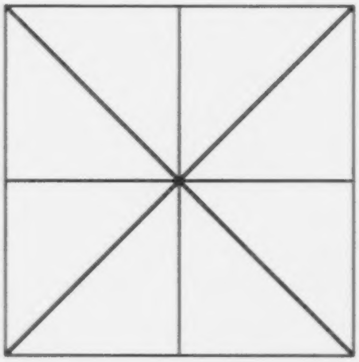
1. Use your crayon to mark two line segments on drawing A that are parallel.

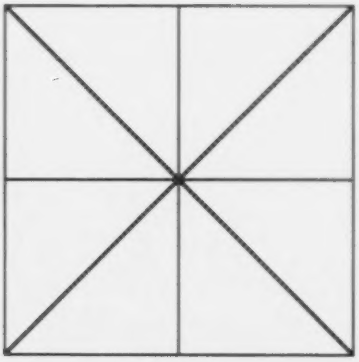
A


B


2. Use your crayon to mark two line segments on drawing B that are perpendicular.

3. Use your crayon to mark two line segments on drawing C that intersect, but are not perpendicular.

C


D


4. Use your crayon to mark a right angle on drawing D.

Activity 3 - Bears

The following task from *Measuring Up: Prototypes for Mathematics Assessment* is representative of the types of tasks found on the Type 3 assessments. It addresses a variety of learning outcomes from the 4th grade proficiency test (see below). It can be used in a variety of ways (students can work individually or in small groups), may take more than one class period, lends itself to a variety of extensions and adaptations, and uses "real" data. The item assumes that studens have experience with analyzing data and drawing graphs. It requires students to decide what kind of analytic approach to take. Students must analyze a fairly complex set of measurements. They must creat a representation that shows data on a reasonable scale in a way that allows comparison of the two groups.

Notice that there are not the same number of bears of each type, which forces the student to consider more that total weights. In fact the data have been adjusted so that the sums of the weights are the same. This will serve as a subtle hint to the student who adds the two columns of figures and is tempted to stop at that point.

The heaviest bear in the entire set is a grizzly, which is the heaviest kind of bear. When looking at student reponses, one must be careful to distinguish between reasoning that relies on the difference between the central values of the weights from reasoning that simply cites the heaviest bear.

Examples of other kinds of questions can be asked about the data include: "What can you say about the weights of male and female bears?" A deeper kind of question deals with age: "Do you think that all these bears are the same age? Explain why you think so."

Additional follow up activities might include oral or written reports to the class using information about other kinds of bears, perhaps illustrated with graphic displays or posters.

This activity provides a "sample" that teachers can use to develop their own assessments. It is important to identify things about this assessment which should be a part of all classroom assessments.

The fourth-grade proficiency learning outcomes which are addressed by this activity are listed below. The range of objectives and the depth at which they are addressed are noteworthy. Assessments which address a variety of objectives, especially objectives from several strands, are important because they help us understand how well students are integrating and applying their mathematical knowledge.

PROFICIENCY OUTCOMES ADDRESSED BY THE BEAR ACTIVITY
4TH GRADE MATHEMATICS

1. Sort or identify objects on multiple attributes (e.g., size, shape, and shading).

2. Use patterns to make generalizations and predictions by

   a. determining a rule and identifying missing numbers in a sequence;
   b. determining a rule and identifying missing numbers in a table of number pairs;
   c. identifying missing elements in a pattern and justifying their inclusion; and
   d. determining missing numbers in a sequence of numbers or a table of number pairs related by a combination of addition, subtraction, multiplication, or division.

3 Select appropriate notation and methods for symbolizing a problem situation, translate real-life situations involving addition and/or subtraction into conventional symbols of mathematics, and represent operations using models, conventional symbols, and words.

4. Identify needed information to solve a problem.

5. Explain or illustrate why a solution is correct.

6. Decompose, combine, order, and compare numbers.

24. Make or use a table to record and sort information (in a problem solving setting using simple and complex patterns in nature, art, or poetry as setting) and make identifications, comparisons, and predictions from tables, pictures graphs, bar graphs, and labeled picture maps.

25. Find simple experimental probabilities and identify events that are sure to happen, events sure not to happen, and those we cannot be sure about.

The data below give the weights of some grizzly bears and black bears living in the Rocky Mountains in Montana.

Grizzly Bears			**Black Bears**		
Bob	Male	220 lbs.	Blackberry	Female	230 lbs.
Rocky	Male	170 lbs.	Greta	Female	150 lbs.
Sue	Female	210 lbs.	Freddie	Male	140 lbs.
Linda	Female	330 lbs.	Harry	Male	230 lbs.
Wilma	Female	190 lbs.	Ken	Male	170 lbs.
Ed	Male	180 lbs.	Hilda	Female	220 lbs.
Glenda	Female	290 lbs.	Grumpy	Male	160 lbs.
Bill	Male	230 lbs.	Blackfoot	Female	150 lbs.
			Marcy	Female	170 lbs.
			Grempod	Male	200 lbs.

1. Organize these data in a way that would help you find which kind of bear is heavier--grizzly bears or black bears. (You can use another piece of paper to do this. Please be sure to put your name on it!)

2. Write down three things you can tell about the weights of the bears. (You may want to use your answer from question 1 to help you.)

3. Based on these data, how much heavier is a typical bear of one kind than a typical bear of the other kind?____________________

How did you figure out your answer?

Activity 4 - GRAPHS

The first task in this group is from *Measuring Up: Prototypes for Mathematics Assessment* available from the Mathematics Sciences Education Board. It is representative of the types of tasks found on the Type 3 assessments. It addresses a variety of outcomes from the 4th grade proficiency test (see below), it can be used in a variety of ways (students can work individually or in small groups), it may take more than one class period, it lends itself to a variety of extensions and adaptations. In fact this assessment provides an example of how teachers might begin with an assessment and "extend" the activity to address specific proficiency objectives. As presented the assessment clearly addresses the following outcomes.

PROFICIENCY OUTCOMES ADDRESSED BY THE TASK
4TH GRADE MATHEMATICS

2. Use patterns to make generalizations and predictions by

    a. determining a rule and identifying missing numbers in a sequence;
    b. determining a rule and identifying missing numbers in a table of number pairs;
    c. identifying missing elements in a pattern and justifying their inclusion; and
    d. determining missing numbers in a sequence of numbers or a table of number pairs related by a combination of addition, subtraction, multiplication, or division.

3. Select appropriate notation and methods for symbolizing a problem situation, translate real-life situations involving addition and/or subtraction into conventional symbols of mathematics, and represent operations using models, conventional symbols, and words.

4. Identify needed information to solve a problem.

5. Explain or illustrate why a solution is correct.

24. Make or use a table to record and sort information (in a problem solving setting using simple and complex patterns in nature, art, or poetry as setting) and make identifications, comparisons, and predictions from tables, pictures graphs, bar graphs, and labeled picture maps.

With the suggested extentions the activity can be used as a basis for addressing a much larger group of objectives in a contextual and meaningful way. Scoring of the basic activity would include both objective and "rubric based" scoring. The first question on page 3 of this activity has "right-wrong" answers: a. Graph 4, b. Graph 5, c. Graph 3, and d. Graph 1. Questions 2 and 3 on this page are probably more important and will be more difficult to score. Question 2 is specifically concerned with telling why it IS the graph of heights of fourth graders. Students may start by their telling their own height in inches, and then in most cases explaining that other students are a few inches taller or a few inches than they are. Question 3 may be easier for students than question 2. They would need to explain that graphs 1, 4, and 5 could not be heights because students in fourth grader are "taller" than indicated on the graphs. Graph 2 presents a different point of discussion. Students would need to explain that most fourth grades do not have students who are six feet tall. It is the case that all of the graphs contain a reasonable number of data points. Students may be accustomed to checking this first when interpreting graphs and in this case it will not provide a clue.

The extentions provide suggestions for teachers who are interested in further exploring graphs or in relating graphs to other topics.

Discussion of Extensions

1. "The heights of the students grandmothers" or "The ages of the students grandfathers" would be appropriate answers, of course there are other possible correct answers.
2. Worksheet one provides a circle with 24 sectors. Students could color 8 of the sectors red and label that part of the graph "Zero Cavities." They could color 3 sections yellow, two sections green, three sections blue, five sections orange, and one section each pink, black and brown. Each section should be correctly labeled.
3. Students will have a variety of answers to this problem. Encourage students to make a table or a chart as well as other types of graphs.
4. This activity would require students to make meaningful interpretations of graphs and to extend their understanding of the data by articulating problem situations related to the data.
5. This activity would require students to make meaningful interpretations of graphs and to use these interpretations in communicating about mathematics.
6. This activity asks that students tie the data in the graph to other situations. Students might make up problems such as:

   If each a filling for a cavity costs 30 dollars, how much money has our class spent on fillings?

   What is the total number of years lived by the fourth graders mothers? How many days it that? (Use your calculators)

   If Graph 1 represents the number of people in the fourth graders' families how many people are in our classes families all together?

Graphs

Activity 4

Task 1

Look at the five graphs on the next page. Each graph shows something about a classroom of fourth graders.

1. Which of the five graphs do you think shows:

   a. The number of cavities that the fourth graders have?______

   b. The ages of the fourth graders' mothers?______

   c. The heights of the fourth graders', in inches?______

   d. The number of people in the fourth graders' families?______

2. Explain why you think the graph you picked for c is the one that shows the heights of the fourth graders.

3. Why do you think the other graphs don't show the fourth graders heights.

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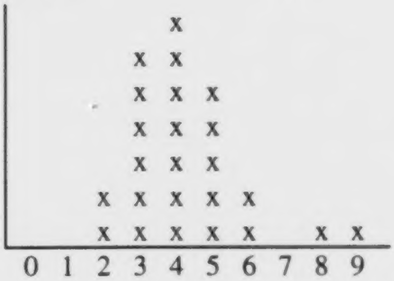
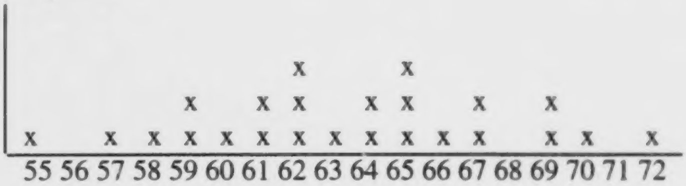
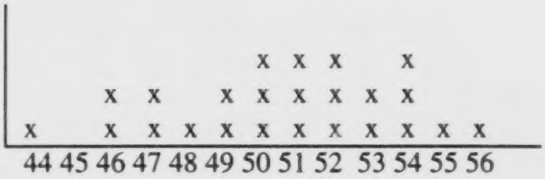
Graph 1



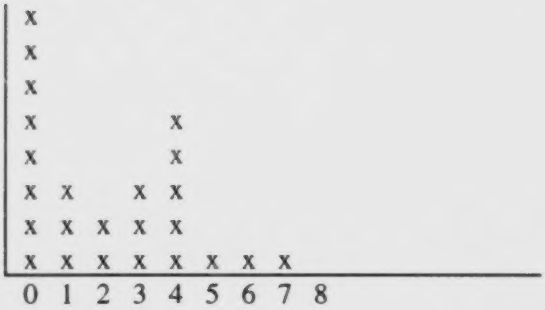
Graph 2



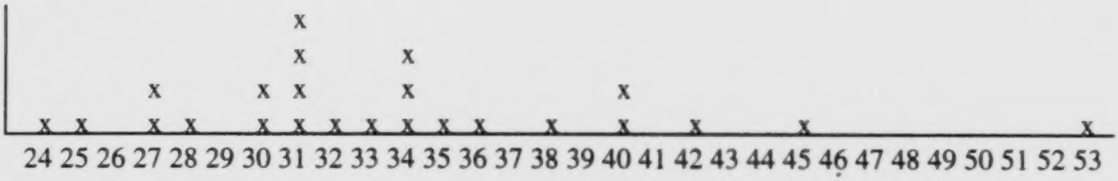
Graph 3



Graph 4



Graph 5



Extentions

What follows are questions, directions or activities for students.

1. Look at graph 2. What would be a reasonable name for this graph?

2. Use Worksheet 1 to create a graph that provides the same information as Graph 4. Be sure to label and title your graph.

3. Choose any graph and make another graph, a table or a chart to represent the same information.

4. Choose any graph and make up four word problems that are related to the graph.

5. Choose any graph and write a "newspaper article" that might accompany the graph.

6. Choose any graph and make up a story problem that uses data from the graph and:

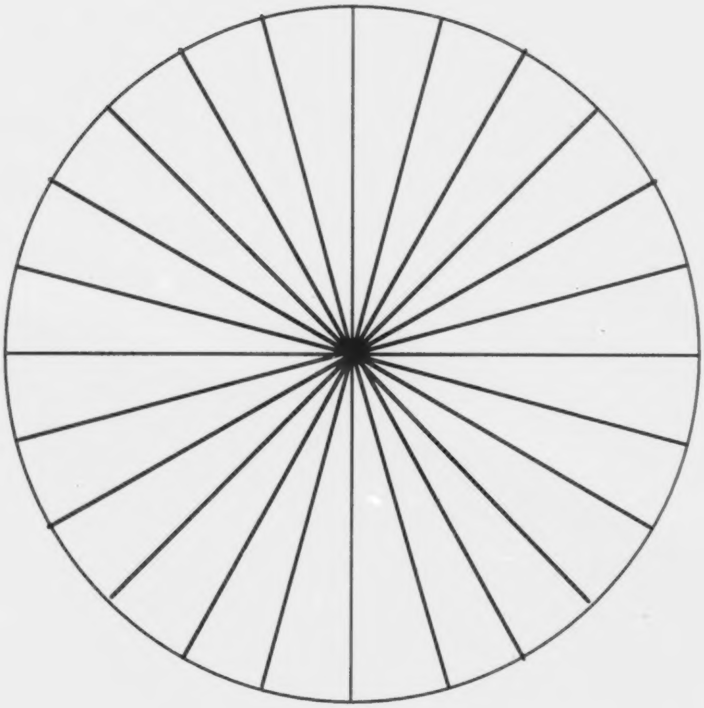
   a. money

   b. time

   c. decimals

   d. multiplication or division

   e. geometry

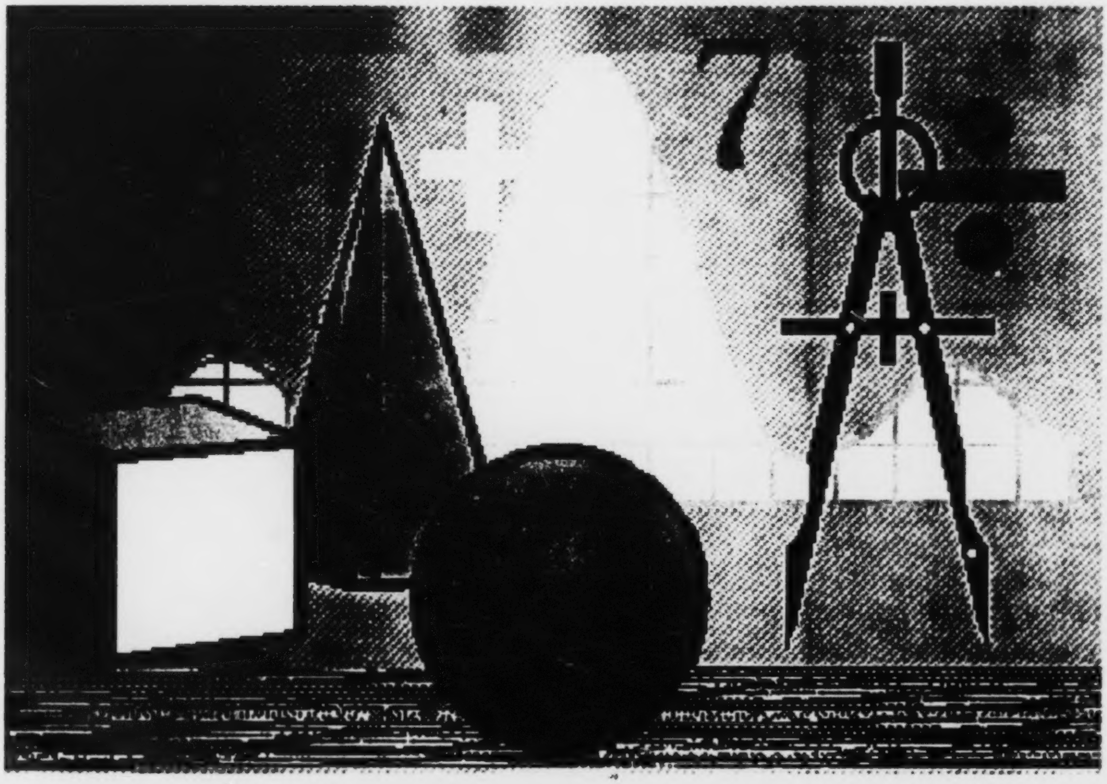


Worksheet 1

Competency-Based Education Assessment Series

Fifth Grade Mathematics

Type 1 Assessment



Developed by

Douglas Darfus

Dan Niswonger

Margaret Kasten

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency test has a traditional design and is intended to be given in one period of approximately 45 to 60 minutes. However, it is also not intended to be a timed test. District or teacher discretion should be used when setting the assessment schedule. Calculators and/or manipulatives are allowed and encouraged to be used by students.

Teacher Notes:

Twenty-seven of the items are multiple choice. It is intended that each of these items would be worth one point.

Items 28-31 require a student-generated response and are worth two points. Items 28 and 31 might be scored with the following suggested rubric:

2 = correct answer and explanation are given
1 = correct answer without thorough explanation or incorrect answer with an appropriate solution method that can produce the correct answer
0 = other

Item 29 could be scored by giving one point for correctly solving the equation (x = 48), and one point for any problem described by that equation. An acceptable answer might be: "After Rudy, Suzi, Bobby and Peggy divided up the candy, they each had 12 pieces of candy. How many pieces of candy were there all together?" An unacceptable answer would be "when you divide by 4 you get 12."

Item 30 requires the student to state two ratios: each should be worth one point.

Item 32 is a more involved item that requires students to draw conclusions or make interpretations from data in a table. The item is worth four points. Districts may have a standard four-point rubric; if so, that should be used in scoring. If not, the rubric on the following page may be adopted or adapted.

Sample rubric for item 32:

0 points
- Nothing is done, except to recopy part or all of the problem.

1 point
- One valid conclusion or interpretation is made with no supporting discussion.
- There is some correct discussion, but no conclusion is drawn.

2 points
- Two valid conclusions or interpretations are given, but there is no discussion.
- Only one valid conclusion or interpretation is given with complete supporting discussion.

3 points
- Two valid conclusions or interpretations are given, discussion is present but incomplete.
- More than two valid conclusions or interpretations are given, but there is no or incomplete discussion.

4 points
- At least two valid conclusions or intrepretations are given with supporting discussion.

Information Sheet

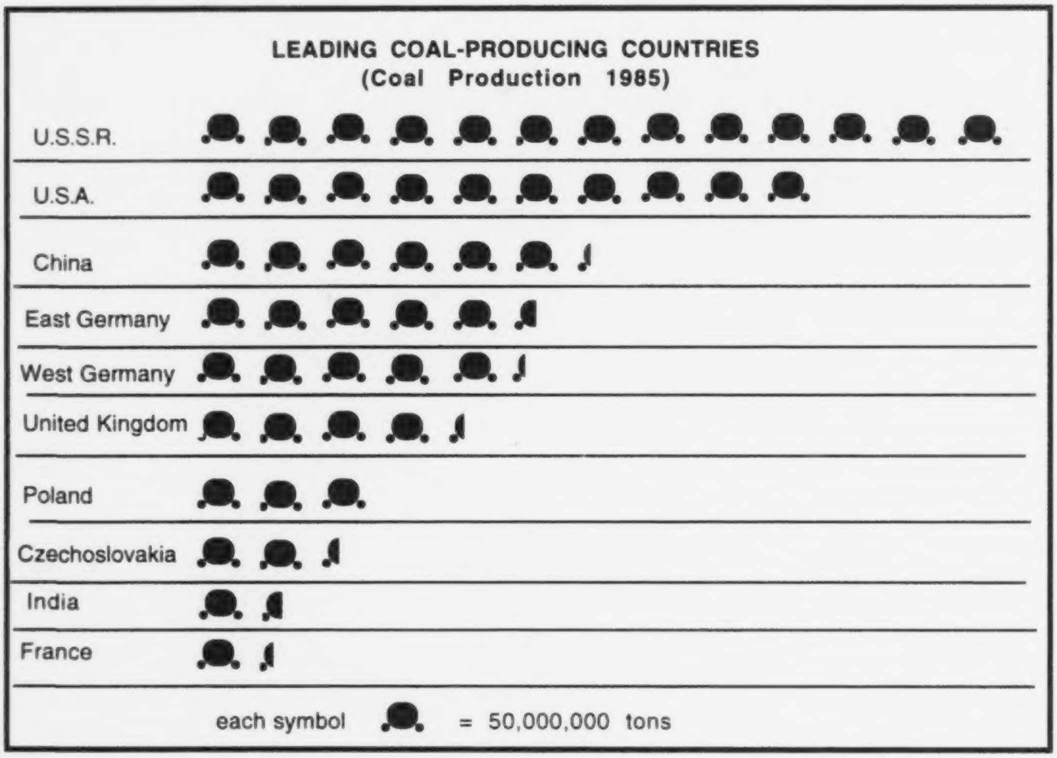
Item	Answer	Objective	Outcome Level *	Critical Objective
1.	C	5-5-4	K/S	Yes
2.	B	5-3-4	K/S	Yes
3.	B	5-7-1	C	No
4.	C	5-6-3	C	Yes
5.	A	5-3-4	C	Yes
6.	A	5-2-2	PS/A	Yes
7.	D	5-8-5, 5-3-3	C, PS/A	Yes, Yes
8.	C	6-3-6	K/S	No
9.	D	6-3-6	K/S	No
10.	C	5-8-5	C	Yes
11.	C	5-6-3	K/S	Yes
12.	D	5-3-6	K/S	Yes
13.	A	5-7-3	C	Yes
14.	D	4-8-4	C	No
15.	C	5-4-1	C	Yes
16.	D	5-4-4	C	Yes
17.	D	5-6-3	PS/A	Yes
18.	B	5-1-2	C	Yes
19.	C	5-3-7	C	Yes
20.	B	5-8-5	C	Yes
21.	B	5-2-2, 5-3-3	PS/A	Yes, Yes
22.	B	5-2-2, 5-3-3	PS/A, C	Yes, Yes
23.	D	5-2-2, 5-3-3	C	Yes, Yes
24.	A	5-3-4	C	Yes
25.	D	5-6-3	C	Yes
26.	D	5-6-3	C	Yes
27.	B	5-3-5	PS/A	No
28.	cube - see "Teacher Notes"	5-6-3	PS/A	Yes
29.	x = 48 - see "Teacher Notes"	5-2-3	PS/A	No
30.	4 to 3, 4/3, or 4:3; 4 to 7, 4/7, or 4:7	3-3-8	C	No
31.	See "Teacher Notes"	3-3-6	C	Yes
32.	See "Teacher Notes"	5-2-2, 5-5-4	PS/A	Yes, Yes

* P S/A = Problem Solving/Application
C = Concept
K/S = Knowledge/Skills

Directions: Circle the letter (A, B, C, or D) corresponding to the correct answer.

1. According to the graph below, how much coal did East Germany produce in 1985?

   A. 25,000,000 tons
   B. 550,000,000 tons
   C. 275,000,000 tons
   D. 250,000,000 tons



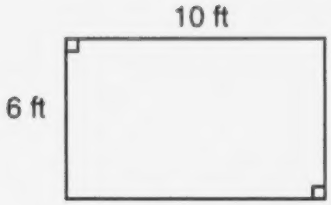
2. In which set are all the fractions greater than $\frac{1}{2}$?

   A. $\frac{2}{3}, \frac{5}{8}, \frac{2}{5}$
   B. $\frac{3}{5}, \frac{5}{9}, \frac{7}{12}$
   C. $\frac{4}{7}, \frac{3}{8}, \frac{7}{9}$
   D. $\frac{3}{4}, \frac{5}{7}, \frac{4}{9}$

3. The unit used in measuring the distance between cities is

A. Centimeters
B. Kilometers
C. Meters
D. Millimeters

4. What is the perimeter of this rectangle?



A. 16 feet
B. 22 feet
C. 32 feet
D. 60 feet

5. If the fraction $\frac{2}{3}$ is changed to an equivalent fraction with 15 as the new denominator, the new numerator must be____.

A. 10
B. 11
C. 12
D. 13

6. Mark needed $74.00 to buy the CD radio he wanted. He already has $38.00. If he wants to get the radio in four weeks, how much must he save on the average each week in order to make his purchase?

A. $ 9.00
B. $ 11.00
C. $ 112.00
D. $ 36.00

7. A dozen oranges were sold for $1.08. What is the price per orange?

A. 12 cents
B. 8 cents
C. 8.25 cents
D. 9 cents

8. Express $\frac{4}{5}$ as a decimal

A. 0.45
B. 4.0
C. 0.8
D. 4.5

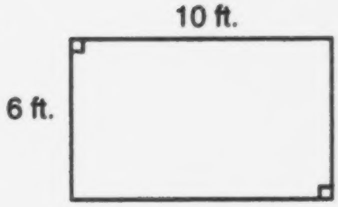
9. Express 0.37 as a fraction

A. $\frac{3}{7}$
B. $\frac{63}{100}$
C. $\frac{7}{3}$
D. $\frac{37}{100}$

10. Figby Middle School contains grades four through eight. If the average enrollment per grade level is 86, what is the total enrollment of Figby Middle School?

A. 91
B. 344
C. 430
D. 460

11. What is the distance all the way around this rectangle?



A. 16 feet
B. 22 feet
C. 32 feet
D. 60 feet

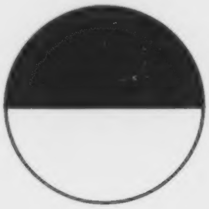
12. Reading left to right, which of these arrangements has the numbers in order from smallest to largest?

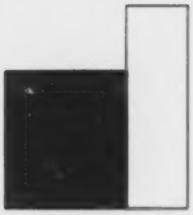
A. $\frac{5}{8}, \frac{3}{10}, \frac{3}{5}, \frac{1}{4}, \frac{2}{3}, \frac{1}{2}$
B. $\frac{1}{4}, \frac{2}{3}, \frac{1}{2}, \frac{3}{5}, \frac{5}{8}, \frac{3}{10}$
C. $\frac{1}{4}, \frac{1}{2}, \frac{2}{3}, \frac{3}{5}, \frac{3}{10}, \frac{5}{8}$
D. $\frac{1}{4}, \frac{3}{10}, \frac{1}{2}, \frac{3}{5}, \frac{5}{8}, \frac{2}{3}$

13. Estimate the answer to $\frac{5}{11} + \frac{4}{7}$.

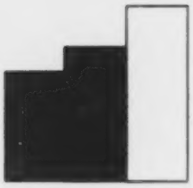
A. 1
B. 2
C. 9
D. 18

14. Which of the following bar graph groups represents the data from the circle graph?

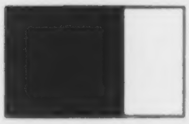




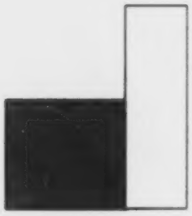
A.



B.



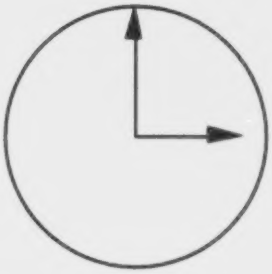
C.



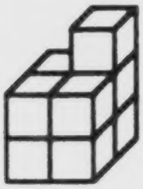
D.

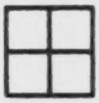
15. The hands of a clock form a 90 degree or right angle at three o'clock. At which of the following times do the hands of the clock form angles greater than 90 degrees?

A. 2:15 p.m.
B. 4:30 p.m.
C. 6:10 p.m.
D. 7:30 a.m.

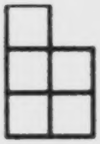


16. Which of the following is not a top view or side view of the solid figure?

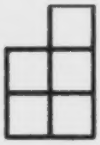




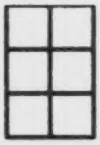
A.



B.



C.



D.

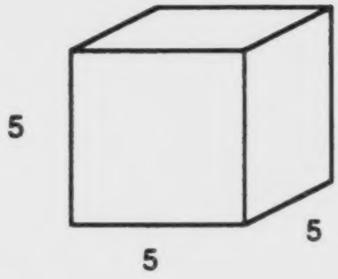
17. You have been asked to wrap the box below for a birthday present. The best dimensions for a single sheet of wrapping paper, allowing for overlap, are:

A. 5 by 16

B. 6 by 14

C. 9 by 18

D. 12 by 22



18. If $\frac{1}{12}$ has a decimal expansion of 0.08333... and $\frac{10}{12}$ has a decimal expansion of 0.8333... , then the decimal expansion of $\frac{1000}{12}$ is

A. 8.333...

B. 83.333...

C. 88.333...

D. 88.8333...

19. Mary and Dale have calculators that use different orders of operation. Both students press these keys on their calculators in the same order: 3 + 2 x 5 =. Mary's calculator displayed 13 as the answer and Dale's calculator displayed 25 as the answer. Who would get the larger answer if they both used the same procedure to do 3 x 2 + 1 =?

A. Mary
B. Dale
C. They would get the same answer on that problem.
D. There is not enough information given to answer the question.

20. At Linn Elementary School, the average height of the second grade class was 37 inches. Over the summer, EVERYBODY grew exactly three inches. What was the average height of the group when they came back to school in the fall?

A. It was still 37 inches because they all grew the same.
B. It was 40 inches because you have to add three.
C. You can't tell the new average without knowing the height of each student.
D. You can't tell the new average without knowing how many students there are in the group.

21. Sarah walks 0.6 mile to school each day in 12 minutes. At that rate, how far can she walk in one hour?

A. 2 miles
B. 3 miles
C. 3.6 miles
D. 7.2 miles

22. The length of Josh's walking pace is 1.5 feet. With that pace length, how many paces does he walk per mile? (Remember one mile = 5,280 feet)

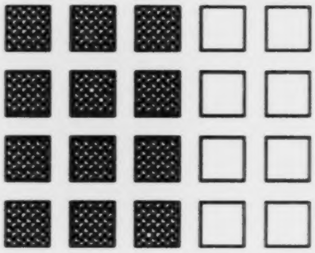
A. 352
B. 3520
C. 5280
D. 7920

23. Tuna costs \$1.29 per can and salmon costs \$3.79 per can. How much would three cans of tuna and two cans of salmon cost?

A. \$ 3.87
B. \$ 5.08
C. \$ 7.58
D. \$ 11.45

24. Which of the following is NOT an equivalent fraction name for the part of the squares shaded below?

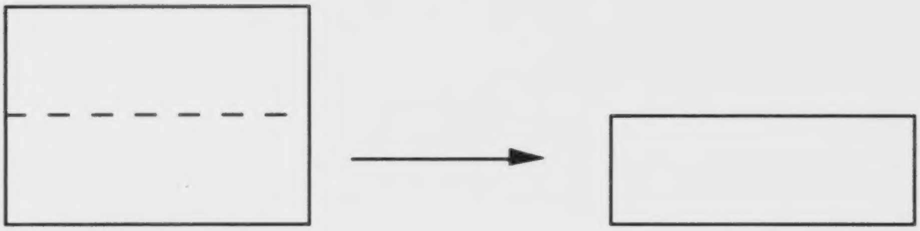
A. $\frac{8}{12}$
B. $\frac{12}{20}$
C. $\frac{3}{5}$
D. $\frac{24}{40}$



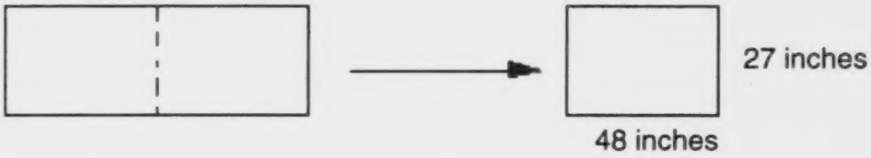
25. A measurement of a rectangular box is given as 48 cubic inches. The measurement could represent the

A. Distance around the top of the box.
B. The length of the edge of the box.
C. The surface area of the box.
D. The volume of the box.

26. Jermaine folded the sheet in half.



Then she folded it again.



How big was the sheet when it was opened?

A. 27 inches X 54 inches

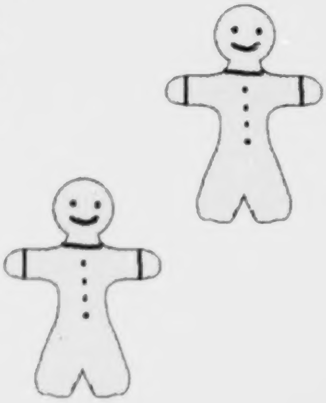
B. 48 inches X 54 inches

C. 27 inches X 96 inches

D. 54 inches X 96 inches

27. Gingerbread people are made by adding water to a mix. The amounts are shown in the table. If you want to make 30 gingerbread people how many cups of water will you need?

Gingerbread People	*Mix*	*Water*
7-8	1 cup	$\frac{3}{4}$
14-15	2 cups	$1\frac{1}{2}$



A. $2\frac{1}{2}$

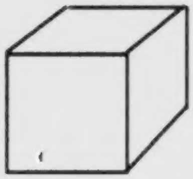
B. 3

C. $3\frac{1}{2}$

D. 4

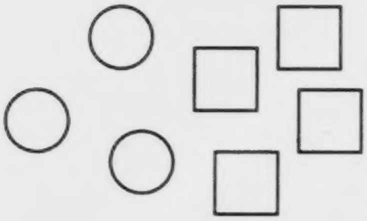
28. Which has the larger volume: an eight-inch cube or a cone with a diameter of eight inches and a height of eight inches?

Explain.



29. Solve this equation: $\frac{x}{4} = 12$. Write a story problem where you would need to solve the equation.

30. Use the pictures to determine the ratios. Write the answers in the spaces provided.



Squares to circles__________________________

Squares to squares and circles________________

31. Write a number that is more than $\frac{7}{8}$ but less than one. Explain how you found the number.

32. Mr. Fisher's sixth grade class took a survey of all 500 students at Susan Q. Hambel Elementary School. The survey was to determine student reading habits. The results of the survey were:

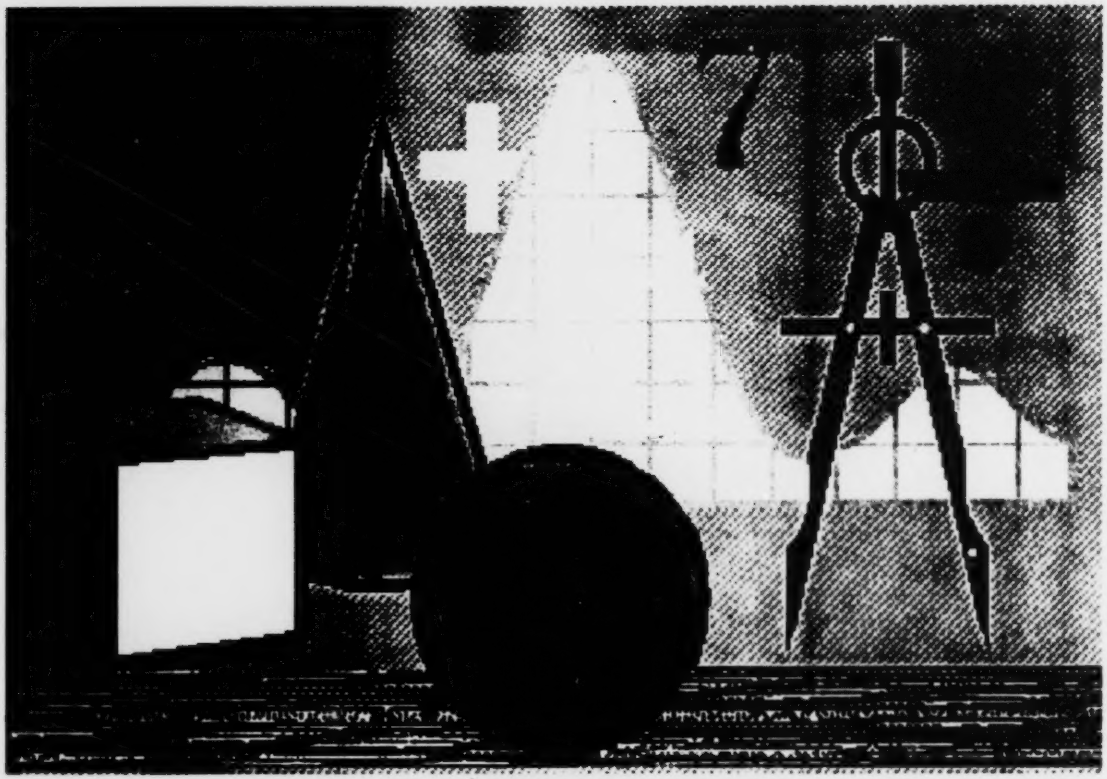
	School Year (Sept.–May)	Summer (June–Aug.)
Read less than one book a month	27	89
Read one book a month	218	111
Read two or three books a month	197	122
Read more than three books a month	58	178

Write a paragraph about two conclusions you can draw from the survey results. Justify your conclusions.

Competency-Based Education Assessment Series

Fifth Grade Mathematics

Type 2 Assessment



Developed by

Douglas Darfus

Dan Niswonger

Margaret Kasten

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

Task Force Members

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Margaret Comstock	The Ohio State University (ret.)
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Jane Stockdale	Columbiana County Schools
Ray Trenta	Akron City Schools

About This Instrument

This model competency assessment instrument is intended to go beyond the traditional multiple choice assessment. The length of time a student will need to complete the assessment may extend beyond the typical 50 minute class period. It is not intended to be a timed test. District or teacher discretion should be used when determining the assessment schedule. The instrument is a mixture of open form items and items requiring extended responses. Calculators and/or manipulatives may be used by students. Some items are direct, straight-forward measures of particular objectives. Some items assess combinations of more than one objective.

The instrument is intended to assess a deeper level of understanding than that being assessed on the Type 1 instrument. Therefore, scoring on this instrument is a bit more complicated than the scoring on the Type 1 instrument. While the entire instrument has a suggested point value of 51, please note that districts are encouraged to consider adapting any or all parts of the assessment, including scoring suggestions.

Discussion

We would suggest that time be taken for class discussion about the problems and solutions after assessment. Additional answers and scoring suggestions:

Item 1: This problem is worth two points. One point should be given for the decimal expansions and one point for the identification of any reasonable pattern.

Item 2: This problem is worth four points.

0 = No work or recopying of problem
1 = Correct answer with no explanation, incomplete or incorrect beginning of a explanation with no answer or incorrect answer
2 = Correct answer with correct but cursory beginning of a set of directions
3 = Correct answer with nearly complete set of directions, only one step or idea omitted
4 = Correct answer, complete set of "directions"

Item 3: This problem is worth two points, two possible combinations give the maximum range: 52.1 X 4.3 and 43.1 X 5.2

2 = Between 224 and 225
1 = Between 214 and 215, between 215 and 216, between 218 and 219, between 220 and 221, between 223 and 224
0 = Any other answer

Item 4: This problem is worth three points, each sum is worth one point.

a. $\frac{1}{2} + \frac{1}{4}$ is one possible answer

b. $\frac{1}{2} + \frac{1}{4} + \frac{1}{16}$ is one possible answer

c. $\frac{1}{3} + \frac{1}{14} + \frac{1}{42}$ and $\frac{1}{4} + \frac{1}{7} + \frac{1}{28}$ are two possible answers (Part c is more difficult and we do not expect every 5th grade student to complete it.)

Item 5: This problem is worth one point: $\frac{2}{3} < .75 < 1$.

Item 6: This problem is worth a total of five points.

Part 1: one of 973, 937, 793 or 739

Part 2: one of the above not used to answer part one

Part 3: one of 379 or 397

Part 4: 4

Part 5: 2

Item 7: This problem is worth four points – two points for each part.

Item 8: This problem is worth four points.

0 = Nothing is done, only an incorrect answer is given
1 = Some interpretation of the given information is made, no subgoal is reached
2 = Evidence of understanding of the problem, a subgoal was reached, correct answer given but it is impossible to tell how it was achieved
3 = Appropriate solution strategy, computational or other minor error or correct answer and justification was weak
4 = Correct answer that has been justified

Item 9: This problem is worth three points, one for each polygon which is correctly drawn.

Item 10: This problem is worth two points.

0 = No work done or recopying of problem
1 = Incorrect answer, but a strategy (computational or drawing) that would have resulted in correct answer with minor error
2 = Correct answer

Item 11: This problem is worth two points.

0 = No work done or recopying of problem, incorrect division of the circle
1 = Correct division of circle, no labeling or labeling and partially correct division of circle
2 = Circle correctly divided and labeled

Item 12: The problem is worth two points.

0 = No work done or recopying of problem
1 = Correct answer without thorough explanation or incorrect answer with an appropriate solution method that can produce the correct answer
2 = Correct answer and explanation are given

Item 13: One point: both length and width must be given 5.5 cm X 11 cm.

Item 14: The problem is worth two points. Zero to 3 correct answers = 0 points; 4 to 7 correct answers = 1 point; and 8 correct answers = 2 points.

Item 15: The problem is worth four points

0 = Nothing is done, only an incorrect answer is given
1 = Some interpretation of the given information is made, no subgoal is reached
2 = Evidence of understanding of the problem, a subgoal was reached, correct answer given, but it is impossible to tell how it was achieved
3 = Appropriate solution strategy, computational or other minor error or correct answer and justification was weak
4 = Correct answer that has been justified

Item 16: 0 points = no attempt or recopying of problem
1 point = correct answer with no work shown; work shown that could have led to correct solution; work shown with incorrect answer due to computational error.
2 points = correct answer given and work shown

Item 17: 0 points = no attempt or recopying of problem
1 point = correct answer with no work shown; work shown that could have led to correct solution; work shown with incorrect answer due to computational error.
2 points = correct answer given and work shown

Item 18: The problem is worth two points for a correct answer. Award one point for a three digit number that meets at least one of the conditions, for example 246. The sum of the digits is 12, but the product is not less than 40.

Information Sheet

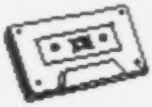
Item	Answer	Objective	Outcome Level *	Critical Objective
1.	See "Dicussion"	5-1-2	C	Yes
2.	$15 -See "Discussion"	5-2-2	PS/A	Yes
3.	224-225 - See "Discussion"	5-3-3	C	Yes
4.	See "Discussion"	5-3-4	C	Yes
5.	$\frac{2}{3} < .75 < 1$	5-3-6	K/S	Yes
6.	See "Discussion"	5-3-6	K/S & C	Yes
7.	See "Discussion"	5-3-7	C	Yes
8.	5 trucks - See "Discussion"	5-3-10	PS/A	Yes
9.	See "Discussion"	5-4-1	K/S & C	Yes
10.	21 cubes - See "Discussion"	5-4-4	K/S	Yes
11.	See "Discussion"	5-5-4	K/S	Yes
12.	area is 70 - See "Discussion"	5-6-3	PS/A	Yes
13.	5.5 cm by 11 cm	5-6-3	PS/A	Yes
14.	5/8, 6/11, 9/17, 6/11; 7/8, 10/11, 9/10, 6/7	5-7-3	C	Yes
15.	See "Discussion"	5-8-5	C	Yes
16.	$ 1.65	5-2-2	PS/A	Yes
17.	$54.00	5-2-2	PS/A	Yes
18.	480	5-2-2	PS/A	Yes

* P S/A = Problem Solving/Application
C = Concept
K/S = Knowledge/Skills

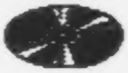
1. Below are the decimal expansions for some fractions with a denominator of 12. When you look at the numbers, you might notice that the decimal expansion in the first row always ends in three; the decimal expansion in the second row always ends in six; and the third row "comes out even." Choose another number, between five and 10. Use it as the denominator, find the decimal expansions, and describe the pattern you find.

$\frac{1}{12}$ = 0.0833...	$\frac{4}{12}$ = 0.3333...	$\frac{7}{12}$ = 0.5833...	$\frac{10}{12}$ = 0.8333...
$\frac{2}{12}$ = 0.1666...	$\frac{5}{12}$ = 0.4166...	$\frac{8}{12}$ = 0.6666...	$\frac{11}{12}$ = 0.9166...
$\frac{3}{12}$ = 0.25	$\frac{6}{12}$ = 0.5	$\frac{9}{12}$ = 0.75	$\frac{12}{12}$ = 1.0

2. Frank bought six cassette tapes and 12 compact disks (CDs) for $222. The CDs all cost the same and each cassette tape costs $7. What was the cost of one CD?



Explain, in words, how you would go about solving the problem above. Solve the problem following your own directions to be sure you have included every step necessary. Revise your explanation if necessary.



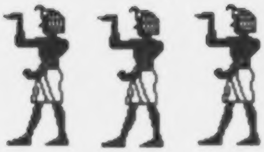
3.

$$\begin{array}{r} \square\,\square\,.\,\square \\ \times\ \square\,.\,\square \\ \hline \end{array}$$

The five digits — 1, 2, 3, 4, and 5 — are placed in the boxes to form a multiplication problem.

If they are placed to give a maximum product, the product will fall between what two whole numbers?

4. Egyptians had special rules for their fractions.



- They used only unit fractions: 1 in the numerator, for example $\frac{1}{2}$ and $\frac{1}{4}$ (except for $\frac{2}{3}$, a special case).
- They expressed numbers like $\frac{3}{4}$ by using only unit fractions: $\frac{1}{2} + \frac{1}{4}$.
- The same unit fraction was never used in an expression more than once.

Think like an Egyptian to provide a name for the following "modern fractions."

a) $\frac{5}{8}$ (using two unit fractions) ________________________

b) $\frac{13}{16}$ (using three unit fractions) ________________________

c) $\frac{3}{7}$ (using three unit fractions) ________________________

5. Place the numbers $\frac{2}{3}$, .75, and 1 in the correct boxes to make the statement true.

$$\square < \square < \square$$

6. Put the digits 9, 7, and 3 in the boxes below so that they form a number that is greater than 468.

☐☐☐ >468

Using the same three digits, 9,7, and 3 make another, different number, that is also greater than 468.

☐☐☐ >468

Now, use the digits 9,7, and 3 to make a number that is less than 468.

☐☐☐ < 468

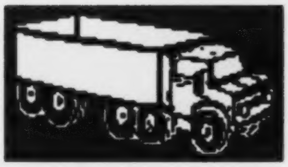
Using the three digits 9, 7, and 3, how many numbers can you make that are greater than 468? ______

7. You and your friend Bobby have different kinds of calculators. The calculators use different orders of operation. Both of you press 5 + 3 x 3 = on your calculators. Bobby's calculator shows 14 as the answer and your calculator shows 24 as the answer.

Make up a problem using at least two different operations where you and Bobby will get different answers.

Make up another problem using at least two different operations where you and Bobby will get the same answer.

8. A group of soldiers needs to be transported by truck to a distant camp. Each truck can carry 36 soldiers at a time. How many trucks are needed if the trucks can make two trips each and there are 300 soldiers to be transported? Explain how you solved the problem and justify your results.

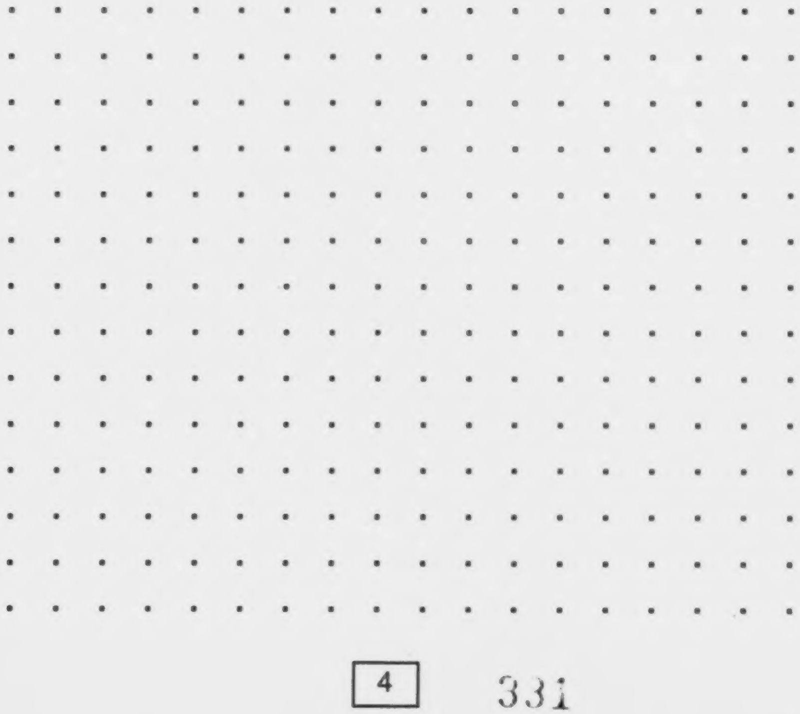


9. Use the dot grid to:

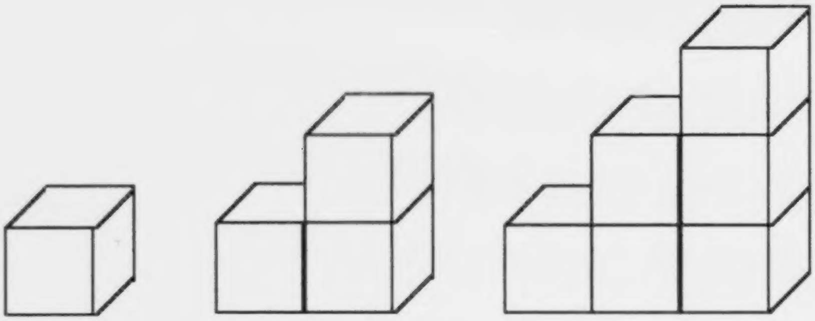
a) Draw a polygon with at least one right angle, label the polygon A.

b) Draw a polygon with all angles less than 90 degrees, and label the polygon B.

c) Draw a polygon with at least one angle greater than 90 degrees, and label the polygon C.



10.



You are building a staircase out of cubes.

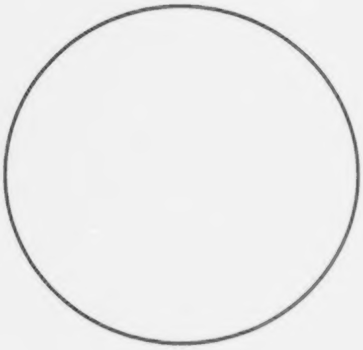
1 step = 1 cube
2 steps = 3 cubes
3 steps = 6 cubes

How many cubes does it take to build a staircase that is six steps high?

11.

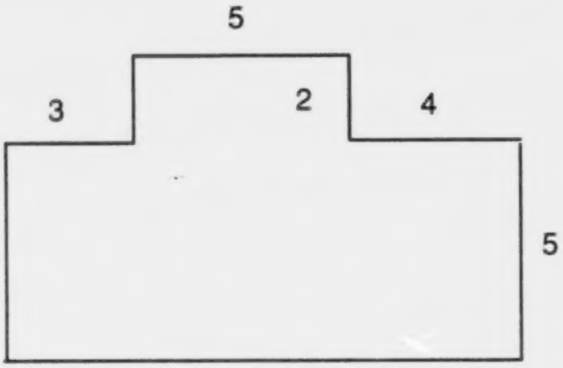
HAIR COLOR SURVEY RESULTS

Color of Hair	Number of Students
Blond	4
Brown	12
Black	8
Total	24

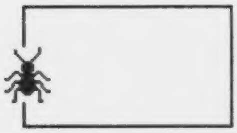


The table above shows the results of a survey of hair color of fifth graders. On the circle, make a circle graph to illustrate the data in the table. Label each part of the circle with the correct hair color.

12. Create another figure with a different shape that has the same area as the given figure. Explain how you know your drawing has the same area as the original figure.



13. An ant travels 33 cm in walking completely around the edge of a rectangle. If the rectangle is twice as long as it is wide, what are the length and width of the rectangle?



14. Finish these fractions so that they are close to, but greater than $\frac{1}{2}$.

$$\frac{}{8} \qquad \frac{}{11} \qquad \frac{9}{} \qquad \frac{6}{}$$

Finish these fractions so that they are close to, but less than one.

$$\frac{}{8} \qquad \frac{}{11} \qquad \frac{9}{} \qquad \frac{6}{}$$

15. Kim doesn't believe that adding a constant (the same number) to every student's test score will simply change the average test score by that same amount. Write an explanation to convince Kim that this is or is not true.

16. The local scout troop sold 4,000 boxes of cookies last year. This year they need to earn $1,600. If they sell the same amount of cookies and the scouts pay $1.25 per box, how much should they charge per box? Show your work.

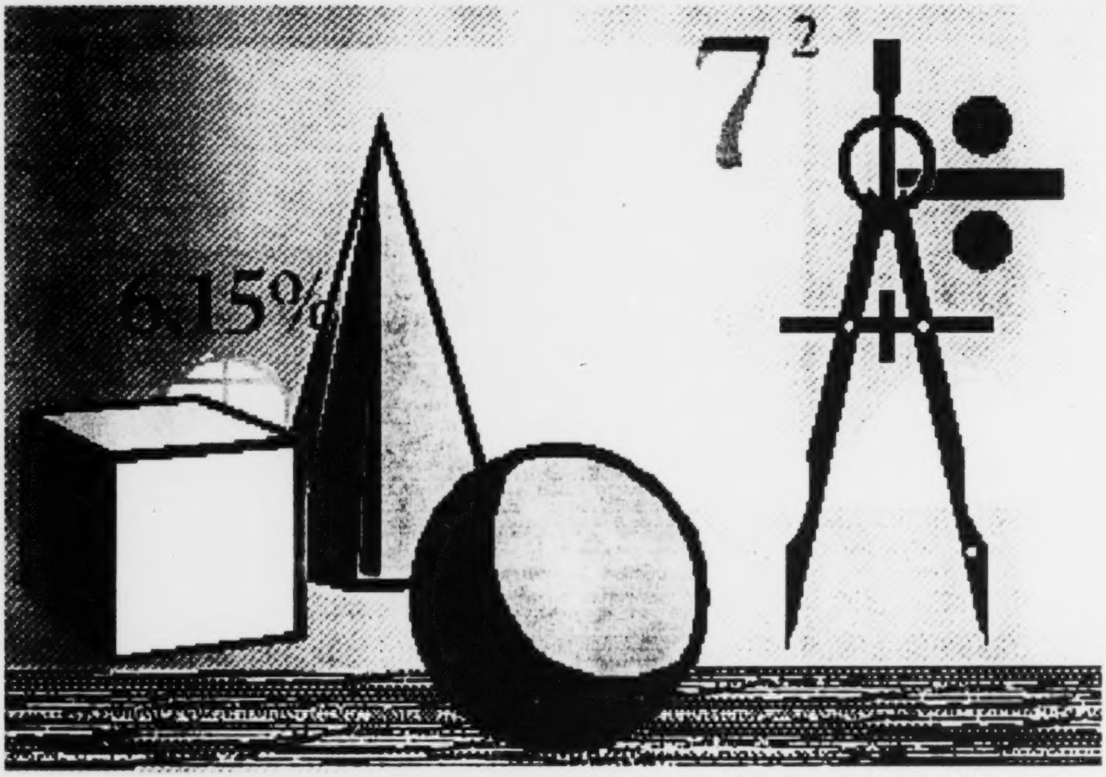
17. Margaret opened her new hat shop on Monday. In the morning, she sold her hats for $3.00 each, totalling $18.00. In the afternoon, she reduced her price to $2.00 each and sold three times as many. How much money should Margaret earn from her first day's sale of hats? Show your work.

18. I am a counting number. All three of my digits are even and different. The sum of my digits is 12. The product of my digits is less than 40. The sum of my hundred's digit, plus my one's digit, is less than my ten's digit. Who am I?

Competency-Based Education Assessment Series

Fifth Grade Mathematics

Type 3 Assessment



Developed by

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Ray Trenta	Akron City Schools

About This Instrument

This model competency assessment instrument is comprised of portfolio tasks keyed to the fifth grade critical objectives form Ohio's *Model Competency-Based Mathematics Program*. These assessments are appropriately used by teachers and districts committed to portfolio assessments. Use of portfolio assessment requires extensive professional development for teachers, specialized instruction and preparation for students and communication with parents and community.

Portfolios offer a good way for students to demonstrate progress in and understanding of mathematics. A portfolio would traditionally include a variety of types of assignments. Those things suggested here are not intended to be a comprehensive or even completely representative set of tasks. An in-depth discussion of the merits of portfolio assessments, appropriate content and formats for portfolios, and the scoring of portfolios exceeds the scope of this document. There are a variety of excellent resources available for those interested in implementing portfolio assessment. Two of those resources are:

> *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions,* edited by Jean Kerr Stenmark and available from the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091
>
> *Assessment Alternatives in Mathematics*, edited by Jean Kerr Stenmark and available from EQUALS, Lawrence Hall of Science, University of California, Berkeley, CA 94720

Various catagorizations for portfolio tasks exist. One such list was developed by the Kentucky Department of Education. Mathematics portfolio entry types are listed as: (1) Investigation/Discovery, (2) Application, (3) Non-Routine Problems, (4) Projects, (5) Interdisciplinary, and (6) Writing. That state department of education also suggests the following as criteria for appropriate portfolio tasks: (1) open-ended, (2) curriculum-based, (3) engages, (4) connects, and (5) extends. District decision to use portfolio assessments must include a commitment to support the process by providing training and time for teachers. Thoughtful consideration of and in-depth dialogue about portfolio entry types and criteria for appropriate folio tasks is essential if portfolio assessment is to be fair and valid. The best portfolio entries are often those that come from the daily curriculum and instruction program. This important point should be central to teacher professional development.

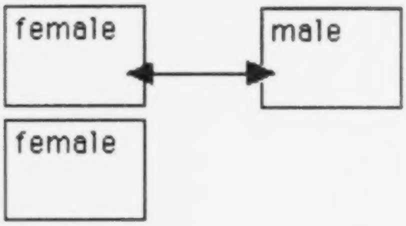
Teachers and districts are invited to consider the following and adopt or adapt the portfolio tasks. These tasks are worthwhile learning opportunities regardless if portfolios are used in the district.

Students should not be left to finish the tasks in this assessment without teacher guidance or assisstance. Other activities should be used in the classroom prior to the tasks in this assessment so students have the mathematical prerequisites required to succeed at these tasks. Examples of the kind of discussion needed is given below for Task 1 only. It has been left to the teacher to use appropriate pre-activities for the other tasks.

Classroom Discussion and Comments for Task 1:

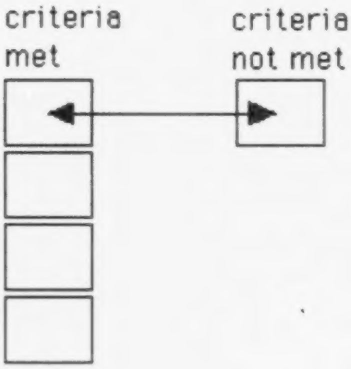
The Fractionville problem could be done as follows: In Fractionville, all adult males are married but only 1/2 of the adult females are married. What fraction of the adult population is married? (or What fraction of the adult population is female?)

A picture can be drawn showing that for each married male there is one adult married female and one single female. So, 2/3 of the adult population of Fractionville is married



You could also change the content of the problem: All 4th graders at Fractionville Elementary take a science proficiency test. All students who needed some extra help preparing for the test were paired with a "study partner" who had already met the science criteria. 1/4 of all the students who had already met the criteria were picked to be one of these "study partners." What fraction of the 4th graders have science study partners? (or What fraction of the 4th graders already met the science criteria?)

Again, with a picture you can show that for each student who hasn't met the science criteria there are 4 students who have met the criteria with only ONE of them serving as a science "study partner." So, 2/5 of these fourth graders have science "study partners" and 4/5 of them met the science criteria.



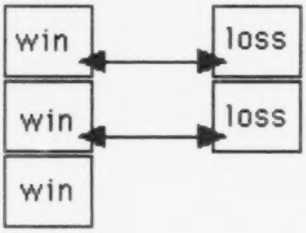
Note: If not interested in the "study partner" relation the previous problem could be reworded as follows:

1) The number of Fractionville Elementary 4th graders who need extra help with the science proficiency is equal to 1/4 the number who have already met the science criteria. What fraction of this 4th grade class have met the science criteria?

2) The number of Fractionville Elementary 4th graders who met the science criteria is 4 times the number who still need help with the science criteria. What fraction of the students still need to meet the science criteria?

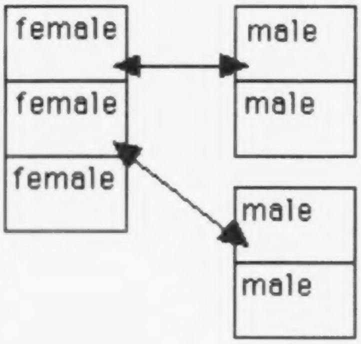
* Next level of difficulty: A baseball team "matches" wins and loses. All of the losses are "matched" but only 2/3 of the wins are "matched." What fraction of the games are "matched?" (or What fraction of the games are won?)

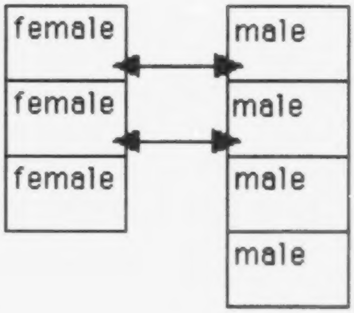
For every two losses there are 3-wins. Two of the wins "match" the two losses but one win stands alone. So 4/5 of games "matched" and 3/5 won.



* Using fractions for the match: In Fractionville 2/3 of the adult females are married but only 1/2 of the adult males are married. What fraction of the adult poulation is married? (or What fraction of the adult population is female?)

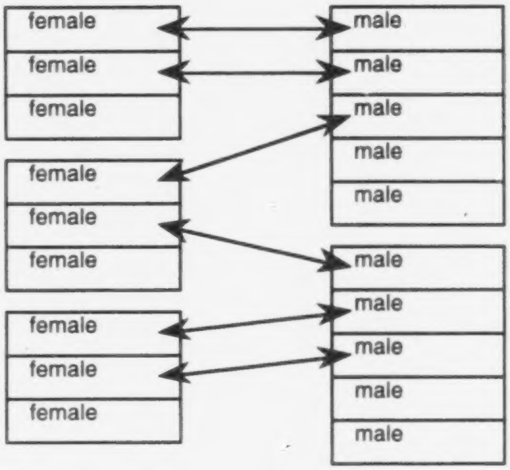
Again a picture is used. 4/7 of the adults are married and 3/7 are female.





The actual question is: In Fractionville, 2/3 of the adult females are married and 3/5 of the adult males are married. What fraction of the adults of Fractionville are married?

Again, a picture can be used to conceptualize the idea:



- Females are added to the picture in groups of 3. Males are added in groups of 5. This continues until 2 out of every 3 females are matched and 3 out of every 5 males are matched.

Note: The previous illustration is just one way to solve the Fractionville problem. There are several methods for solving this problem, some of which are very creative.

Some of the concepts needed prior to the Fractionville problem:

- Amount shared depends on both size of fraction and total amount to be shared. For example, Sam has 1/2 of a pie and Mary has 2/3 of a pie. Sam claims that he has as much pie as Mary. How could that be true?
- Given a fractional amount, find the whole amount. For example, 3/5 of the cost of an item is $75. What is the full price?
- Fractions, multiples and least common multiples.
- The general ability and disposition to use numbers, words, symbols, pictures, charts, graphs, manipulatives and so on to conceptualize, pose, represent, model, solve, compare, contrast, and communicate about problem situations.
- Teachers should go with the rich ideas that come from student discussion. For example, some students find the average of 2/3 and 3/5 in the original Fractionville problem as the fraction of population married. The teacher could explore why or why not this is true.

A Few Notes for the Teacher ...

The Teacher Notes that follow list the objectives that are addressed by each assessment task. There are brief discussion notes for those types of assessment tasks that may be less familiar to most teachers.

TEACHER NOTES: Activity 1

5-2-2: read a problem carefully and identify subgoals that need to be attained in order to solve the problem
5-3-4: find equivalent fractions

Discussion: This task requires an understanding of both equivalent fractions and the relative size of fractions. Students probably will use a variety of strategies to solve this problem. The correct answer is 12/19.

TEACHER NOTES: Activity 2

5-1-2: investigate the patterns of digits formed when fractions are changed to decimal form

Discussion: By fifth grade most students understand that the decimal form of a fraction can be obtained by dividing the denominator into the numerator. They may be especially adept at doing this with calculators.

TEACHER NOTES: Activity 3

5-4-1: compare and contrast angles in relation to right angles
5-4-4: build models of previously encountered shapes and figures and describe the process in words

Discussion: This type of task is rapidly becoming a fairly standard performance measure. Precise use of technical language may not be as important as clear understanding of the spatial ideas embedded in the task.

TEACHER NOTES: Activity 4

5-5-4: interpret tables that describe problem situations
5-8-5: explore the concept of average and calculate the arithmetic mean of a given set of numbers

Discussion: This task will probably take several class periods to complete. It may best be worked on in groups

TEACHER NOTES: Activity 5

5-3-3: multiply and divide decimals

Discussion: Students should be encouraged to develop problems that are substantively different from one another. It is useful to have students share the problems. Discuss which problem is the hardest, which problem is the easiest, and which problem is the most fun.

TEACHER NOTES: Activity 6

5-3-6 order combinations of whole numbers, fractions, and decimals using the symbols, <, ≤, >, ≥, and = and by placing them on the number line

5-3-10 round, as appropriate to a problem situation, to the nearest thousand, hundred, ten, one, tenth, or hundredth

5-7-3 round fractions to 0, 1/2, and 1 and use these values to estimate sums and differences of fractions

Discussion: These tasks might form the basis for class presentations. Students should be encouraged to support their arguments with pictures and/or diagrams.

TEACHER NOTES: Activity 7

5-6-3: determine what to measure and measure in order to determine perimeters, areas, and volumes of simple shapes and solids

Discussion: This is also a multiday task. It would appropriately be done by both groups and individuals.

TEACHER NOTES: Activity 8

5-2-2: read a problem carefully and identify subgoals that need to be attained in order to solve the problem

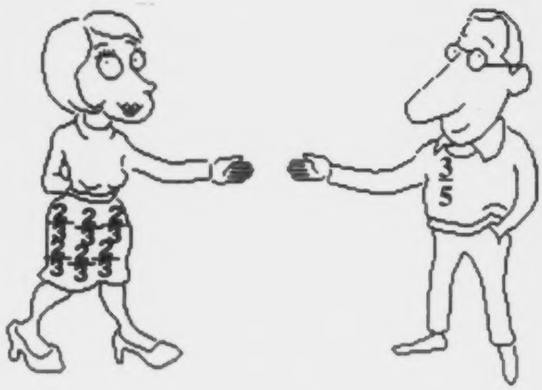
5-2-5: validate and generalize solutions

Discussion: This task could serve as an introduction to a discussion of the importance of percent.

Activity 1

Fractionville

In Fractionville, $\frac{2}{3}$ of the adult females are married and $\frac{3}{5}$ of the adult males are married. What fraction of the adults in Fractionville are married? You may want to draw a picture to help you decide.



Activity 2

Fractions and Decimals

One way to change a fraction to a decimal is to divide the denominator into the numerator. Use a calculator to do this if one is available. Whenever you change a fraction to a decimal one of two things happens. The decimal stops (like $\frac{1}{2} = 0.5$ or $\frac{1}{4} = 0.25$ -- mathematicians say "terminates") or it repeats (like $\frac{1}{3} = .333...$ or $\frac{1}{9} = .111...$).

$\frac{3}{4}$

.333... $\frac{5}{6}$

0.5 $\frac{4}{9}$

$\frac{2}{3}$ *.1212...*

0.25 $\frac{7}{8}$

Write three fractions, with different denominators, as decimals that **terminate**.

	Fractions		Decimal Equivalents
1st fraction:		=	
2nd fraction:		=	
3rd fraction:		=	

Write three fractions, with different denominators, as decimals that **repeat**.

	Fractions		Decimal Equivalents
1st fraction:		=	
2nd fraction:		=	
3rd fraction:		=	

What method did you use for finding the terminating decimals?

What method did you use for finding the repeating decimals?

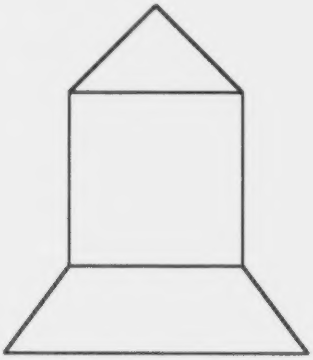
State a rule or rules for finding terminating and repeating decimals?

Repeating decimals are in some ways more interesting than terminating decimals. First of all, they don't always repeat just one number. For instance the decimal form of $\frac{4}{33}$ is .121212... and the decimal form of $\frac{6}{11}$ is .545454... . Sometimes there are "several" digits before a pattern starts to repeat, $\frac{1}{7} = .142857142857142857...$. Before calculators, when people had to divide by using pencil and paper there was great interest in how "far out" one would have to divide before a repeating pattern would begin. It turns out there is a rule. Tell what you would do to try to figure out the rule, and if you can tell what the rule is.

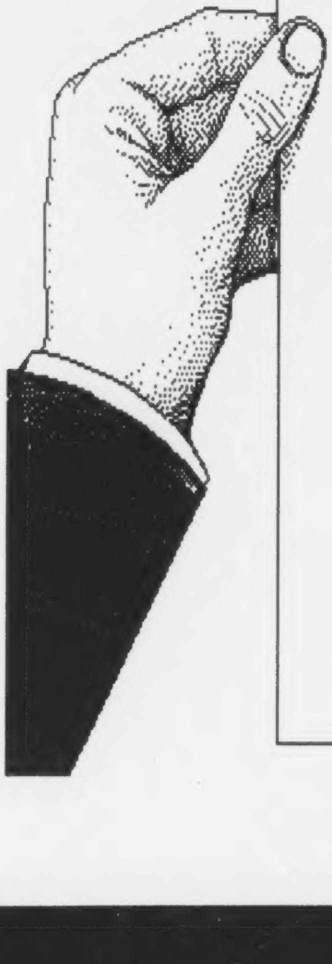
Activity 3

Dear Steve

Write a description to your friend Steve on how to draw the figure shown. Don't use any pictures in your description.



Dear Steve,



Activity 4

Who Should It Be?

There are ten girls on the Fifth Grade Girls Basketball Team. At the end of the season the coach was trying to decide who would get the "Most Valuable Player" award and what four other players would get award ribbons. She looked at the following statistical chart.

Name	Number Games	Total Minutes Played	2 PT Field Goals Attempted	2 PT Field Goals Made	3 PT Field Goals Attempted	3 PT Field Goals Made	Free Throws Attempted	Free Throws Made	Total Points
Lisa	10	181	56	24	5	2	41	21	75
Emily	7	135	49	18	7	0	23	11	47
Sarah	10	104	122	48	8	3	43	19	124
Betsy	5	27	15	3	6	2	5	1	13
Amy	9	110	13	6	0	0	12	7	19
Brandy	8	83	12	4	2	0	6	2	10
Shonda	6	64	15	5	1	0	5	2	12
Liz	10	164	35	10	4	1	8	3	26
Kathy	8	100	19	8	0	0	3	1	17
Chantel	6	32	3	1	0	0	4	1	3

The Coach doesn't want to hurt anyone's feelings, so she would like to base her decisions on the data in the table. The problem is that she is not sure just what information to use.

Use the table to organize the data in a way that will help you decide who should get the awards. Use at least one of the columns and all three if possible. Write down three things that you think should be considered in the selection of the player to be named "most valuable" and the other players to recieve recognition.

NAME			
1.			
2.			
3.			
4.			
5.			
6.			
7.			
8.			
9.			
10.			

Students should use another sheet of paper to answer the following questions:

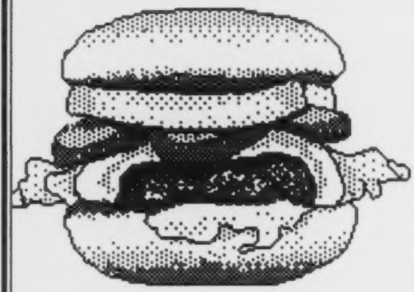
Based on these data, who would you select to be named "most valuable" and what four players should recieve the other recognition?

How did you decide?

Activity 5

McProblems

Make up 3 problem situations that would require the use of the data in the table below. Each problem must require either multiplication or division of decimals. Solve your own problem situations.

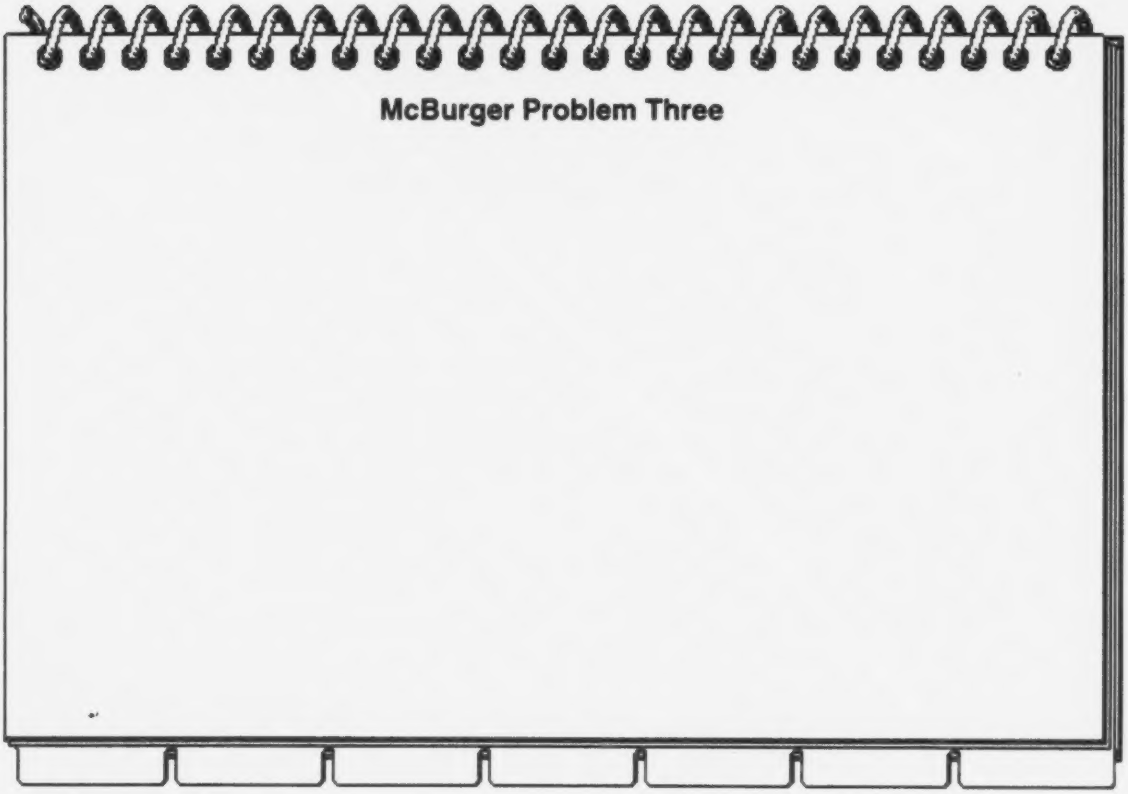


McBurger's Menu

Hamburger	.99
Cheeseburger	1.09
Big Stuff	1.39
Fish-o-let	1.19
Beefsteak	1.79
Cookie	.59
Pie (apple or cherry)	.79
Large fries	.99
Regular fries	.69
Shakes (vanilla, strawberry, chocolate)	1.29
Cola Sugar-free 7-Down Root Beer	.55/.79/.99
Hot Chocolate	.69
Milk	.59
Coffee	.69/.99

McBurger Problem One

McBurger Problem Two



McBurger Problem Three

Activty 6

How Do You Know?

Justify your thinking about mathematical concepts related to estimation and ordering. Write a paragraph to answer each of the following.

a. How do you know that $2/3 < 3/4 < .8$?

b. How do you know that $.56 > .506$?

c. How do you know without doing the computation that the sum 7/8 + 9/10 will be closer to 2 than to any other whole number?

d. How do you know when it is appropriate to "round numbers" and when it is important to use exact amounts? Give an example when "rounding" is better than using exact numbers and give an example when using exact numbers is better than "rounding."

Activity 7

PERIMETERS AND AREAS

Your class has been asked by the principal to figure out how much sod the school should purchase to landscape the "green areas" around the building. There are seven areas that will need to have new sod. Before proceeding with the job, you realize that you need to know the area of each one of these grassy areas. The principal has provided your class with a scale drawing of the area around the school. The seven "green areas" are labeled A through G (look at the next page).

What is the most specific geometric name for each Green Area shape?

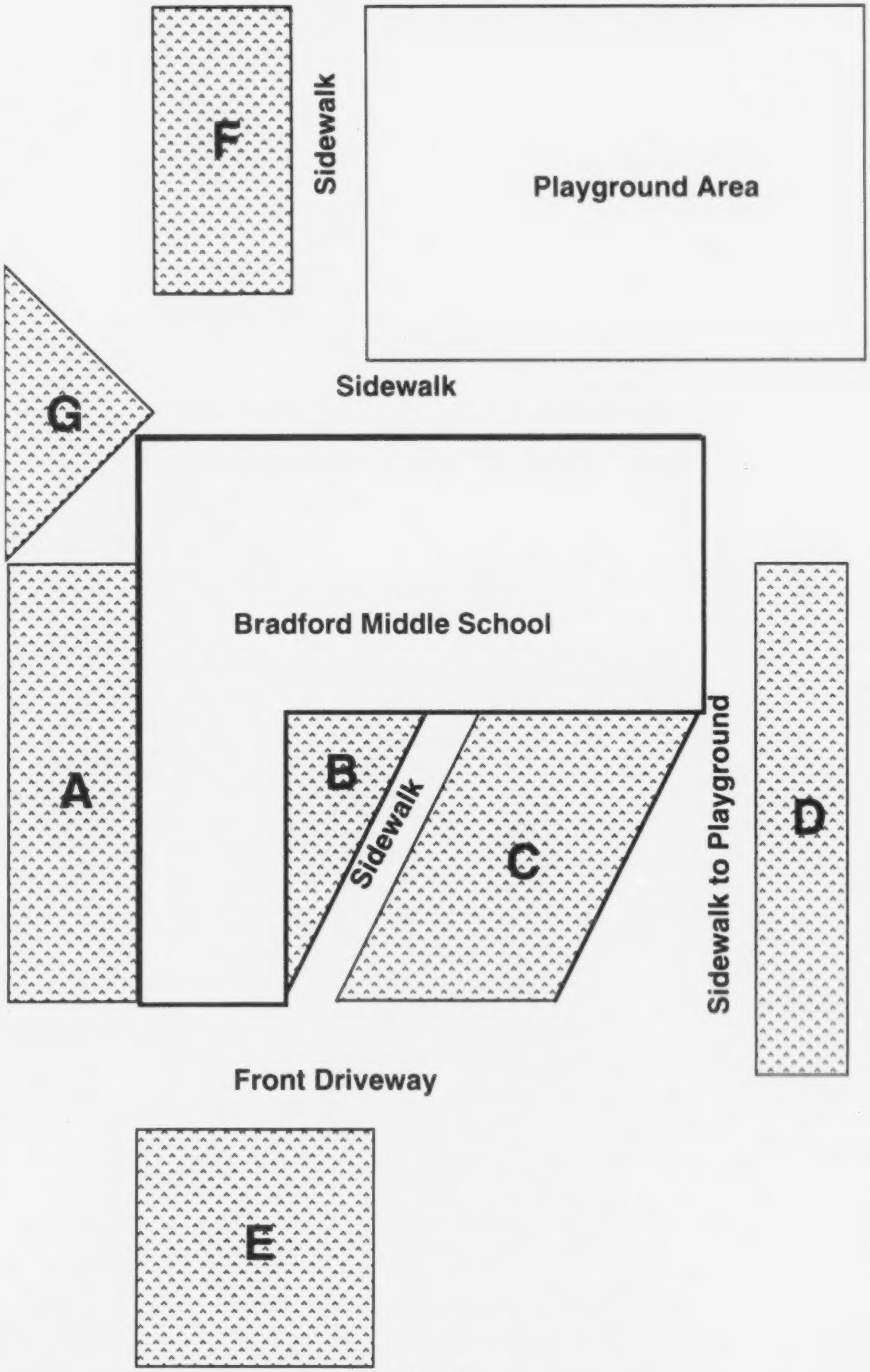
A ______________________ B ______________________

C ______________________ D ______________________

E ______________________ F ______________________

G ______________________

Use the scale drawing on the next page, a centimeter ruler, and centimeter grid to find the perimeter and area of Green Area shapes A, D, and F. Which is largest A or D or F, or do they have the same area? Explain.


F
Sidewalk
Playground Area
G
Sidewalk
Bradford Middle School
A
B
Sidewalk
C
Sidewalk to Playground
D
Front Driveway
E

Investigation: Use the ruler, grid paper and scale drawing to complete the following:

Shape A (model): length (l) = __________; width (w) = __________;
perimeter = __________; area = __________.

Green Area A: actual perimeter = ____________________;
actual area = ____________________.

Shape D (model): length (l) = __________; width (w) = __________;
perimeter = __________; area = __________.

Green Area D: actual perimeter = ____________________;
actual area = ____________________.

Shape E (model): length (l) = __________; width (w) = __________;
perimeter = __________; area = __________.

Green Area E: actual perimeter = ____________________;
actual area = ____________________.

Shape F (model): length (l) = __________; width (w) = __________;

perimeter = __________; area = __________.

Green Area F: actual perimeter = ____________________;
actual area = ____________________.

Conjecture:

In terms of l and w, the perimeter of a rectangle is____________________

and the area of a rectangle is____________________

Conjecture:

In terms of its sides, the perimeter of a square is____________________

and the area of a square is____________________

If the perimeters of two shapes are equal, do they have equal areas? Explain and illustrate.

If the areas of two shapes are equal, do they have equal perimeters? Explain and illustrate.

Investigate with your scale drawing how to find the area of shape B.

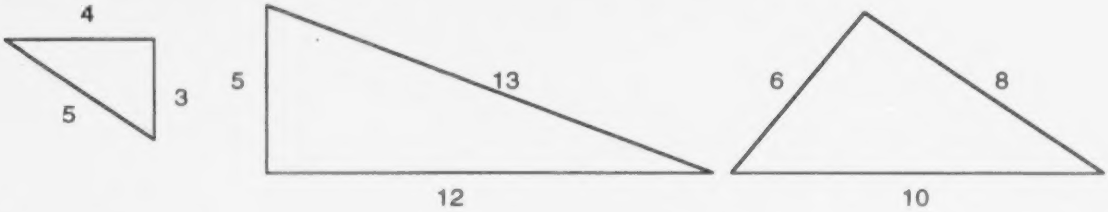
What is the length of its base (b)?_______ Its height (h)_______?

How is shape B related to Shape F? Explain and justify your conclusion.

What is the area of shape B?__________

Does your method for finding the area of shape B work for other right triangles? Why or why not?

Consider the right triangles below. Find the area of each and justify your conclusion.



Conjecture:

The area of a right triangle with base (b) and height (h) =____________

You have 60 meters of fencing and want to fence a garden in the shape of a rectangle against a wall. Draw a picture of the shape of the fenced garden with the largest area (give the dimensions).

Activity 8

Increase or Decrease?

The Facts:

In 1990 the education budget of a certain community was $30 million out of a total budget of $500 million. In 1991 the education budget of the same community was $35 million out of a total budget of $605 million. The inflation rate for that one year period was 10%.

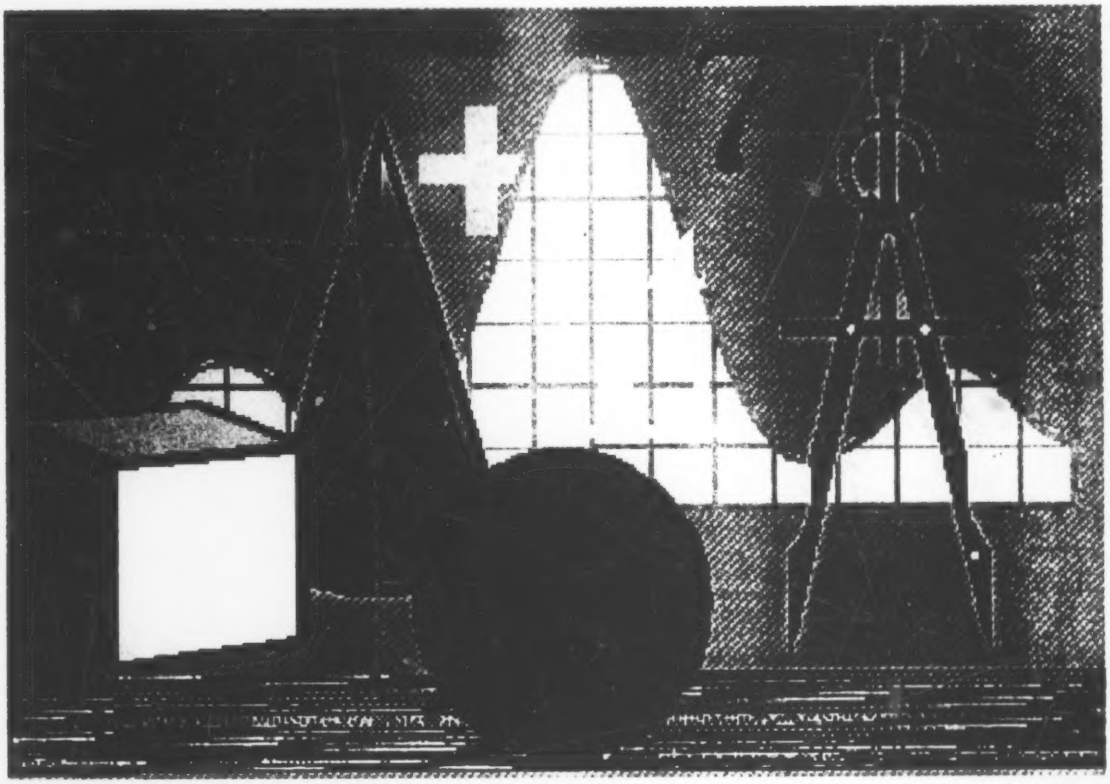
The Tasks:

1. Use the facts to argue that the education budget increased from 1990 to 1991.
2. Use the facts to argue that the education budget decreased from 1990 to 1991.

Competency-Based Education Assessment Series

Sixth Grade Mathematics

Type 2 Assessment



Developed by

William A. Bowman

Linda Diltz

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Steven Meiring

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The model competency-based education assessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency assessment instrument was constructed by a committee of sixth grade teachers who would like to share the following information with other teacher users. The instrument is designed to be given in two parts, each requiring one period of approximately 45 minutes for completion. The instrument is a mixture of more traditional design items (multiple choice), open-form, short answer items, and items requiring extended responses. Some items are direct, straight-forward measures of particular outcomes. Other items assess combinations of two or more outcomes.

Since sixth grade is also a proficiency test grade, we have chosen to align this instrument most closely with the mathematics outcomes from the sixth grade proficiency test (please see ***Sixth Grade CBE Assessment Outcomes*** which follows). These outcomes are chosen (with some modifications) from Ohio's *Model Competency-Based Mathematics Program*, published by the Ohio Department of Education. All references to strands and outcome levels pertain to the strands and outcome levels described in that document.

The ***Information*** pages that follow provide most of the answers to items, cite the relevant outcome(s) assessed by item, specify the outcome level that is being assessed by the item (**K**nowledge, **C**oncept, or **P**roblem **S**olving), and give an instrument summary of items and scoring weight. The ***Discussion*** pages that follow give the answers that require more space, provide item scoring suggestions, discuss possible student responses to items, and offer some teacher comments.

Use of this instrument is voluntary. Its purpose is to provide a model instrument for meeting the CBE (Competency Based Education) requirement to assess student progress relative to the district's CBE outcomes, once during the year by standardized means. Since the district program may deviate from the outcomes and/or philosophy upon which this instrument is based, decisions may be made to modify and/or use any part of this instrument as it matches the district program.

Other decisions may be made concerning the conditions under which assessment occurs. The development committee constructed this instrument on the assumptions that students will have the same access to calculators and appropriate manipulative and measuring tools that they ordinarily have for learning. Some item performances may be negatively impacted by the availability of calculators (#5, 24, 32). Other items will place a difficult computation and time burden on students who do not have access to calculators (#8, 9, 14, 18, 25, 28, 30, 37). Item 40 requires students to use a protractor to measure angles and to produce accurate drawings with rulers. Item 39 requires students to construct a bar graph. Several items portray situations that students might understand more easily if they are able to represent these problems with manipulatives (#10, 11, 12, 20, 31, 34, 36).

Teachers are encouraged to examine carefully the incorrect choices that students select for multiple choice items. Distractors have been created to identify specific sources of student misunderstanding – for example, place value (#1, 26), meaning of operations (#10), and careful reading of questions (#2, 9, 29). Many items are direct measures of specific outcomes at the knowledge or concept levels. Other items reward student conceptual thinking rather than procedural approaches (#7, 8, 37). Several items assess real-world applications of mathematics (#8, 9, 14, 25, 30, 37). And several items assess combinations of ideas in one item (#6, 9, 15, 29, 30, 39).

As a final note of encouragement to teachers, the committee wishes to point out the inherently different character of this type of assessment instrument from other testing means. A large number of outcomes is being assessed that span a whole year's learning. Multiple levels of understanding are assessed through situations that are phrased largely in worded formats. The nature of the items are such that they highlight as much what many students have yet to learn as they validate what students have already learned. Consequently, aggregate scores tend to be lower than those for unit tests or other standardized tests that the district might employ.

The value of this type of assessment is that it can help spotlight gaps in students' knowledge and abilities that need to be addressed as students progress further through the district CBE program. Item analysis and other summaries of group performance can be quite useful in making instructional decisions to strengthen a local program. Evaluation decisions (e.g., grading, promotion) should reflect the much more comprehensive documentation that occurs throughout the school year. Such decisions are best left to the classroom teacher and should be subject to district policies. Whereas the results from this CBE instrument might be used, in part, to inform such decisions, they are not intended for that exclusive purpose or use.

Sixth Grade CBE Assessment Outcomes

The committee of practitioners which determined the outcomes for the sixth grade mathematics proficiency test selected those outcomes from the state mathematics model course of study, *Ohio Competency-Based Mathematics Program*. The committee revised some outcomes and combined others into the following list. We have elected to use the same list and statement of outcomes to develop the CBE assessment instruments, Types 2 and 3 for sixth grade. To correlate these outcomes with those from the model, we are using the code X-Y-Z where the first number X references the grade level from the model, the second number Y the strand from the model, and the third number Z the outcome number from that strand. A designation of (C) following this code identifies the outcome as a critical outcome from the model.

Any word, phrase, or outcome written in *italic print* indicates something which has been added to the sixth grade proficiency outcomes to reflect better their intent or the scope of the sixth grade program.

Number	Outcome Statement	Correlation to Model
	Strand One	
1.	Apply the relation between doubling the side of a regular figure and the corresponding increase in area.	6-1-2 (C)
2.	Determine the rule, identify missing numbers, and/or find the nth term in a sequence of numbers or a table of numbers involving one operation or power.	4-1-2
	Strand Two	
3.	Apply appropriate notations and methods for symbolizing the problem statement and solution process.	4-2-4
4.	Identify needed and given information in a problem situation as well as irrelevant information.	6-2-3 (C)
5.	Validate and/or generalize solutions and problem-solving strategies.	6-2-4
	Strand Three	
6.	Compute with whole numbers, fractions, and decimals.	6-3-1 (C)
7.	Find equivalent fractions.	5-3-4 (C)
8.	Change freely between fractions and decimals.	6-3-6 (C)
9.	Order combinations of whole numbers, fractions, and decimals by using the symbols <, ≤, >, ≥, and = and/or by placing them on a number line.	5-3-6 (C)

10.	Use ratios, proportions, and *percents* in a wide variety of applications.	6-3-3

Strand Four

11.	Visualize and show the results of rotations, translations, reflections, or stretching of geometric figures.	6-4-4 (C)
12.	Recognize, classify, and/or use characteristics of lines and simple two-dimensional figures including circles; and apply models and properties to characterize and/or contrast different classes of figures including three-dimensional figures.	6-4-7 (C)

Strand Five

13.	Use the distributive property in arithmetic computations.	6-5-1
14.	Explain and reflect differences between calculators with arithmetic logic when symbolizing a keying sequence and in the display as each key is pressed.	4-5-3 (C), 5-5-1, and 5-5-2
15.	Use variables to describe arithmetic processes, to generalize arithmetic statements, and to generalize a problem situation.	6-5-3
* *25.*	*Solve linear equations using concrete representations.*	6-5-6 (C)

Strand Six

16.	Determine perimeters, areas, and volumes of common polygons, circles, and solids using counting techniques or formulas.	6-6-1 and 6-6-2
17.	Convert, compare, and compute with common units of measure within the same measurement system.	6-6-3 (C)
18.	Measure angles with a protractor.	6-6-4

Strand Seven

19.	Apply appropriate strategies to find estimates of sums, differences, products, and quotients of whole numbers and determine whether the estimate is greater than or less than the exact result.	4-7-3 and 4-7-4

20.	Estimate the sum, difference, product, or quotient of decimal numbers by rounding, and the sum, difference, or product of fractions and/or mixed numbers by rounding the fractions to 0, 1/2, or 1.	6-7-5 (C)

Strand Eight

21.	Collect data, create a table, picture graph, bar graph, circle graph, or line graph and use them to solve application problems. and 6-8-2	4-8-1,4-8-4, 5-8-1,6-8-1,
22.	Read, interpret, and use tables, charts, maps, and graphs to identify patterns, note trends, and draw conclusions.	6-8-3 (C)
23.	Apply the concept of average and calculate the arithmetic mean, mode, *median, and range* of a given set of numbers.	6-8-4
24.	Make predictions of outcomes of experiments based upon theoretical probabilities and explain actual outcomes.	6-8-9 (C)

---

* Outcome #25 was added to the original list of 24 proficiency outcomes. It properly belongs under Strand V. We elected to preserve the original numbering of outcomes to correspond to the proficiency outcome numbering.

Discussion

Additional Answers and Suggestions

2. Some students will choose distractor B mistaking a "faster" time to mean addition.

7. 1 mile in 7.5 minutes is the same as 2 miles in 15 minutes or 8 miles per hour. Some students will "build" the ratio to answer the question.

8. This item rewards the student who converts to yards first (4 x 3.5). Distractor D is division by 3 instead of 9. Distractor C is the semiperimeter with 6 inches mistaken as 0.6 feet.

9. Distractor A is 1 1/2 times the unit price. Distractor C is (3 x $12.99) + 1/2($12.99).

12. Accept any reasonable procedure such as: (a) remove 5 oz. weight from each side; (b) divide both sides of the balance into two equal groups; (c) since 2 turtles weigh 2 x 11 oz, then 1 turtle weighs 11 oz.

13. A circle graph must represent 100% of whatever it displays – in this case, a complete day. The sum of the hourse indicated is 22, not 24 as required. (Note that a student could answer "Yes" to the first part correctly IF they indicate in their explanation that another 2 hours must be accounted for.

14. The answer of $430. 42 rounds off the payment to the nearest cent. This amount also assumes no interest and 36 equal payments. Assign full credit if the student simply indicates that the selling price must be divided into 36 equal monthly payments.

15. 6 x 2 L = 12 000 mL; 110 students x 100 mL = 11 000 mL. So she will have 12 000 mL of fruit drink and require 11 000 mL for students. She should have 1 000 mL or 1L of drink left over.

17. Students are likely to get this whole chart or not at all. So score 2 points for everything correct; 1 point for 1-3 numbers incorrect; and 0 points for anything else.

Number of bags	1	2	3	5	20
Suckers	3	6	9	15	60
Candy bars	2	4	6	10	40
Bubble gum	5	10	15	25	100

Students can solve this problem by noticing that a row must increase by the same ratio; so the sucker's row is 3 6 9 ? 60. Then they may notice that for a column it is the case that the number of suckers and candy bars equals the number of bubble gums. Having made that identification, they can fill in the other numbers.

18. Count correct for the range either listing it from 54 - 92 or the difference 38.

19. Score 1 point each for the two calculations of area and 1 point for associating the correct part of the distributive property with its area calculation. E.g.

Think of one big rectangle.
$7 \times (6 + 8) = 7 \times 14$
$= 98$ sq. in.

$a \times (b + c)$

Think of two rectangles.
$(7 \times 6) + (7 \times 8) = 42 + 56$
$= 98$ sq. in.

$(a \times b) + (a \times c)$

20. Score 1 point each for parts (a) and (b) and 2 points for part (c).
To answer part (a), students may observe the pattern that there will be 2 rows of 3 and 8 rows of 5.
To answer part (b), students may observe the pattern that there will be 2 rows of 3 and 98 rows of 5.
To answer part (c), students can subtract 6 for the top two rows of 3 and then divide the remaining 62 blocks by 5 to determine that there can be 12 additional rows of five with 2 blocks left over.
There are many other possible patterns and ways of answering this question – using columns instead of rows, for example.

21. Distractor D will identify students who do not regroup as they multiply.

26. Students should notice that there are only three dots for the four numbers; but that .3 and .300 name the same number and therefore must be located at the left-most dot. The largest number remaining is .35 and must therefore be located at the right-most dot. Therefore P must name the position of the remaining number .345.

29. Distractor B might be chosen by students who do not read carefully enough to realize that the average is to be computed only for the days that the school was *open*, four days rather than five.

30. The diameter of the circular garden is 21 feet; so the circumference is $\pi \times 21'$ or $32.97'$. Dividing by 2 ft intervals gives 32.97 which rounds to 33 bush locations.

31. Score 1 point for recognizing that $3\ \square = 138 + 594$ and 1 point for the answer.

32. The student calculator does the calculation from left to right as $(7 + 14) \div 3 \times 6$. The teacher's calculator follows the order of operations and does the calculation as $7 + [\,(14 \div 3) \times 6\,]$. Some calculators *may* give this result as 35.000002 rather than 35.

33. Score this problem according to a 2-point rubric: 2 points for the correct answer and correct reasoning or explanation; 1 point for the correct approach but an a minor computation error; 0 points for anything else.

    Note that students may work this problem several ways: (1) noticing the pattern for successive miles is \$1.50, \$2.50, \$3.50, \$4.50, \$5.50; (2) filling in the amounts in 25¢ increments up to 5 miles; (3) working backward and recognizing that the meter starts at 50¢ for zero miles + 25¢ for each additional quarter-mile; and others. An incorrect answer that will require some explanation (perhaps using a ruler example) occurs if a student adds the amount for a 2-mile trip (\$2.50) to the amount for a 5-mile trip (\$3.50) and gives \$6.00 for a 5-mile trip.

34. Score just one point for part (a); if a student answers the first part of the question correctly, they are likely to answer the second part so as well. Acceptable answers for part (b) should state the notion that there are 7 chances in 21 to draw a green marble – more than any other color; there are 2 chances in 21 to draw a yellow marble – fewer than any other color. Part (c) can be answered by saying, "add or take away marbles so that there is the same number of each color" or any answer that is equivalent to this idea.

35. Score 1 point for part (a) if each ratio is correctly written and identified by person: Ms. Jones, 6 : 40; Principal, 7 : 50. Score 2 points for a correct explanation in part (b) that compares the two ratios by converting to equivalent ratios and identifying Ms. Jones' ratio as higher: 6 to 40 is 3 to 20 or 15 to 100; 7 to 50 is 14 to 100.

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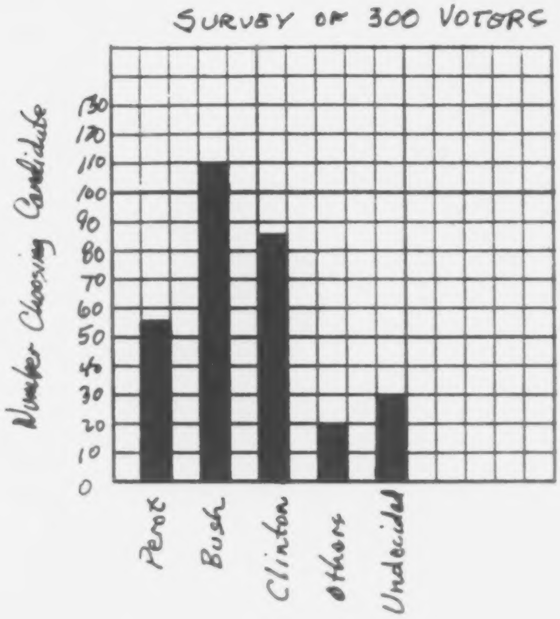
36. Score 1 point for the three correct answers and 1 point for a correct explanation or drawing. Students can answer this question by drawing symbols to represent the pets and the conditions. For example,

the simplest combination that gives	**d**		**d d**
the correct ratios of pets to each	**sss**	but this is only 8,	**sss sss**
other is this:	**cccc**	so trying again	**ccccccc**

37. Score this problem with the same 2-point rubric as question #33. Students who divide this rectangle into squares will see that 12 squares fit along the width and 16 squares along the length which enables them to multiply 12 x 16 to get the correct answer and avoid more difficult calculations.

38. The graph should look something like the following. Score 2 points for bars correctly drawn and identified by candidate and the vertical axis appropriately calibrated. Score a third point for labeling the vertical axis and giving the graph a name. (Students may draw a horizontal bar graph rather than vertical, if they so choose.)



39. Score 2 points for a correct procedure and answer and 1 point for a correct procedure but incorrect calculation. Students may either find the two missing sides by subtraction or recognize that the unknown sides could be "folded out" to make a rectangle of base 6.2 m and height 7.3 m and calculate the perimeter from these values.

40. (a) Score 1 point if each of these values is within 2° of the answers given. (b) Score 1 point for the answer and 1 point for the reason. The answer to (b) *could* vary depending on how the students measured the angles in (a). For example, if a student finds a variation in the measures of angles B and C, then they would correctly select "scalene" as their answer to (b) and supply a corresponding reason. (c)) Score 1 point for the answer and 1 point for the reason. (d) Students must discern that the two triangles are congruent to answer this question correctly. (e) Score 2 points for a correct answer (including labeling the triangle and action) from among these choices of actions: *reflection* - flip original triangle over a line; *contraction* - smaller triangle, similar to original; *expansion* - larger triangle, similar to original.

Information

Item	Answer	Outcome	Outcome Level	Point Value
1.	B	9	K, C	1
2.	A	4, 6	K, PS	1
3.	C	8	K	1
4.	B	10	K	1
5.	C	19	C	1
6.	D	17	K, C	1
7.	A	10	K	1
8.	A	16	PS	1
9.	D	6	PS	1
10.	C	6	C	1
11.	4 vertices; 6 edges	12	K	(a) 1; (b) 1
12.	A turtle weighs 11 oz. See Discussion Page.	25	C	(a) 1; (b) 1
13.	No. See Discussion Page.	4, 21	PS	2 pts; 1 each part
14.	$430.42; see Discussion Page.	4, 6	PS	2 pts; 1 each part
15.	Yes. See Discussion Page.	17	PS	2 pts; 1 each part
16.	10' by 10' square; 100 sq. ft.; 4	1, 16	C, PS	3 pts; 1 each part
17.	See Discussion Page.	2, 10, 22	PS	2 point rubric
18.	(a) 76; (b) 78; (c) 88; (d) 54 - 92 or 38	23	K	4 pts; 1 each part
19.	(a) 98 sq. in.	5, 13, 16	C	(a) 2; (b) 1
20.	(a) 46; (b) 496; (c) 14; See Discussion Page.	2, 5	PS	(a) 1; (b) 1; (c) 2
21.	A	6	K	1
22.	C	10	K	1
23.	B	8, 9	C	1
24.	C	20	C	1
25.	B	8	K	1
26.	D	9	C	1
27.	B	7	C	1
28.	C	6	PS	1
29.	C	23	PS	1
30.	C	16	PS	1
31.	244 ounces	15, 25	K	2
32.	1st uses arithmetic logic; 2nd uses algebraic	14	C	2 pts; 1 each explan.
33.	$5.50; see Discussion Page.	2, 22	K	2 point rubric
34.	(a) green; yellow; see Discussion Page.	24	C	3 pts; 1 each part

Item	Answer	Outcome	Outcome Level	Point Value
35.	(a) Jones 6:40; principal 7:50; (b) Jones higher	7, 10	C	(a) 1; (b) 2
36.	(a) 6 snakes; (b) 2 dogs; (c) 7 cats	3, 10	PS	2 point rubric
37.	192 tiles	16	PS	2 point rubric
38.	See Discussion Page.	21	K	3 point rubric
39.	27 m	6, 16	PS	2
40.	(a) 52°-64°-64°; (b) D; two equal angles (c) A; each angle less than a right angle; (d) A (e) various answers	11, 12, 18	K, C	(a) 1; (b) 2; (c) 2; (d) 1; (e) 2

75 points possible

Items by Outcome Level	K: 15 C: 15 PS: 15	*Point Weighting by Strand* *	1: 7.5% 2: 9% 3: 29%	4: 11.5% 5: 9% 6: 14%	7: 3% 8: 18%

* Sum is 101% due to round-off error.

Part One

Circle the correct answer for questions 1 to 10.

1. Which of these numbers is closest to zero?

A. .06

B. .026

C. .206

D. .260

2. Chris and Joni ran a 50 meter race. The wind was blowing against them at 1.8 kilometer per hour. Chris finished with a time of 6.5 seconds. Joni's time was 0.43 seconds faster. What was Joni's time for the race?

A. 6.07 seconds

B. 6.93 seconds

C. 7.21 seconds

D. 10.8 seconds

3. The meteorologist reported that 0.79 of an inch of rain fell in a 30-minute time period. This is closest to which fraction of an inch below?

A. $\frac{2}{3}$

B. $\frac{3}{4}$

C. $\frac{4}{5}$

D. $\frac{5}{6}$

4. Angie's volleyball team played 20 games and won 65% of those games. How many games did they win?

A. 11

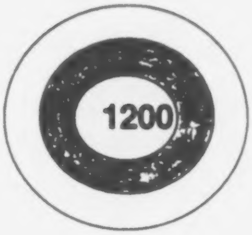
B. 13

C. 15

D. 85

5.

426		19	
	62		32



Which two numbers from the box above give a product closest to the target number 1200?

A. 62 and 32

B. 426 and 19

C. 62 and 19

D. 426 and 32

6. What fraction of a yard is 2 feet 4 inches?

A. $\frac{3}{4}$

B. $\frac{4}{5}$

C. $\frac{2}{3}$

D. $\frac{7}{9}$

7. If Betsy's boat travels one mile in 7.5 minutes, how many miles will her boat travel in one hour?

A. 8

B. 13.3

C. 67.5

D. 450

8. Martha wants to put carpeting in her bedroom. The length of her room is 12 feet and the width is 10 feet 6 inches. How much carpet will she need to buy?

A. 14 square yards

B. 10.5 square yards

C. 22.6 feet

D. 42.4 square feet

9. Joe went to *CD Express* to take advantage of their big sale. The sale stated:

Buy three CDs and get three more for half off of the regular price.

The regular price of CDs is $12.99. How much did it cost Joe to buy six CDs?

A. $ 19.49

B. $ 38.97

C. $ 46.47

D. $ 58.46

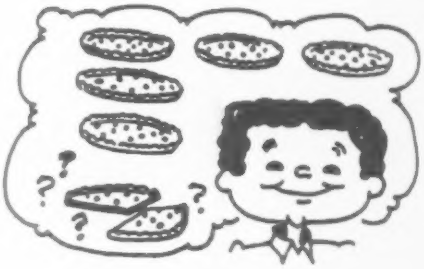
10. When you divide 5 by $\frac{2}{3}$ you get $7\frac{1}{2}$.
What does the $\frac{1}{2}$ actually mean?

A. $\frac{1}{2}$ of one

B. $\frac{1}{2}$ of 5

C. $\frac{1}{2}$ of $\frac{2}{3}$

D. $\frac{1}{2}$ of 7

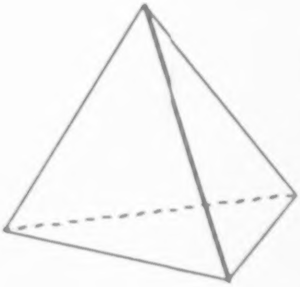


Hint: Think of eating pizzas.

Answer questions 11 to 20 in the space provided. Show your work.

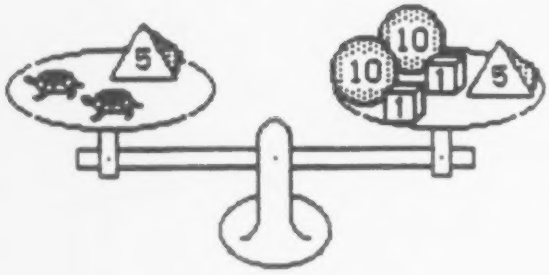
11. Study the picture of a solid pyramid shown to the right. It has four faces.

a) How many vertices does it have?



b) How many edges does it have?

12. Study the balance shown at the right. All weights are in ounces and all similar objects weigh the same.

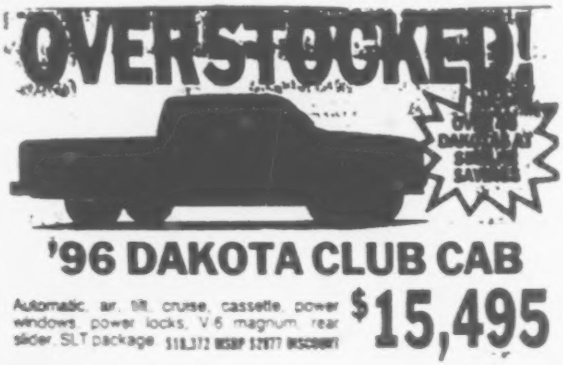


a) Explain how you could find the weight of one turtle.

b) How much does one turtle weigh? ___________

13. Erin wants to make a circle graph to represent a typical day of activities. She includes 7 hours of sleeping, 1 1/2 hours of eating, 7 hours of school, 2 hours of basketball, 2.5 hours of homework, and 2 hours of other activities. Will her circle graph be accurate? Why or why not?

14. You want to buy the car that you see in the advertisement below. How much will your monthly payments be if you pay for the car over a three-year period of time? Explain your solution.


OVERSTOCKED!

'96 DAKOTA CLUB CAB

Automatic, air, tilt, cruise, cassette, power windows, power locks, V-6 magnum, rear slider, SLT package $18,372 MSRP $2877 DISCOUNT **$15,495**

15. Jennifer is doing a survey to determine how many of the students in her grade will like a new fruit drink. She plans to bring to school six 2-liter bottles of the fruit drink. Will she have enough to give each of the 110 sixth grade students a sample cup containing 100 milliliters of the fruit drink? Explain your answer.

16. George has a square garden with an area of 25 square feet. He doubles both the length and width of his garden.

    a) Draw and label a diagram to show George's garden after its length and width are doubled.

    b) What will be the new area of his garden? ____________

    c) How many times greater is his garden now than it was? ____________

17. Jenny is putting together bags of candy. She made a chart to represent her data but forgot to take the paper out of her pocket before her jeans were washed. Below are pieces of her chart. Fill in the missing numbers.

Number of bags	1	2	3		
Suckers	3		9		60
Candy bars		4		10	
Bubble gum		10			100

18. There are nine students in Ms. Garcia's math class. Their scores on the last quiz are listed in the chart shown.

Student	**Score**
Tom	78
Eliza	65
Kim	54
Eric	88
Lisa	92
Sam	75
Chris	60
Sara	88
Juanita	83

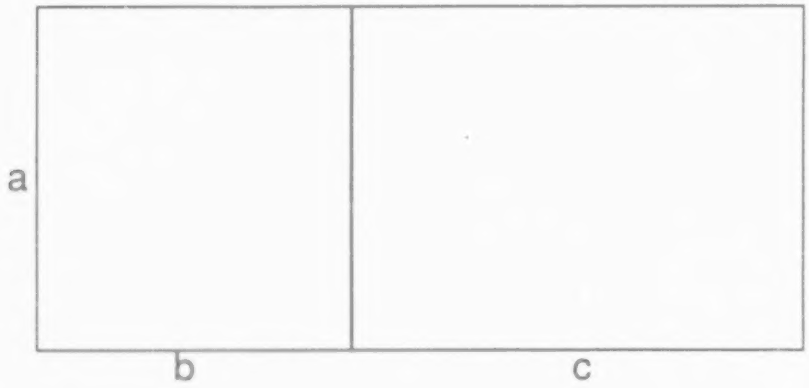
a) What is the mean (average) score?

b) What is the median score?

c) What is the mode score?

d) What is the range for the scores?

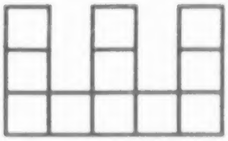
19.



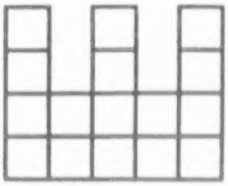
a) Show two ways to find the area of the rectangle if: $a = 7$, $b = 6$, $c = 8$.
All measurements represent inches.

b) The distributive property is: $a \times (b + c) = (a \times b) + (a \times c)$
Match each side of this equation with one of the ways that you calculated the area in part (a).

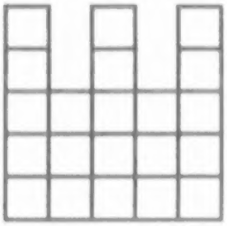
20. A wall 3 blocks high looks like this and takes 11 blocks to build.



A wall 4 blocks high looks like this and takes 16 blocks to build.



A wall 5 blocks high looks like this and takes 21 blocks to build.



a) How many blocks are needed to build a wall 10 blocks high? ____________
Show your work.

b) How many blocks are needed to build a wall 100 blocks high? ____________
Show your work.

c) How high of a wall could you build with 68 blocks? ______________
Why do you think so?

Part Two

Circle the correct answer for questions 21 to 30.

21. What is .4 of a quarter?

    A. $ .10

    B. $ .15

    C. $ .65

    D. $ .820

22. If $\frac{7}{8} = \frac{a}{6}$, then a = ?

    A. 42

    B. 4.5

    C. 5.25

    D. 48

23. Which of the following is true?

    A. $\frac{1}{4} > .25$

    B. $\frac{5}{5} > .62$

    C. $\frac{1}{6} > 1.5$

    D. $.9 > \frac{3}{2}$

24. Choose the best estimate for this fraction sum: $\frac{9}{10} + \frac{4}{9}$

    A. 5

    B. 2

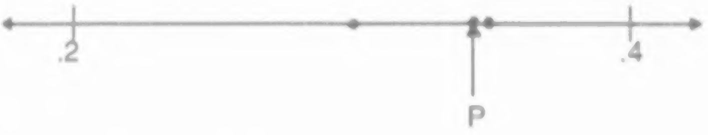
    C. $1\frac{1}{2}$

    D. 1

25. Ramon Gonzalez has been at bat (AB) 393 times during his baseball career. He has made 97 hits (H). Ramon's batting average is determined by finding the ratio of "Hits to At Bat" (H/AB). State Ramon's batting average as a decimal to the thousandths place.

A. 0.198
B. 0.247
C. 0.393
D. 0.490

26. If the numbers { .35 .3 .300 .345 } are put in order on the number line below (at the dots), which number would be located at P?



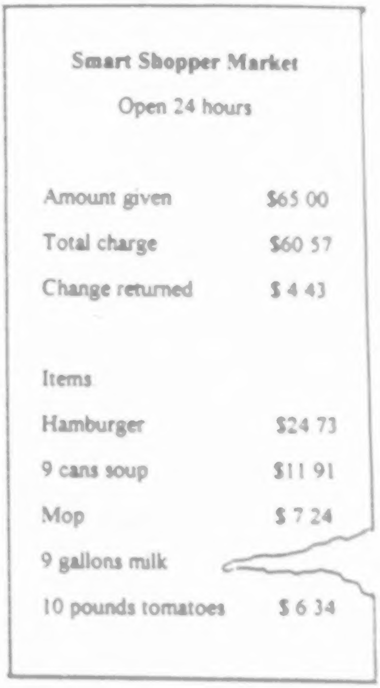
A. .35
B. .3
C. .300
D. .345

27. Ashley ran 3/4 mile on Monday, 7/8 mile on Wednesday, 2/3 mile on Thursday and 4/5 mile on Saturday. On which day did she run the greatest distance?

A. Monday
B. Wednesday
C. Thursday
D. Saturday

28. The grocery tape at the right was torn. How much was spent for milk?

A. $ 4.43
B. $ 5.92
C. $ 10.35
D. $ 14.78



Smart Shopper Market

Open 24 hours

Amount given	$65 00
Total charge	$60 57
Change returned	$ 4 43
Items	
Hamburger	$24 73
9 cans soup	$11 91
Mop	$ 7 24
9 gallons milk	
10 pounds tomatoes	$ 6 34

29. Mr. Turner's sixth grade class made a survey for the school cafeteria. On Monday, 14 out 30 students bought lunch. On Tuesday 9 of them bought lunch. On Wednesday one-third of the class bought lunch. School was closed on Thursday due to weather, but on Friday 7 students bought lunch. All students were present each day.

    What was the average number of students that bought lunch each day that the school was open?

    A. 6

    B. 8

    C. 10

    D. 12

30. Diane wants to plant bushes around the edge of a round flower garden. The radius of the garden is 10 1/2 feet. How many bushes should she buy if she wants to plant one every two feet?

    A. 16

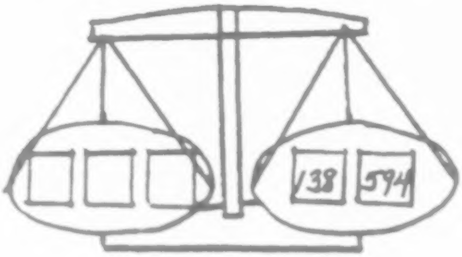
    B. 21

    C. 33

    D. 173

Answer questions 31 to 40 in the space provided. Show your work.

31. The unknown boxes on the left side of the scale weigh the same amount in ounces. Find the weight of each box if the scales balance. Show your work.



32. You pushed the following buttons on your calculator:

7 + 14 ÷ 3 x 6 = **Result 42**

Your teacher told you that her calculator showed a different answer when she used the same keying sequence:

7 + 14 ÷ 3 x 6 = **Result 35**

Explain why the results are different by telling how the two calculators operate.

33. The table below shows the amount of fare a taxi driver charges (before tip) for varying length rides. Find the cost of the fare for a 5-mile trip.

Miles	1	1 1/4	1 1/2	1 3/4	2	2 1/4	2 1/2	2 3/4	3
Cab Fare	$1.50	$1.75	$2.00	$2.25	$2.50	____	____	____	____

34. A bag contains these marbles: **2 yellow, 6 red, 3 blue, 7 green, 3 white**

a) If you picked a marble from this bag without looking, which color are you most likely to pick? _______________

Which color are you least likely to pick? _______________

b) Explain your answers to **part a.**

c) What would you have to do to make the chances equally likely for drawing any color?

35. The school principal made 7 baskets out 50 free throw shots attempted with a basketball. Ms. Jones, the sixth grade teacher, made 6 baskets out of 40 free throw shots attempted.

    a) Write a ratio for each person comparing the number of baskets made to their total number of free throw shots attempted.

    b) Who had the higher ratio of baskets made, the principal or Ms. Jones? Explain your answer.

36. Raul has more pets than any student in his class. Find out how many pets of each type he has by using the clues given. Show or explain how you got your answers.

Raul's Collection

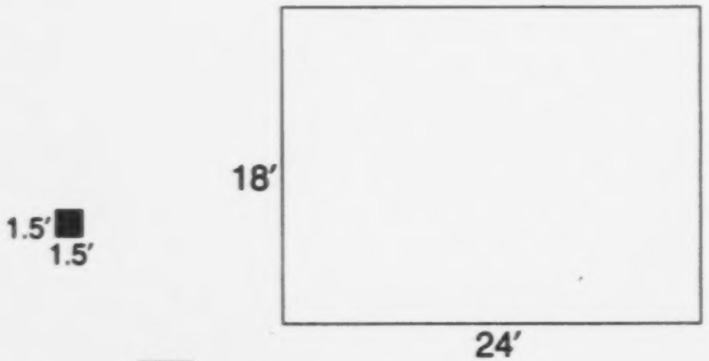
3 times as many snakes as dogs
1 more cat than snakes
15 pets in all

a) How many pets are snakes? _________

b) How many pets are dogs? _________

c) How many pets are cats? _________

37. The large rectange below represents a floor. The small square represents a tile. How many tiles will it take to completely cover the floor if there is no space between the tiles?

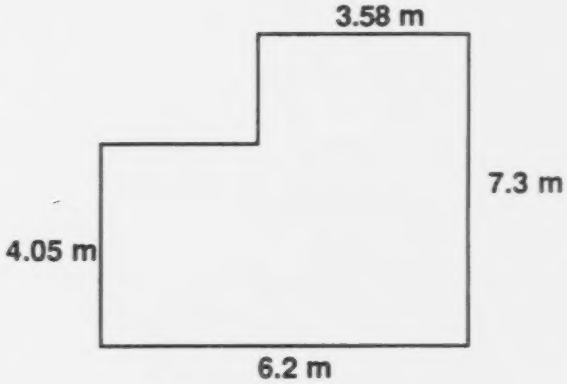


38. The table at the right shows the results of a survey of 300 registered voters on their presidential choice before the 1992 election. Make a bar graph in the space below to display this data. Label all parts of the graph and show the units chosen on the axis to indicate the lengths of the bars.

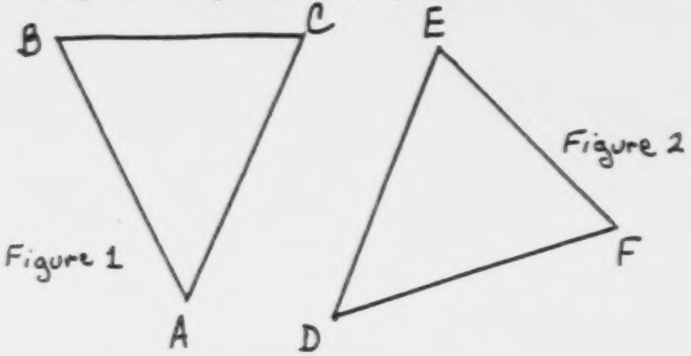
SURVEY RESULTS FOR 300 REGISTERED VOTERS

Candidate	No. Favoring
Perot	55
Bush	110
Clinton	85
Others	20
Undecided	30

39. What is the perimeter of the playground region shown to the right? Show your work.



40. Given the following two triangles, answer questions a. - e.



a) Measure the number of degrees in each angle of the triangles.

∠A = __________

∠B = __________

∠C = __________

∠D = __________

∠E = __________

∠F = __________

b) The triangles above can be classified as __________ triangles.

A. scalene
B. equilateral
C. quadrilateral
D. isosceles

Why do you think so? Explain your thinking.

c) The triangles above can also be classified as __________ triangles.

A. acute
B. obtuse
C. reflex
D. straight

Why do you think so? Explain your thinking.

d) To change **Figure 1** into **Figure 2** is an example of a(n) ___________ .

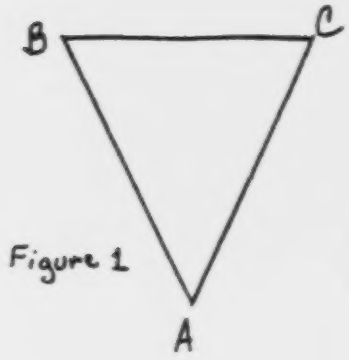
A. rotation

B. reflection

C. contraction

D. expansion

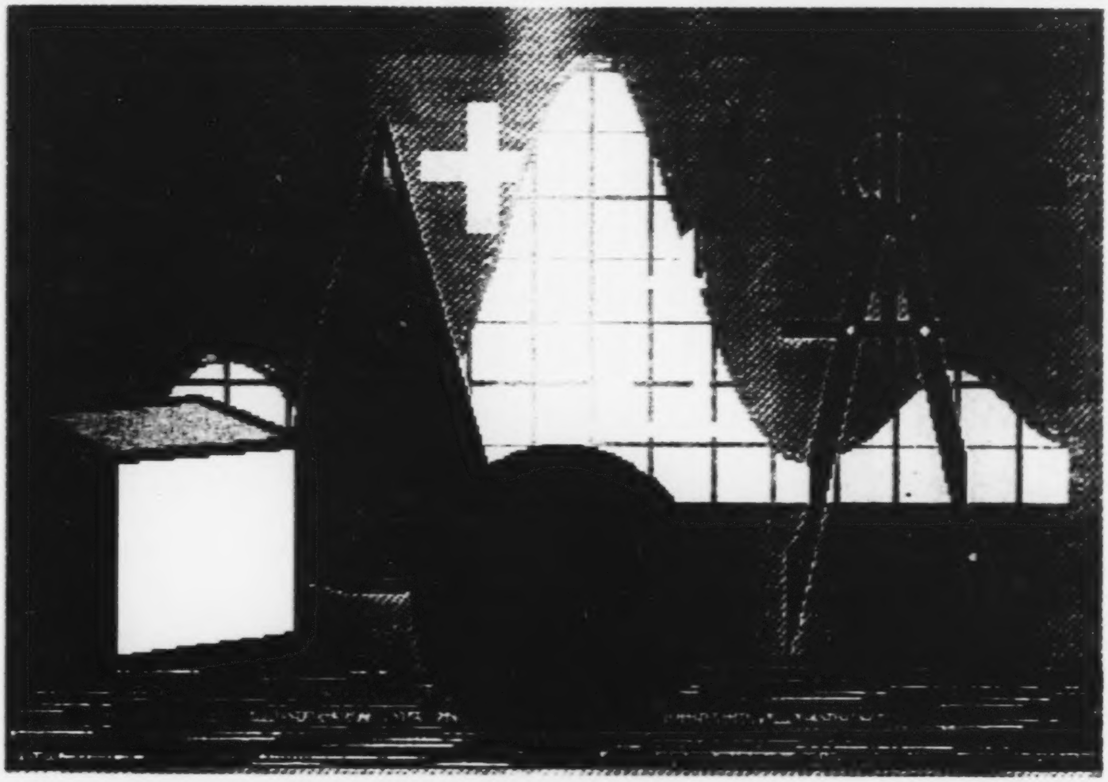
e) Choose an action from **part d** above that you did NOT select as the answer to **part d.** Make your own drawing to show what **figure 1** would become when changed by this action. Be sure to label the action that you are applying.



Competency-Based Education Assessment Series

Sixth Grade Mathematics

Type 3 Assessment



Developed by

William A. Bowman

Linda Diltz

Jennifer L. Salvino

Steven Meiring

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The model competency-based education assessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency assessment instrument consists of performance tasks in the form of activities. Each requires a substantial period of time for the student to complete. The activities involve more authentic-type situations that reflect the ways in which mathematics is applied. Multiple skills, understandings, and thinking are required to complete each activity. Rather than assessing single objectives, the activities target strands of the sixth grade mathematics curriculum. Several objectives may be assessed within the scope of a single activity and cut across more than one strand. The manner in which the assessment is carried out may also reflect upon the outcome levels measured relative to those objectives.

For example, Activity 1 that follows, *Shape Changes in Geometry for a Holiday*, addresses objectives 1-2, 4-4, 5-3, and 6-2 from the sixth grade strands of the *Model Competency-Based Mathematics Program*. Activity 6 *(Corral the Area)* addresses objectives 1-1, 2-4, 6-1, 6-2, and 8-3 in an activity that explores maximizing area for a given perimeter, showing connections among patterns, tables, and graphs, and finally an extension to a different figure. Activity 9 *(M & M Investigation)* consists of several connected lesson/assessments that investigate the colors of M & Ms in a package from the viewpoints of experimental and theoretical probability. The results are then summarized in graphical form, using both circle and bar graphs for comparison, and drawing upon ideas such as ratios, proportions, percents, scales and the processes of estimation, sampling, and compilation of data. In the course of these investigations, teachers can choose at various assessment points whether to focus upon how students set about the investigation (requiring a process rubric) or assess the results achieved (requiring an outcome rubric).

It is the expectation that these performance tasks will be given to students throughout the year at appropriate intervals. Multiple forms of assessment will be employed – for example, student products, teacher observations, interviews, self-assessment, and journal writings. Some tasks may be given individually and some in group settings. Products and records of students' activity may be kept in individual portfolios. At a suitable point toward the end of the year, a thorough review of the work and records within the portfolio can be used to determine how well the student is progressing relative to the competency-based outcomes of the sixth grade.

It is characteristic of performance tasks that dividing lines among instruction, learning, and assessment become fuzzy. Students will engage many of the following activities within a learning and doing, rather than a testing, environment. They will have the opportunity to converse with others, use appropriate manipulatives, employ calculating tools, consult resources, reflect upon results, and write out their findings. The activities are not intended to be time-restrictive for completion, although choices will have to be made as to which and how much of each activity can be fit within practical time constraints. Since these activities permit multiple avenues of approach and individual styles, they are more equitable and accessible to the diversity of students than some means of assessment.

For more information about performance assessment, consult *the Performance Assessment* section that follows and other publications such as the National Council of Teachers of Mathematics resource, *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark, 1991).

Performance Assessments

Overview

"A performance assessment in mathematics involves presenting students with a mathematical task, project, or investigation, then observing, interviewing, and looking at their products to assess what they actually know and can do.

A performance task can:

- allow the examination of the process used as well as the answer or finished project;
- be used with groups as well as individuals;
- document, through observation records or student products, accomplishments not revealed by ordinary tests."

source: *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*

Performance assessments give students an opportunity to display a full range of ability. They emphasize better the nature of mathematics, its processes, and practice. The tasks are interesting and motivational. Assessment does not have to interrupt student learning. Richer and more complete information becomes available for making instructional decisions. A more engaging record of the broad character of the mathematics program is available for parents, administrators, and citizens.

Assessing Performance Tasks

As teacher, you must decide upon which outcomes you wish to focus for a particular task and then choose how to assess those outcomes. For example, let us consider Activity 5 *(Devising an Economical Transportation Plan)* from this instrument. The goals of the activity are for students to:

- systematically investigate the transportation costs for various combinations of vehicles;
- recognize that the problem is not well-defined and different cases must be considered;
- decide various beginning assumptions and organize solutions that correspond;
- write up their findings in a report to the principal for selecting the appropriate plan.

You could choose to focus upon how students go about the *process of investigation* and design a corresponding rubric:

1 - Needs significant instruction and direction
2 - Needs some instruction but initiates own investigation
3 - Conducts investigation and develops one plan based upon one set of assumptions
4 - Meets the expectations of the activity, reflecting more than one plan
5 - Exceeds expectations; making unanticipated discoveries or explorations

Alternatively, you could focus upon outcomes the student achieves in problem solving:

1- Unsystematic in investigation; considers only one case; minimal thinking
2- Somewhat systematic; recognizes more than one situation, but arbitrarily chooses only one for consideration
3 - Systematic approach in calculating per person cost by vehicle, but overlooks some obvious constraints in developing a plan(s)
4 - Conducts thorough, systematic consideration of options with acceptable written plan
5 - Exceeds expectation level in reasoning, developing alternative plans, and presentation in writing

Another possibility is to focus upon the quality of the response:

Minimal Response. Only some of the essential considerations of the task are met.

Nearly Proficient Response. Most essential considerations are met with some small errors in reasoning or calculation, incomplete explanations, analysis, or writing.

Proficient Response. Full application of ideas and skills required in the activity, consideration of appropriate options, and adequate written communication.

Advanced Response. Goes well beyond all expectations with displays of creativity, elegance, exceptional reasoning, and presentation of ideas.

Alternatively, a teacher can assess *attitudes* (interaction and participation):

0 = Dependent 1 = Needs Support 2 = Independent

ATTITUDES	Date			Comments
Cooperates				
Shares/Collaborates tries, contributes ideas				
Questions Peers encourages others to participate				
Takes Risks confident in own ability				
Stays on Task perseveres				

source: *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*

Discussion

Activity 1 Assesses Strands 1, 4, 5, and 6. (See previous discussion on Performance Assessment.)

Solution: a. Figure 1 is that of a pumpkin of normal dimensions.

b. Figure 2 is the same pumpkin which has been stretched *vertically* by a factor of 2.

c. The rule **(2x, y)** means to double the x-coordinate for each pair of numbers while leaving the y-coordinate unchanged. Figure 3 is the original pumpkin stretched *horizontally* by a factor of 2.

d. Figure 4 is the original pumpkin after it has been stretched *horizontally* and *vertically* by a factor of 2 in each direction.

e.

Figure	Rule	Area of STUV (figure's nose)
1	(x, y)	2 square units
2	(x, 2y)	4 square units
3	(2x, y)	4 square units
4	(2x, 2y)	8 square units

f. 2 g. 2 h. 4

i. Doubling the values for locating points in one direction caused the nose area to double. Doubling the values for locating points in both directions caused the nose area to double twice or by an overall factor of 4.

j. It is an example of an expansion. The base and left side of the pumpkins stay on the edges (axes) of the graph. The figure is simply stretched.

k. The new figure would be an example of a translation. It would be the same size and shape, but moved one unit to the right and two units up.

Comment: This activity may be done in whole or only partially depending upon the prior experience and understanding of students. The total activity touches upon these objectives from the sixth grade – 6-1-2, 6-4-4, 6-5-3, 6-6-2. A key outcome from this activity is how geometric figures can be described by the coordinates of their vertices (corners) and then transformed by changing this rule to cause a stretch or translation of the figure (other types of changes of coordinates can cause a rotation or reflection). You may wish to tell students that this is how computerized animation is done, in principle. The second key outcome is that *each* doubling of a side of a rectangular figure has a doubling effect on its area. So that doubling two dimensions causes a four-fold change in the area.

Activity 2 Assesses Strands 1 and 2.

Solution: a. 220 toothpicks

b.

size	1x1	2x2	3x3	4x4	5x5	6x6	7x7	8x8	9x9	10x10
toothpicks	4	12	24	40	60	84	112	144	180	220

Students will see various patterns as they do this exercise, depending upon whether they look for the patterns as they build the squares from toothpicks or from the table of results.

A pattern from the table is that the last row can be rewritten as multiples of 4 in this way:
1•4 3•4 6•4 10•4 15•4 21•4 28•4 36•4 45•4 55•4

Another pattern that uses the number of toothpicks in a side as one of the factors is:
1•2•2 2•3•2 3•4•2 4•5•2 5•6•2 6•7•2 7•8•2 8•9•2 9•10•2 10•11•2

A pattern that comes from building the squares is to consider the toothpicks around the perimeter and the ones inside the square separately. Then this pattern can be seen:

outside	4•1	4•2	4•3	4•4	4•5	4•6	4•7	4•8	4•9	4•10
inside	0	2•2	4•3	6•4	8•5	10•6	12•7	14•8	16•9	18•10

A pattern that considers building the second square *by adding to* the first with an L-shaped region, then building the third by adding an L-shape to the second, and so on, is:

4   4 + 8   4 + 8 + 12   4 + 8 + 12 + 16   4 + 8 + 12 + 16 + 20   4 + 8 + 12 + 16 + 20 + 24

c. 312 toothpicks are needed to make a 12 by 12 square.

d. 5100 toothpicks are needed to make a 50 by 50 square.

e. A 15 by 15 square could be made from 480 toothpicks.

Comment: Many other patterns are possible in this activity. The important aspects of the activity are to give students the opportunity to discover patterns through multiple ways (e.g., building or constructing, tables, observations of numbers) and then to use their discovered patterns to make predictions which they can then check for verification. Parts **d** and **e** of this activity are for optional use with students who can benefit from being challenged. The overall activity involves these objectives: 6-1-1, 6-2-1, 6-2-2, and 6-2-4.

Activity 3 Assesses Strand 3.

Solution: a. The old checking plan works best for one check ($1.80 + .15) versus ($2.40 + .10).

b. The new checking plan is cheaper for 20 monthly checks ($2.40 + 2.00) versus ($1.80 + 3.00).

c. The two checking plans charge the same for 12 checks ($1.80 + 1.80) and ($2.40 + 1.20).

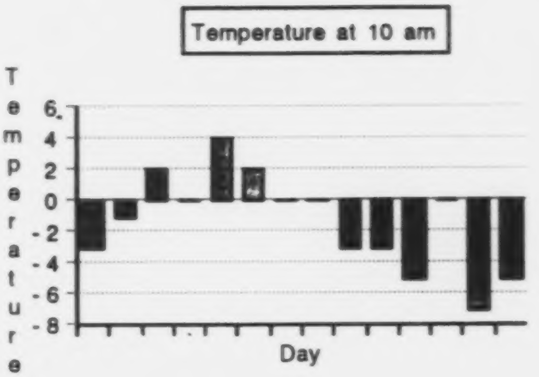
d. A person should look at their past record of using checks. If the number written averages less than 12 per month, they should use the old checking plan. If the number written averages more than 12 per month, they should use the new checking plan.

Activity 4 Assesses Strands 5 and 8.

Solution:

a.

Date	Temp	Date	Temp
1st	- 2	9th	?
2nd	- 3	10th	- 3
3rd	- 1	11th	- 3
4th	2	12th	- 5
5th	?	13th	?
6th	4	14th	- 7
7th	2	15th	- 5
8th	0		



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b. The average daily temperature was - 1.75° or - 1.8°. Note that some students may be tempted to divide the sum of the temperatures by 15 instead of 12.

c. 5th: 3° (2° or 4° are also reasonable); 9th: anything from 0° to -3° is acceptable;
13th: - 6° (-5° or -7° are also reasonable).

d. By selecting a temperature in between two known values for one that is unknown, students are making a guess that no unusual or irregular behavior occurred. This is reasonable given temperature patterns.

e. This activity is designed to show students that an *average* value is not always a good representative for individual data that are unknown in a collection of values IF nearby values indicate a different trend. When no additional information is available (we don't know the pattern of nearby values), then we can employ the average because we have nothing better. In this data set, only the data around the 9th suggests that the average value might be a good estimate.

Activity 5 Assesses Strand 3.

Solution: This activity tries to show a very practical illustration that different answers are possible depending upon decisions that may not all be specifically stated at the beginning of a real-world problem. For example, any plan should show a consideration of the unit cost for transporting an individual by that kind of vehicle.

Vehicle	Per Person Cost
Car (4)	$ 3.75
Van (8)	$ 2.75
Minibus (12)	$ 2.67
Bus (26)	$ 2.88

If there are no other considerations, we should use as many minibuses as possible. However, since the vehicles hold different numbers, we will not find it economical to use a vehicle unless it is mostly filled. Now, other assumptions come into play. IF all atheletes can be transported as a total set of 64 students (regardless of which team they belong to), then we can use 5 minibuses (holding 60) and one car (4) at a cost of $175.

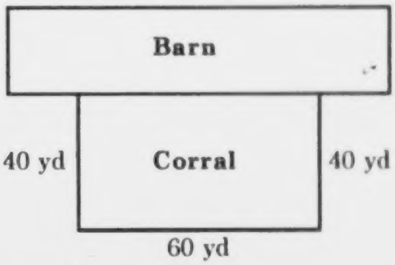
However, it is quite reasonable also not to mix team members together. In that case, we need 2 minibuses (holding 24) and 1 van (8) for the 29 girls, at a cost of $86; and we need 3 mini-buses (holding 36) for the 35 boys, at a cost of $96. The total cost, in this case, is $182.

Principal FussBudget may have other considerations in mind, as well, such as the safety of using larger vehicles, which would involve the use of one or more buses.

Comment: Our advice is NOT to provide all of these decisions in advance. Let the students, working in groups, come to these realizations and provide *alternatives* for the principal's consideration, based upon different constraints. The focus in this activity is upon analyzing the situation, considering different possibilities, and then communicating these options in writing to another party who is not present.

Activity 6 Assesses Strands 6 and 8.

Solution: a.

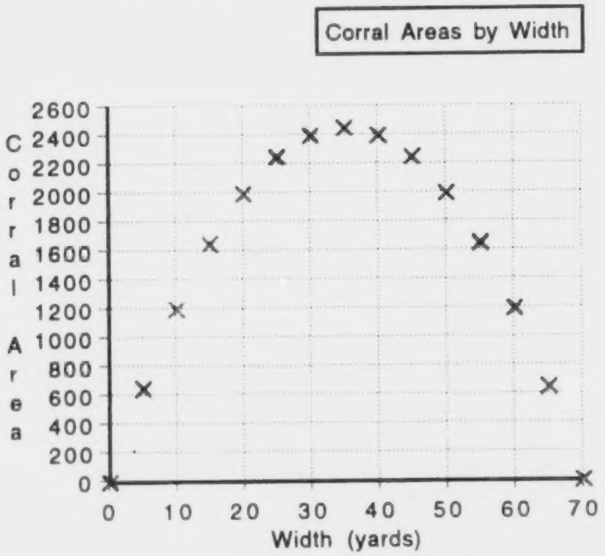


b.

Length	Width	Total Fence Used	Area
130 yd	5 yd	(130 + 5 + 5) yd	650 sq. yd.
120	10	(120 + 10 + 10)	1200
110	15	(110 + 15 + 15)	1650
100	20	(100 + 20 + 20)	2000
90	25	(90 + 25 + 25)	2250
80	30	(80 + 30 + 30)	2400
70	35	(70 + 35 + 35)	2450
60	40	(60 + 40 + 40)	2400
50	45	(50 + 45 + 45)	2250
40	50	(40 + 50 + 50)	2000
30	55	(30 + 55 + 55)	1650
20	60	(20 + 60 + 60)	1200
10	65	(10 + 65 + 65)	650

c. length: 70 yd width: 35 yd

d.



e. As the width changes from zero by 5 yd amounts, the length of the corral decreases by 10 yd amounts and the area increases. This occurs until a width of 35 yd and length of 70 yd when the area is 2450 sq. yd. – the maximum. Then as the width increases further, the area begins to decrease again until it goes back to zero for a width of 70 yd.

Comment: This activity is intended to give students experience with the difference between perimeter and area (e.g., the perimeter of the corral stays constant at 140 yd while the area varies). Students need not fill in as many table values as shown in the solution, but should zero in around the values 35 yd and 70 yd where the maximum occurs. They should recognize the symmetry in the table to know to do this. You might wish to draw attention to the ratio of the width to the length for a corral of maximum area (1 : 2) and ask students who can benefit from an extension whether the same ratio would have occurred if all of the fence had been used to make a four-sided corral. (No, the maximum will occur for a square corral that is 35 yd on a side.)

Activity 7 Assesses Strands 1, 2, and 5.

Solution: a. 56 games; each team plays 7 games against every other team for two times – which is 8 x 7 x 2, but games are counted twice in this tally, so the total is 56.

b.

Teams	1	2	3	4	5	6	7	8	9
Games	0	2	6	12	20	30	42	56	72

c. 156 games – (12 x 13) + 2

d. The rule is: No. of Games = □ x (□ + 1)

Comment: This activity assesses objectives 6-1-1, 6-2-1, 6-2-4, 6-5-2, and 6-5-3.
You may wish to assist students in counting the number of games played by a given number of teams by suggesting that dots represent the teams with a line segment connecting two dots as a game played between those teams.

Activity 8 Assesses Strands 1 and 6.

Solution: a. The effect is that doubling the length of a side of a cube increases the volume by a factor of 8. The volume pairs are: 1 and 8; 8 and 64; 27 and 216; 64 and 512; 125 and 1000.

c. V = s x s x s; doubling each side makes this formula V = 2s x 2s x 2s or V = 8 x s x s x s

d. The effect is that doubling the length of a side of a cube increases the surface area by a factor of 4. Surface area (for a cube) behaves the same way as area for a 2-dimensional figure. The surface area pairs are: 6 and 24; 24 and 96; 54 and 216; 96 and 384; 150 and 600.

f. S = 6 x s x s; doubling each side makes this formula S = 6 x 2s x 2s or S = 24 x s x s which is the same as S = 4 x (6 x s x s).

Comment: This activity is an extension of objective 6-1-2 to three dimensions. For assessment purposes, you can learn how well students can apply their understanding of these ideas when they consider a new object (3-dimensional). The distinction between volume and surface area is problematic for students of all ages. Therefore, drawing the similarity of the behavior of surface area to the area of plane figures can be quite beneficial to students.

Activity 9 Assesses Strand 8.

Solution: a. The key to how well this portion of the activity works is the thorough mixing of M & Ms each time between draws. Unless all of the M & Ms have an equal chance of being drawn, this sampling procedure will NOT give a good indication of the contents. If a trial before class indicates a problem with mixing, you might wish to use small sacks, pouring the contents of the package into the sack, and then drawing without looking inside.

b. Part of this activity involves discovering how few or how many M & Ms need to be drawn to get a reliable sample from which to guess the colors and relative ratios.

c. Part **b** can give an indication of the ratios of different colors to one another, but not of the total number of M & Ms in the original package. Some students might suggest that the least common multiple of the ratios is a good guess.

d. The ratios calculated here represent the *empirical* probability (determined by experiment) for drawing a particular color. These values are to be compared to the calculated probability in the next section of this activity.

g. The decimal comparisons between experimental and calculated probabilities will range in their accuracies. Taking several samples is more reliable than using one. Including more trials (picks) in each sample improves accuracy.

i. The estimated circle graph will give you considerable information in how well students are able to visualize the comparison of parts to the whole for individual colors and then translating that to a portion of a circle. This is a step which is not frequently practiced in mathematics applications because we tend not to think about estimating except with respect to calculations.

j. Parts **j** and **m** will be confusing to students because they are converting the color ratio to two other ratios (part of a circle in **j** and percent of the whole in **m**). This requires division of **n** by two different numbers – 360° and 100. To help students understand, explain that all ratios are comparing the PART to the WHOLE. When we are talking about numbers, the whole is the total of the M & Ms in the package. When we are considering shading in part of the circle, we measure our angles in degrees, so the whole is 360°. Later in part **m**, we want to label the circle sectors as part of the whole where we are using percents as the measure – therefore, the whole is represented by 100%.

o. Bar graphs make it easy to compare the color frequencies to one another. But it is difficult to get an idea of how a particular color frequency compares to the total for all colors. The bar graph does permit you to determine the number of each color for the package.

Circle graphs permit comparisons of one color frequency to another and also as a part of the whole. But the graph does not show the number of M & Ms of each color nor the total number of M & Ms in the package (although this kind of information is often added to circle graphs, in some form).

Comment: This activity will span several days and involves multiple objectives from Strands 7 and 8. An important aspect of the assessment, in addition to process outcomes during the activity, is to ask students to reflect on the activity as a whole and to explain what they have learned individually and as groups.

Activity 10 Assesses Strands 4, 6, and 7.

Comment: There are multiple discrete tasks to this total activity. These may be done individually or in combination without the need to do each part. Or the tasks can be spread over the year. The discrete tasks involve different mathematical concepts (and therefore objectives), but each reflects process aspects of measuring, estimating, and applying mathematics ideas to real-world events.

As in the preceding activity, it is important to assess students as the activity is in progress for certain types of outcomes. It is equally important to assess students as they reflect back on what they have discovered or applied in the course of the activity. Written responses by students to the questions posed for each measurement task are extremely important for assessing communication objectives.

Activity 11 Assesses Strand 4.

Solution: b.

	F	E	V
Tetrahedron	4	6	4
Octahedron	8	12	6
Hexahedron	6	12	8
Dodecahedron	12	30	20
Icosahedron	20	30	12

c. $\mathbf{F + V = E + 2}$ or some equivalent form of this equation.

Activity 12 Assesses Strands 2 and 5.

Solution: a.
1. $5.6 + y = 13.4$
2. $a + a + a = 30$ or $3a = 30$
3. $8 + w + 15 + z = 36\ 1/2$
4. $7.1 + r + 4 = 2r$
5. $c + c + c + x + x = 25$ or $3c + 2x = 25$
6. $2b + 6 = b + 21$
7. $3x + 1 + x = 40$

b.
7. $90 + 90 = x + 3$
8. $10 + 10 + 10 + 10 + 10 = y + y + 15$
9. $40 + 60 + y = x$
10. $60 + 60 + 60 + y + y = 440$
12. $x + .1 + 5 = y$; the two sides are equal
13. ■ weighs as much as two ● 's.
14. ● weighs as much as three ■ 's.
15. ▲ weighs as much as two ■ 's ; ■ weighs as much as two ● 's.
16. three; there are multiple ways to decide – one is to add a nail to both sides of the first balance and then replace the bolt and nail on the left with a cube, leaving three cubes balanced by nine nails.

Comment: There are two key ideas in this activity to assess. First, an equation gives two names for the same amount as shown on the two sides of the equal sign. Second, balances permit us to do certain things without changing the balanced condition. We can add the same amount to both sides or take the same amount away from both sides. We can replace something on one side with something else that is the same amount. We can double or halve the amounts on both sides as long as we do the same thing. We can add the amounts from one balance to the respective sides of another balance. Each of these actions performed with a physical balance sets the stage for what we can do later with an equation and symbols.

Activity 13 Assesses Strand 4.

Solution: b. squares: 90° + 90° + 90° + 90° = 360°
pentagons: 108° + 108° + 108° = 324° with a gap
equilateral triangles: 60° + 60° + 60° + 60° + 60° + 60° = 360°
octagons: 135° + 135° + 135° = 405° with an overlap

c. Tessellation can occur when the angle sum is exactly 360°; otherwise, it does not occur.

d. Place the triangles so that three different angles come together at a vertex.

e. Place the quadrilaterals so that four different angles come together at a vertex.

g. Sum for 3-4-4-6: 60° + 90° + 90° + 120° = 360°
Sum for 12-3-4-3: 150° + 60° + 90° + 60° = 360°

ACTIVITY 1

Shape Changes in Geometry for a Holiday

You are going to draw four related figures in this activity by plotting and connecting points. On page 3 of this activity, you will find a table of points organized into columns and sets of points. Look at the table of points while you read these directions.

a. A completed figure will be designated by the number at the top of a column (for example, **Figure 1**). The separate parts of the figure are drawn by plotting Sets I, II, III, IV, and V of the points and connecting as indicated. On page 4 of this activity, plot and connect the points for Figure 1 on the graph labeled *Figure 1.* DO THAT NOW.

b. After you have drawn Figure 1, move to the next column of the table of points and fill in the coordinate values for each point by following the pattern given by the rule at the top of the column. For example, the rule for Figure 2 is to take the coordinate pair from the first column and to *double* the second number for that pair. (The first number is unchanged.)

Example

C becomes (13, 6); D becomes (13, 14); but A and B remain the same since doubling zero is still zero: A(4, 0) and B(9, 0)

After you have filled in all of the coordinate values for column 2, then draw Figure 2 in a similar way on the graph for *Figure 2,* which you will find on page 4 of this activity.

c. After you have drawn Figure 2, move to column 3. What do you think the rule **(2x, y)** means? Which number of the pair (x, y) from the first column is being changed? By how much?

Fill in the coordinate values for points in column 3. Then plot and connect points as before on the graph for *Figure 3,* which you will find on page 4.

d. Similarly, complete the table for column 4 and plot and connect points on the graph for *Figure 4,* which is found on page 5.

---

e. After you have finished your graph work, then turn to page 6 of this activity and complete the directions and questions that you find on that page.

Figures	1	2	3	4
Rules	(x , y)	(x, 2y)	(2x, y)	(2x, 2y)
Set I	A (4, 0)			
	B (9, 0)			
	C (13, 3)			
	D (13, 7)			
	E (11, 9)			
	F (8, 8)			
	G (9, 10)			
	H (7, 10)			
	I (6, 8)			
	J (3, 9)			
	K (0, 7)			
	L (0,3)			
	Connect to A			
Set II	M (4, 7)			
	N (3, 5)			
	O (5, 5)			
	Connect to M			
Set III	P (9, 7)			
	Q (8, 5)			
	R (10,5)			
	Connect to P			
Set IV	S (6, 5)			
	T (6, 3)			
	U (7, 3)			
	V (7, 5)			
	Connect to S			
Set V	W (2, 4)			
	X (4, 2)			
	Y (9, 2)			
	Z (11, 4)			

Figure 2

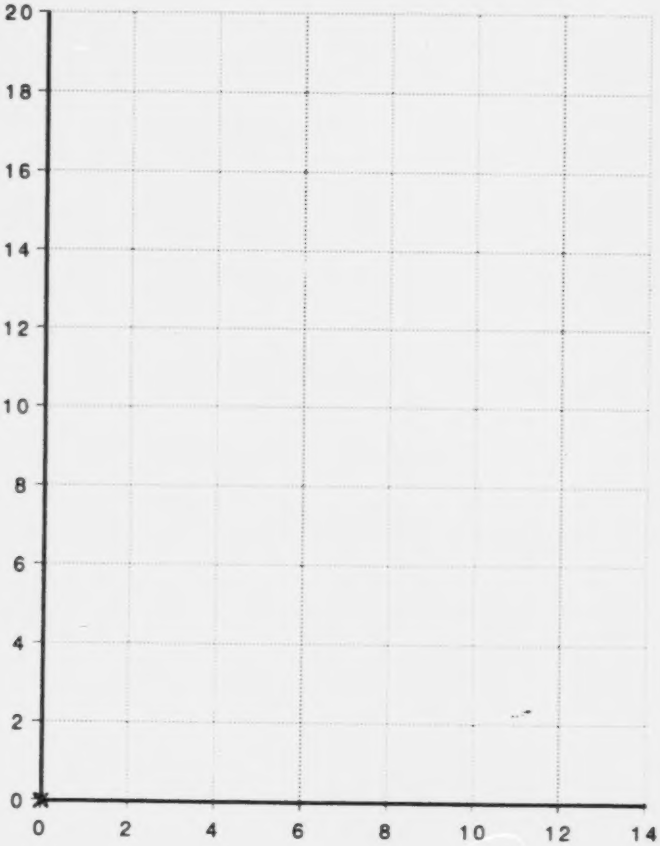


Figure 1

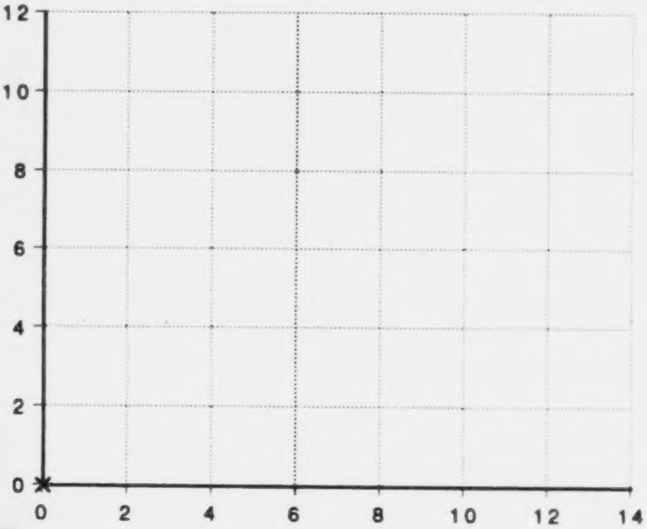


Figure 3

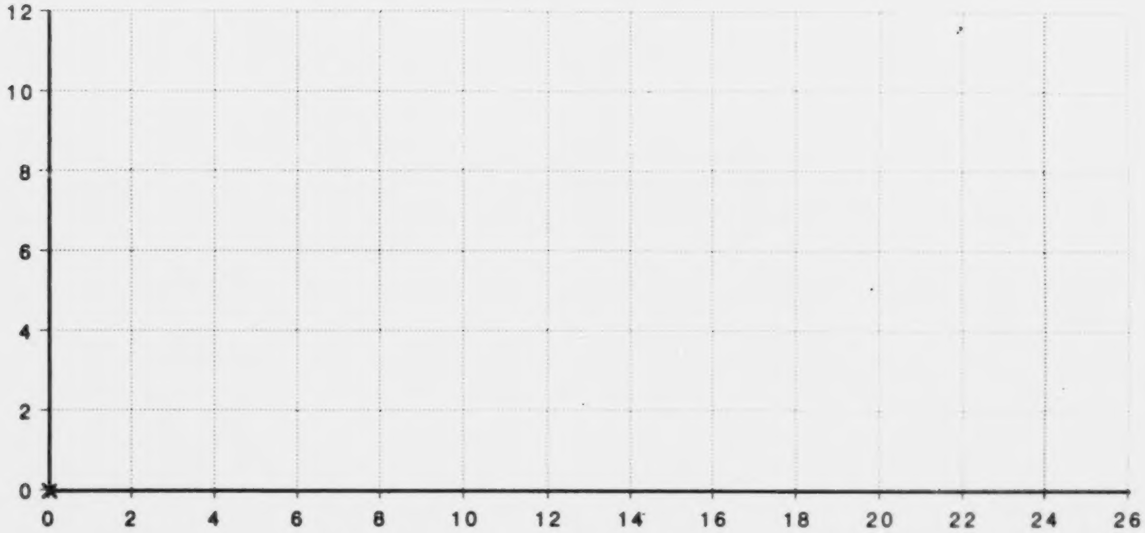
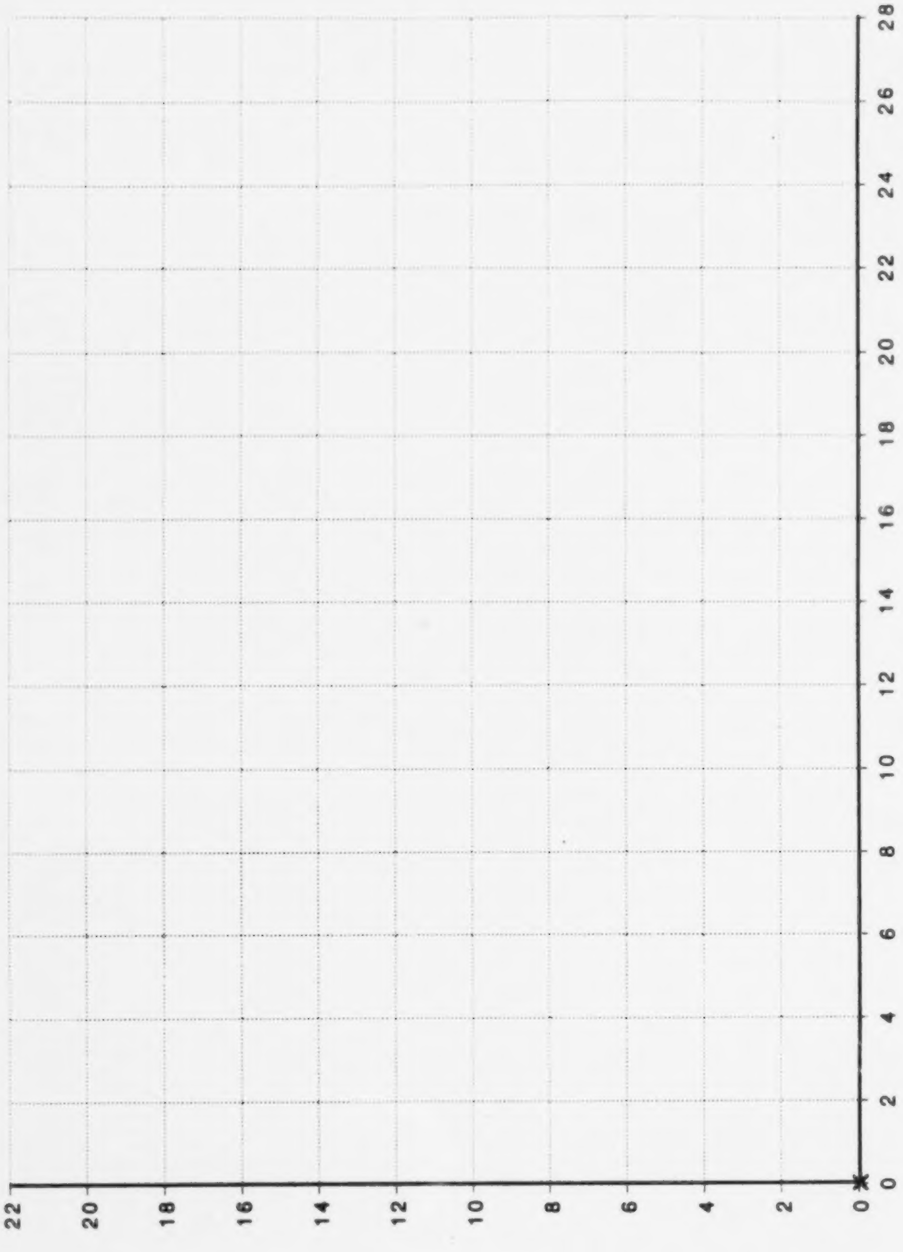


Figure 4



Fill in the chart below. It involves looking at the *nose* on each of the figures that you drew on Graphs 1-4. *Use* the chart to answer the questions below.

Figure	Rule	Area of STUV (figure's nose)
1		
2		
3		
4		

f. How many times larger is Figure 2 than Figure 1? ______________

g. How many times larger is Figure 3 than Figure 1? ______________

h. How many times larger is Figure 4 than Figure 1? ______________

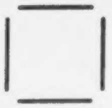
i. Compare the rules and the areas of the noses. Explain how the rules caused the areas to change.

j. Compare Figures 1 and 4. Is this an example of a rotation, reflection, translation, or expansion? ______________ Why?

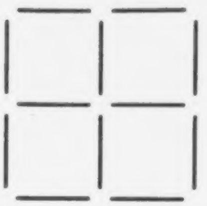
k. If Figure 5 used the rule **(x + 1, y + 2)**, would the new figure be a rotation, reflection, translation, or expansion? Explain.

ACTIVITY 2

Patterns Among the Toothpick Squares

A 1 by 1 square looks like this  and is made up of 4 toothpicks.

A 2 by 2 square looks like this when made up of 1 by 1 squares.



a. How many toothpicks are needed to make a 10 by 10 square made up of 1 by 1 squares? How do you know?

b. Make a chart that answers the question above for squares up to 10 by 10. Then state any patterns that you find in your chart.

c. Use a pattern that you found in **part b** above to predict the number of toothpicks needed to make a 12 by 12 square of toothpicks in this manner. Then make a drawing to check your prediction

*prediction:*____________________

drawing:

d. Use a pattern that you found in the table above to predict the number of toothpicks needed to make a 50 by 50 square made up of 1 by 1 squares. Then state the pattern that you used.

*prediction:*____________________

pattern:

e. What size square like those above could you make from 480 toothpicks? How do you know?

ACTIVITY 3

What's the Best Plan?

Your local bank is advertising new rates for checking accounts. Use the information below to answer the questions.

OLD CHECKING PLAN	NEW CHECKING PLAN
$1.80 - Service Charge per Month	$2.40 - Service Charge per Month
$0.15 - Charge for Each Check Written	$0.10 - Charge for Each Check Written

a. Which plan has the better monthly charge if you write only one check per month? Explain your answer.

b. Which plan has the better monthly charge if you write 20 checks per month? Explain your answer.

c. For how many checks written per month will the plans charge the same? Show your results organized in a table.

d. What do you conclude about which plan is best to use?

ACTIVITY 4

A Matter of Temperature

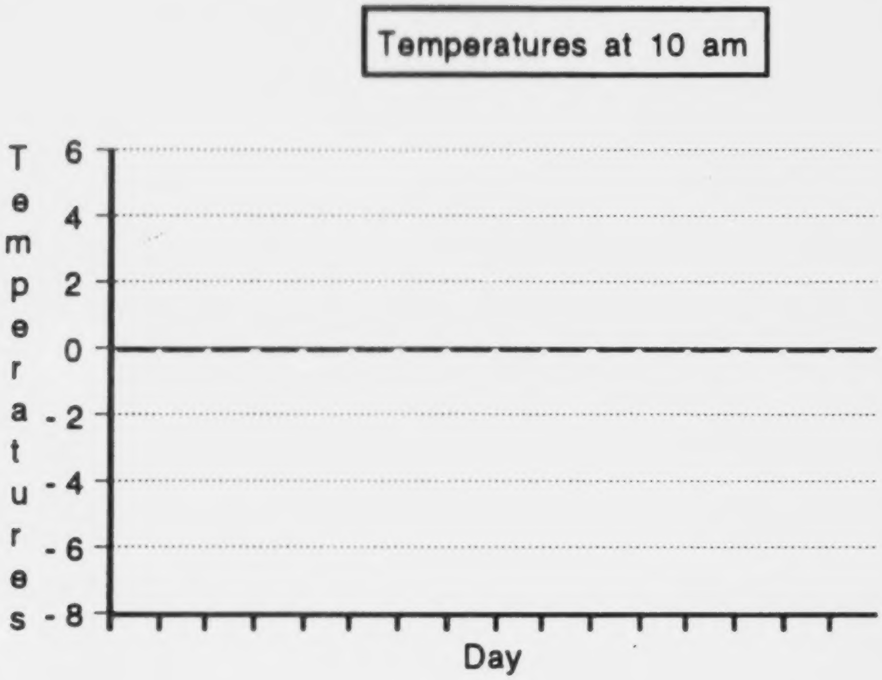
Ms. Sanchez' sixth grade class recorded temperatures for 15 consecutive days at 10:00 am each day. Students wrote the temperatures on slips of paper, and now they need to organize them. Three of the temperatures for given dates came up missing (indicated by the question marks).

4th: 2	2nd: - 3	10th: - 3	8th: 0
1st: - 2	9th: ?	3rd: - 1	11th: - 3
5th: ?	13th: ?	12th: - 5	6th: 4
15th: - 5	7th: 2	14th: - 7	

a. Organize the information above, and plot it on the bar graph below.

Organized Data

Bar Graph



b. What was the average daily temperature? ____________

c. What could you fill in for the missing temperatures with data that is reasonable?

d. Why do you think your values for the missing temperatures in **Part c.** are reasonable?

e. Why might the *average* temperature calculated in **Part b.** NOT be a good value to use for the missing temperatures?

ACTIVITY 5

Devising an Economical Transportation Plan

The girls' and boys' soccer teams from Needmore Middle School need to get their players to the state tournament. There are 29 players on the girls' team and 35 players on the boys' team. Using the information in the chart below, find the most economical and easiest way to get all of the players to the tournament. Explain your solution and write it up as a report to convince Principal FussBudget how this will save money.

Vehicle	Capacity	Cost
Car	4 people	$ 15.00
Van	8 people	$ 22.00
Minibus	12 people	$ 32.00
Bus	26 people	$ 75.00

ACTIVITY 6

Corral the Area

Mr. Jones plans to use his barn as one side of a rectangular corral. He has 140 yards of fencing to use to form the other three sides of the corral. Mr. Jones wants to find the length and width combination that would give him the greatest area for his corral.

a. First draw a sketch of the situation.

b. Begin your investigation by filling out a table that compares areas for different length and width combinations. (Use a separate piece of paper for your table.)

Length	Width	Total Fence Used	Area
120 yd.	10 yd.	(120 + 10 + 10) yd.	1200 sq. yd.

c. What length and width combination encloses the greatest area?

length: ________________ *width:* ________________

d. On a graph with width as the horizontal axis and area as the vertical axis, plot each pair (width, area) from your table of values to make a scatterplot of the data from the table.

e. Describe how the pattern of data points behaves. For example, what happens to the area as width increases from 0 through various values until it reaches 70 yd?

ACTIVITY 7

Badminton Tournament

Hiroshi was asked to organize a badminton tournament. Each team is made up of two players and each team will play every other team twice.

a. If there are eight teams in the league, how many total games will be played?

b. Complete the following table for games played for various numbers of teams. Look for a pattern in the table.

Number of Teams	1	2	3	4	5	6	7	8	9
Number of Games									

c. Predict the number of games that would be played if 12 teams were in the league.

d. Explain a rule that could predict the number of games that would be played if ☐ teams were in the league.

ACTIVITY 8

Doubling on the Cube

In this activity, you are going to study the effects on volume and surface area of doubling the length of a cube.

a. Complete the following chart which summarizes the effects on **volume** of doubling the length of the side of a cube. For the first two length combinations, *build* the cubes indicated from unit cubes supplied by your teacher.

Length	1 unit	*Double the Length*	2 units
Volume	1 cu. unit	*New Volume*	_____ cu. units
Length	2 units	*Double the Length*	4 units
Volume	__________	*New Volume*	______________
Length	3 units	*Double the Length*	6 units
Volume	__________	*New Volume*	______________
Length	4 units	*Double the Length*	8 units
Volume	__________	*New Volume*	______________
Length	5 units	*Double the Length*	10 units
Volume	__________	*New Volume*	______________

b. What can you conclude about the relationship that exists between doubling the length of the side a cube and its volume?

c. Write the formula for the volume of a cube.What happens if you replace *s* in this formula with *2s*? (Be sure to replace each *s* with *2s*.)

d. Complete the following chart which summarizes the effects on **surface area** of doubling the length of the side of a cube. For the first two length combinations, *build* the cubes indicated from unit cubes supplied by your teacher.

Length	1 unit	*Double the Length*	2 units
Surface Area	6 sq. units	*New Surface Area*	_____ sq. units
Length	2 units	*Double the Length*	4 units
Surface Area	__________	*NewSurface Area*	_______________
Length	3 units	*Double the Length*	6 units
Surface Area	__________	*New Surface Area*	_______________
Length	4 units	*Double the Length*	8 units
Surface Area	__________	*New Surface Area*	_______________
Length	5 units	*Double the Length*	10 units
Surface Area	__________	*New Surface Area*	_______________

e. What can you conclude about the relationship that exists between doubling the length of the side a cube and its surface area?

f. Write the formula for the surface area of a cube. What happens if you replace *s* in this formula with *2s*? (Replace each *s* with *2s*.)

ACTIVITY 9

m & m Investigation

Use your group's package of **m & m's** as the data needed to answer the following questions and complete the activities.

Sampling

a. Tear off a small corner of the package and pour out just one **m & m**. Record its color, put it back, and then shake the package. Continue doing this a number of times until you can write out a guess as to the colors of **m & m's** that your package contains and the ratios of these colors to one another. For example, *1 blue, 3 reds, 2 yellows, ...*

Color	**Frequency**

Total m & m's Drawn: _______

b. Write out your guesses for your package's colors and ratios.

c. Do you have any guess about the total number of **m & m's** that your package might contain? How did you decide your guess?

d. For each color, write the ratio of the frequency of times that color was pulled from the package compared to the total number of times that an **m & m** was drawn from the package.

ratio for drawing Red = _____
ratio for drawing Blue = _____
ratio for drawing Brown = _____
ratio for drawing Yellow = _____
ratio for drawing Orange = _____
ratio for drawing Green = _____

Calculated Probability

e. Now pour out the contents of your package and record its actual contents by color, frequency, and total.

Color	Frequency

Total m & m's in Package: _______

f. If you put all of your **m & m's** back into the package and drew one out at random, what is the probability of getting a particular color?

probability of a Red = _____
probability of a Blue = ______
probability of a Brown = _____
probability of a Yellow = ______
probability of an Orange = _____
probability of a Green = ______

g. Compare the ratios that you found by *sampling* in **Part d** to the *calculated probabilities* that you determined in **Part f.** (You may have to convert to decimals for comparison.) Is there any reason why these ratios for a particular color might be close? Would drawing a larger number of **m & m's** in **Part a** have made a difference?

Graphing

h. Use the data from **Part e** to create a bar graph to represent your group's package of **m & m's**. Label the horizontal axis with the colors and the vertical axis with the numbers. Color your bar graph and give it a title.

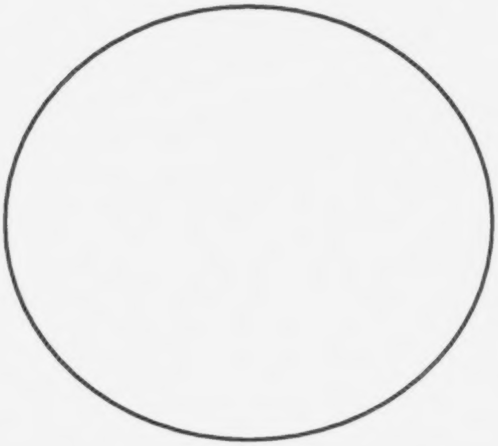
i. Your data can also be shown by a circle graph. But you must first find out what part of a circle each color will be. In your group, make a rough sketch to estimate how you think your finished circle graph will look.

j. An accurate circle graph can be constructed by using the following proportion to change your data from **Part e** to the number of degrees in the angle for each colored section.

$$\frac{\textbf{number of each color}}{\textbf{total number of m \& m's}} = \frac{\textbf{n}}{\textbf{360°}}$$

Why is 360° used in the second ratio? When you solve for the value of **n**, what information will you have? How can this help you create a circle graph to display your data?

k. Create your circle graph here to represent the **m & m's** in your package. Color and label each section. Give your graph a title.



l. Compare your constructed circle graph with your estimated sketch. Are the two graphs similar? Explain why you drew the sketch the way you did.

m. Percents are often used on circle graphs. You can label your sections with percents instead of the actual numbers of **m & m's** by using this proportion.

$$\frac{\textbf{number of each color}}{\textbf{total number of m \& m's}} = \frac{\textbf{n}}{\textbf{100}}$$

Why is 100 used in the second ratio? When you solve for the value of **n**, what information will you have?

n. Use the proportion in **Part m** to find the percent each color is of the total package.

Red: _____ % **Blue:** _____ %

Brown: _____ % **Yellow:** _____ %

Orange: _____% **Green:** _____ %

Fill in the information from above onto the circle graph that you made in **Part k.**

o. Compare the two graphs that you made in **Parts h** and **k**. What kinds of information about the contents of your **m & m** package are presented in the bar graph that are NOT presented in the circle graph?

What kinds of information about the contents of your **m & m** package are presented in the circle graph that are NOT presented in the bar graph?

ACTIVITY 10

Outdoor Measurement

How Fast Is It Going?

Materials Needed: Stop watch or watch with second hand, trundle wheel, calculator

Description: You are going to take measurements as a team to determine the speed of cars passing the school. Using the trundle wheel, measure out a convenient distance (e.g., 50 m) along the street or roadway identified by your teacher. Have one student at the beginning mark of the measured distance and a second one with the stop watch at the finishing mark. Using hand signals and the stop watch, measure the time for three cars and a truck to travel this distance. Record in the table below and make the calculations.

Vehicle	Time (in seconds)	Speed (km/h)	Vehicle	Time (in seconds)	Speed (km/h)
car 1			**car 3**		
car 2			**truck**		

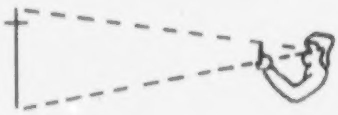
Discussion: Suppose *car 1* travels 50 m in 6 sec. What proportion will let you find how far the car travels in 60 sec (1 min)? Next, how far would it go in 60 min (1 h)? How many km is that? Show all your calculations on another sheet of paper for your work group.

How Tall is That?

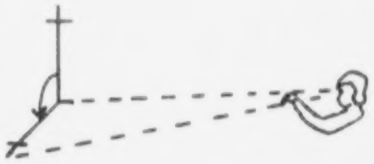
Method 1

Materials Needed: measuring tape, pencil or ruler, stake

Description: Working in teams you are going to find the height of a tall object by indirect measurement. You will transfer the vertical height to an equivalent horizontal distance that is easier to measure.



Have one person on the team stand at the base of the object to be measured. The rest of the team should stand a distance from the object and a "sighter" will begin backing up while holding a pencil (or ruler) at arm's length in front of him/her.



Sighting over the pencil while backing to the correct distance, the sighter can make the pencil appear to be the same height as the vertical object. Then by rotating the pencil 90°, one end of the pencil should sight the base of the vertical object and the other end represents the top of the object.

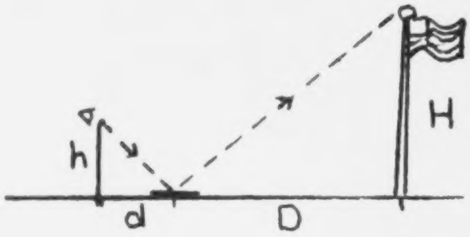
Now, signal the team member at the vertical object to walk away from the object, perpendicular to the line of sight (line from sighter to the object) until that team member appears to be at the other end of the pencil. Place a stake at that point. The vertical distance is now conveniently laid out on the ground for easy measurement with the measuring tape. [Your team might like to use several "sighters" and repeat this process for each, taking the average for the team's report.]

Vertical Object: ____________________ *Determined Height:* ________________

Method 2 (Measure the same object as that for Method 1)

Materials Needed: measuring tape, 2 yard or meter sticks, mirror, marker pen

Description: Place a mirror flat on the ground at a spot some distance from the object whose height is to be found. Use a marking pen to draw a line across the mirror (perpendicular to the line of sight from the mirror to the object). The "sighter" now backs away from the mirror and object until she/he can just view the top of the object touching the line drawn across the mirror.



Carefully, measure the following three distances: (1) **h** - eye height of the sighter from the ground; (2) **d** - distance along the ground from the sighter to the line in the mirror; (3) **D** - distance from the base of the object to the line in the mirror. The following proportion can then be written to find the height **H** of the vertical object. Three parts of this proportion have been measured, so **H** can be calculated.

$$\boxed{\frac{H}{D} = \frac{h}{d}}$$

[Again, team members may wish to take turns being the sighter and having the measurements **h** and **d** taken. Then an average calculated value for **H** should be reported for the team.]

Discussion: Which method (1 or 2) do you think was more accurate? Why? What are the advantages or disadvantages of each method? Why would an average calculated result for the team be more reliable than a single result?

How Thick is That?

Materials Needed: Measuring tape, calculator

Description: The diameters of many round objects, unless they are thin like a coin, are difficult to measure. So instead, we can measure their circumference and then use the formula $C = \pi \times d$ to find the diameter. (π is approximately 3.14)

Select three circular objects of varying sizes which have diameters that would be difficult to measure directly. Then make the appropriate measurements and calculations in the following table.

Name of Object	Measured Circumference	Calculated Diameter

Discussion: The accuracy of a calculated result, like the diameter, is affected by both your ability to use the measuring tool and the size of the measurement. A small error in the measured quantity (circumference) compared to a large calculated quantity (diameter) is said to be very accurate. A large error for the measured quantity (diameter) compared to a small calculated quantity (quantity) is said to be less accurate. Discuss the accuracy of the diameters that your team determined for the three objects in the above table. Start by estimating how much your calculated circumferences might be "off" by.

How Large is That?

Materials Needed: Measuring tape (or trundle wheel), large signs with planet and sun names, and stakes

Description: Your class is going to lay out a scale model of the solar system. You will need about two football fields in distance to accomplish this. Start by going to one end of the fields and placing a disk representing the sun (constructed to the scale below) and place the sun sign. Then measure each of the following scale distances for the planets from this location, leaving a sign next to the stake to mark that spot. [It will help if your signs are attached to the stake which can be driven into the ground.] Continue until you have a scale model of the solar system.

Sun	**Diameter:** 1,400,000 km	**Scale:** 4.3 cm
Planet	*Kilometers from the sun*	*Scale Distance*
Mercury	**57,900,000**	**1.80 m**
Venus	**108,000,000**	**3.35 m**
Earth	**149,500,000**	**4.65 m**
Mars	**228,000,000**	**7.10 m**
Jupiter	**778,000,000**	**24.15 m**
Saturn	**1,430,000,000**	**44.3 m**
Uranus	**2,870,000,000**	**89.14 m**
Neptune	**4,495,000,000**	**139.65 m**
Pluto	**5,910,000,000**	**183.50 m**

Discussion: What observations can you make about the solar system and space from this model? Why is laying out a model more helpful than just reading a column of distances from the sun (such as that in the table above)? The sun is the largest body by far in our solar system. What does that tell you about the sizes of the planets in the scale model that you have laid out?

How Many is That?

Materials Needed: Measuring tape, calculator

Description: You are going to work in teams to take measurements and estimate the number of bricks on the outside of your school building (or other building indicated by your teacher). To do that, you will need to measure a **sample section** of a wall and count the number of bricks in that amount of area.

Next, make a sketch of your building and label the outside walls that you are going to measure. Then by taking measurements of these walls, find: (1) the areas of these walls; and (2) use the number of bricks in your counted sample section to compute the bricks for these walls. Finally, sum the number of bricks for all outside walls of the building and report this result to your teacher. Use the following chart to record your measurements and calculations.

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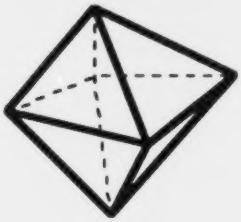
Sample Section Dimensions	Area of Sample Section	Number of bricks
Wall Dimensions	**Area of Wall**	**Number of Bricks**
#1		
#2		
#3		
#4		
#5		
#6		
#7		
#8		
#9		
#10		

Total Number of Bricks in Building Walls: ____________________

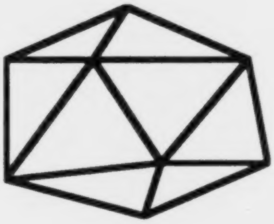
Discussion: Assume that you do these kinds of measurements for a job. What would you do differently for the next job? What improvements would you make?

Regular Polyhedra

Polyhedra are solids whose **faces** (sides) are polygons like squares or pentagons. The faces meet along **edges** and the edges meet at corners called **vertices.**



regular octahedron



regular icosahedron

There are only five polyhedra that are regular (all edges are the same length and all faces are identical). You can make models of them from *rods* and *connectors* by knowing what a face looks like and how many faces come together at a vertex.

Your teacher will give you materials to use for rods (*toothpicks,* for example) and connectors (*gum drops,* for example). Start by trying to make a **tetrahedron.** In a tetrahedron, 3 equilateral triangles come together at each vertex.

Begin by forming one equilateral triangle. Then add two more rods and one connector to form a second equilateral triangle along one edge of the first one you already formed (figure 1 below). Next add one more rod to figure one (see dashed edge in figure 2 above) to form a pyramid. The pyramid is the top "half" of the tetrahedron. Finish the lower half.

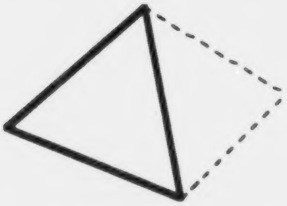


figure 1

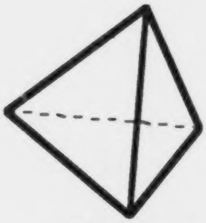


figure 2

a. Make the following polyhedra. They will be made correctly if they look the same no matter how they are turned or positioned.

b. After making each one, count the number of faces (F), edges (E), and vertices (V) and record in the table below. Why do you suppose each has the name it has? (Hint: it has something to do with the prefix of each name.)

Tetrahedron - 3 *equilateral triangles* come together at a vertex.

Octahedron - 4 *equilateral triangles* come together at a vertex.

Hexahedron (cube) - 3 *squares* come together at a vertex.

Dodecahedron - 3 *pentagons* come together at a vertex.

Icosahedron - 5 *equilateral triangles* come together at a vertex.

	F	E	V
Tetrahedron			
Octahedron			
Hexahedron			
Dodecahedron			
Icosahedron			

c. Can you think of a formula that is true for each polyhedron? (Hint: the formula should involve F, E, and V.)

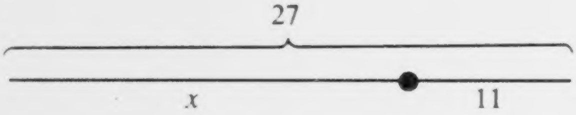
ACTIVITY 12

Algebra through Diagrams and Balances

Two Names for the Same Amount

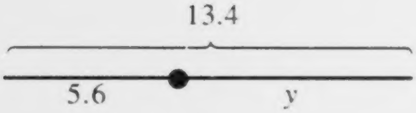
a. Describe the lengths of these line segments in two ways and then write an equation that sets them equal.

Example:



$x + 11 = 27$

1.



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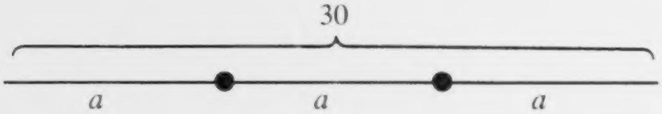
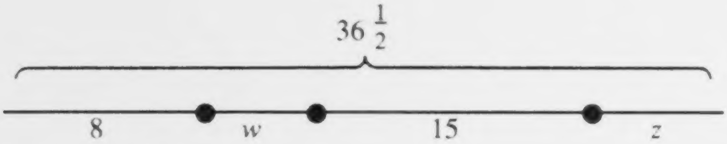
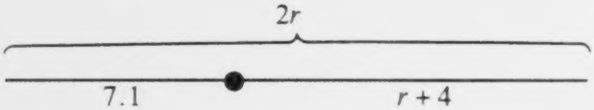


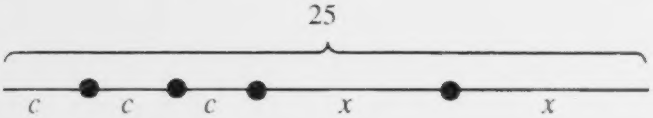
3.


$36\frac{1}{2}$
8
w
15
z

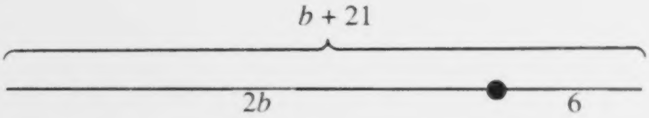
4.



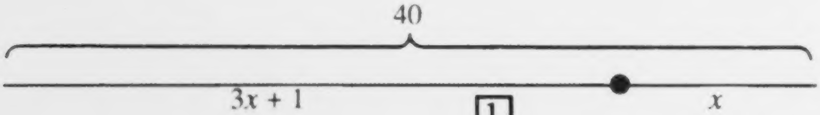
5.



6.

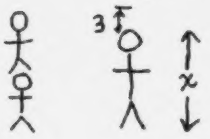


7.

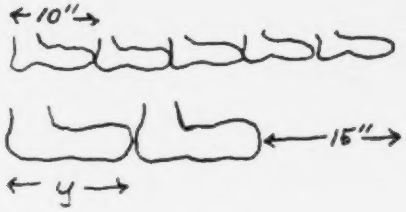


b. Each situation below is described by words and a drawing. Write an equation that fits the situation.

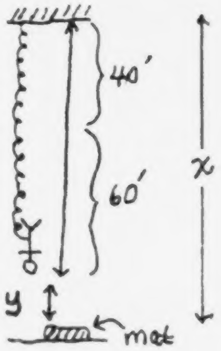
7. Aloke is 90 centimeters tall. Her father is twice her height, less 3 centimeters.



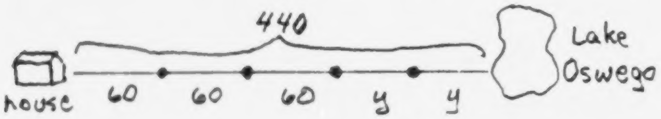
8. Ravi made a cast of Bigfoot's imprint. Two of Bigfoot's imprints plus 15 inches was the same distance as five of Ravi's 10-inch foot.



9. The bungee jump at the fair has a cord that is 40 feet long at rest. It will stretch another 60 feet at its furthest reach when someone jumps. To be safe, the maximum stretch of the cord and jumper needs to still be y feet above the mat. The jumping platform is x feet above the mat.



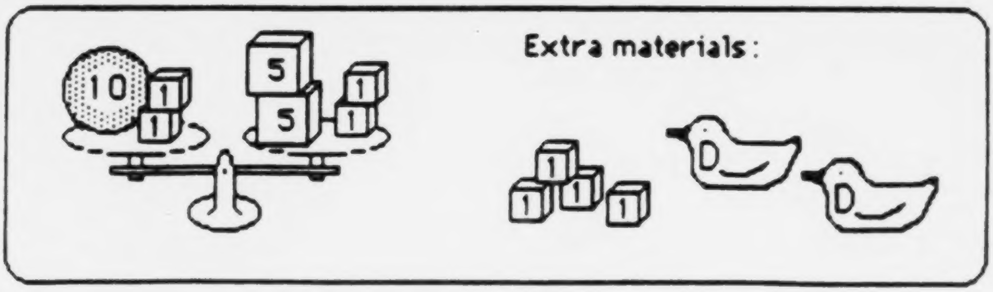
10. The distance from Lake Oswego to Jinwoo's house is 440 miles. She drove for 3 hours at 60 miles an hour and 2 hours at y miles an hour and was still 150 miles away.



Operations with Balances

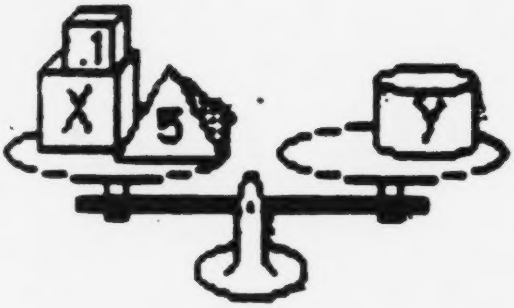
Answer the following questions for the balance situations shown. All objects that appear to be similar are of the same size and weight.

11. Consider the scale balance shown in the picture.



- Why is the balance level? How do you know?
- What will happen if you add 1 to both sides? Will it still balance?
- What will happen if you remove two 1's from both sides?
- What will happen if identical toy ducks are added to each side?

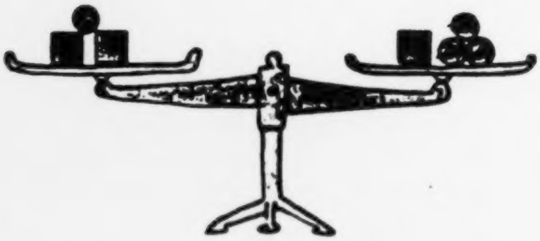
12. Write an equation for the balance shown to the right. How is the equation similar to the balance?



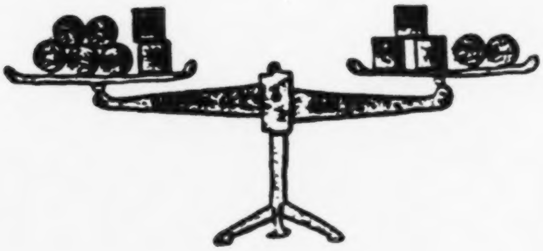
Working with a Balance Model

Set up a balance model for each situation shown on the next page. (Two similar-sized and shaped boxes placed on either end of a ruler can model the balance. Choose manipulatives to represent the objects shown in the situation. Place them in the appropriate boxes.) Answer each question by adding or removing objects so that the balance would be kept.

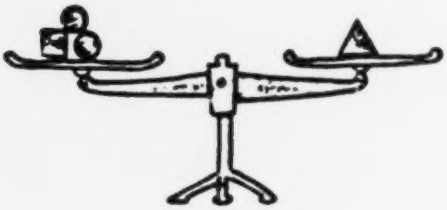
13. How much does a ■ weigh?

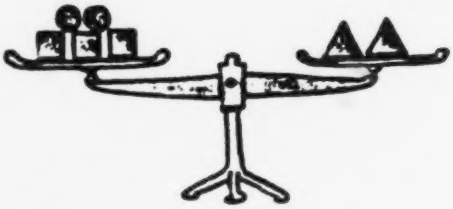


14. How much does each ● weigh?



15. The same kinds of weights are used in both pictures.

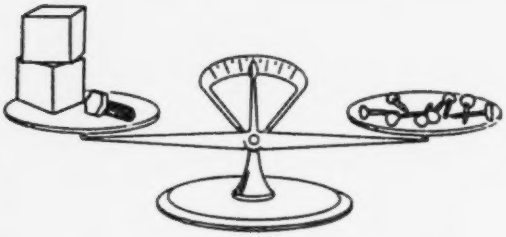


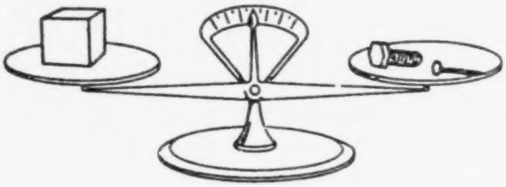


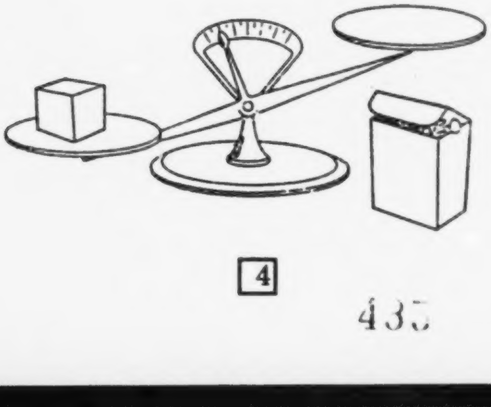
A. ▲ = ? ■

B. ■ = ? ●

16. How many nails will balance a cube? Explain how you know.





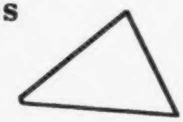


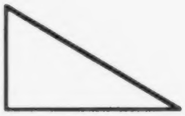
ACTIVITY 13

Tessellating

Covering a plane surface with a regular pattern of polygons is called tiling or *tessellating*. Some polygons will tessellate a surface and some will not. Which ones of the following polygons do you think will tessellate (tile) the table top? Get some pattern blocks from your teacher and try.

Triangles

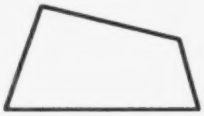

acute-angled


right-angled

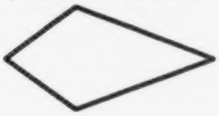
obtuse-angled

Quadrilaterals

Basic quadrilateral shape:



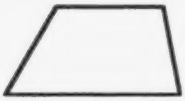
Specific quadrilaterals:


kite

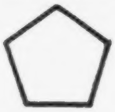
parallelogram

square

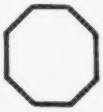
rectangle

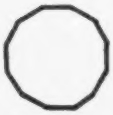

trapezoid

Other Polygons


pentagon

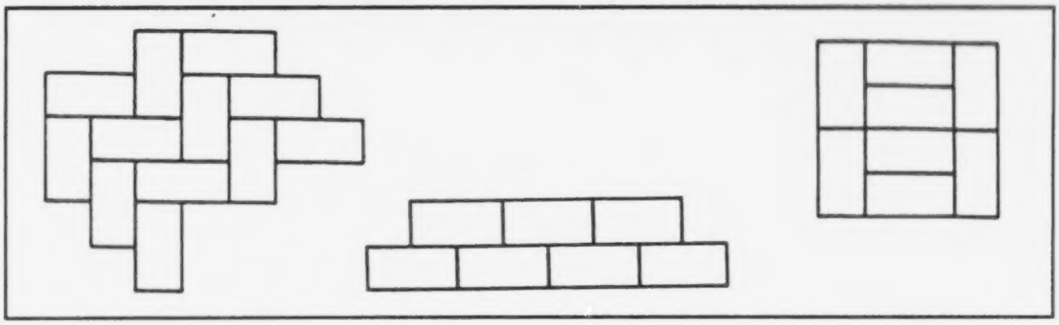
hexagon


octagon

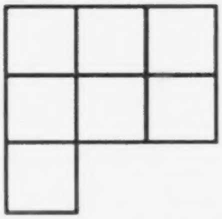

dodecagon

a. Draw a sketch of any interesting tessellating patterns that you can find with any of these shapes.

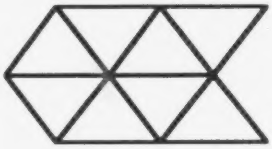
Here are some tessellations involving the rectangle.



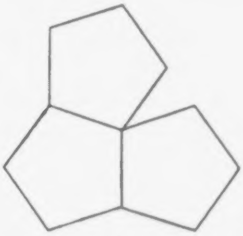
Not all shapes tessellate.



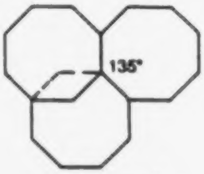
squares tessellate



equilateral triangles tessellate



pentagons leave gaps



octagons overlap

b. For each of the figures above, select a vertex where the polygons come together. Find the sum of the angles about that vertex.

Angle sum

for squares: ______

for equilateral triangles: ______

for pentagons: ______

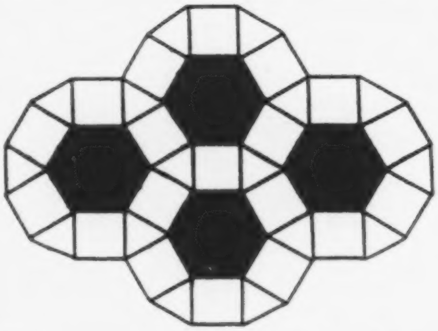
for octagons: ______

c. What can you conclude about the angle sum about a vertex if tessellation occurs? does not occur?

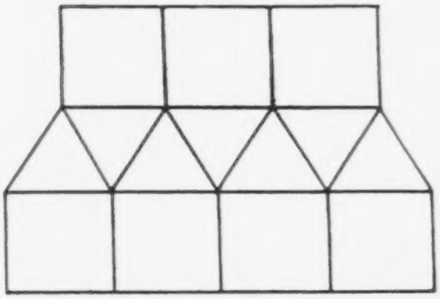
d. All triangles and all quadrilaterals can tessellate. To see how this occurs, cut an acute or obtuse triangle from tagboard and use it to trace congruent triangle shapes on the tagboard. Cut several out and then arrange them so that they tessellate. Make a drawing below to show how triangles tessellate.

e. Repeat this activity for a general quadrilateral (other than square or rectangle). Make a drawing below to show how quadrilaterals tessellate.

More interesting tessellating patterns can be formed by using more than one shape in the pattern (floor designs, for example). One way to describe a tessellating pattern is to list the *number* of sides for the regular polygons that come together at each *vertex* (corner).



Hexagons, squares, equilateral triangles
(6-4-3-4)



Squares, equilateral triangles
(4-4-3-3-3)

Some other interesting tessellating patterns are these:

8-4-8 6-3-6-3 12-3-12
12-4-6 6-3-3-3-3 12-3-4-3
3-3-6-6 3-4-4-6 4-3-4-3-3

f. Tessellate the top of the table with the following patterns and then make a drawing of each pattern: 8-4-8 and 6-3-6-3

g. Explain by showing the angle sums about a vertex why these two patterns will tessellate: 3-4-4-6 and 12-3-4-3

h. How does the pattern 3-3-6-6 vary from the pattern 6-3-6-3? Show your results with a drawing below.

Extension: Look up the topic of *Escher drawings* and report on how the Dutch artist Escher used tessellations to create famous paintings.

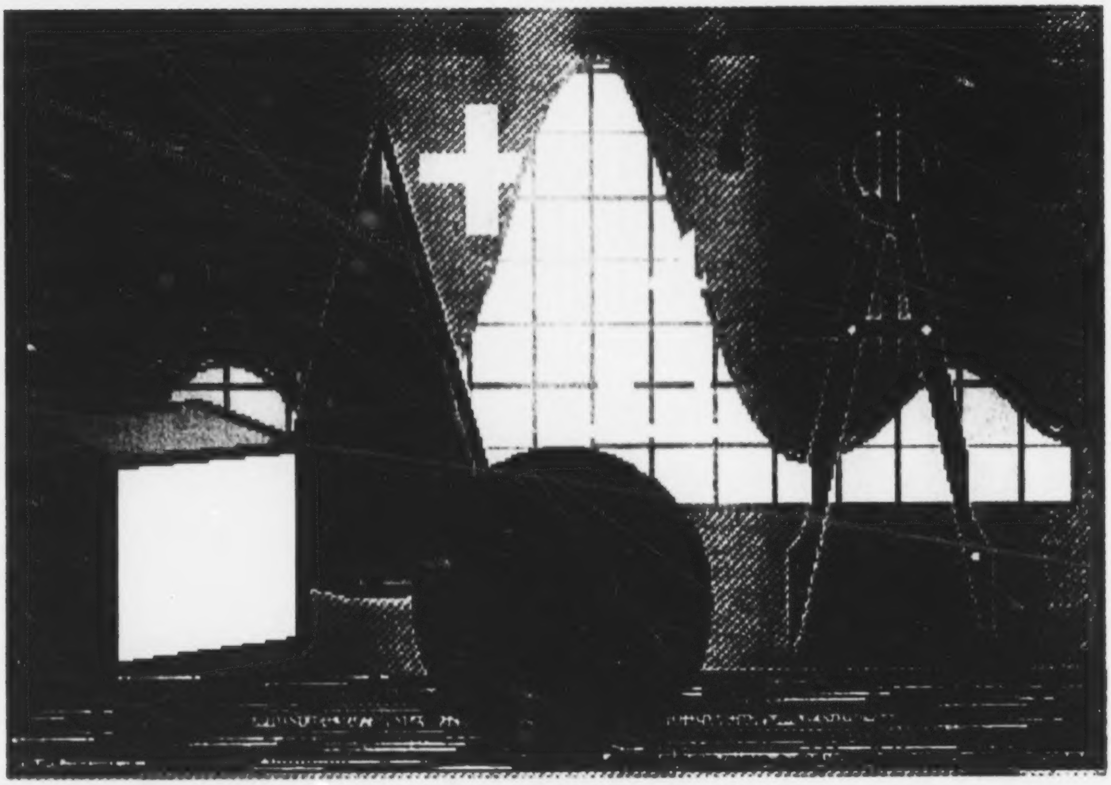
Summary of Type 3 Assessment Activities

Strands	Activity Name	Strand Tally
1, 4, 5, 6	1. Shape Changes in Geometry for a Holiday	1. ////
1, 2	2. Patterns Among the Toothpick Squares	2. ////
3	3. What's the Best Plan?	3. //
5, 8	4. A Matter of Temperature	4. ////
2, 3	5. Devising an Economical Transportation Plan	5. ////
6, 8	6. Corral the Area	6. ////
1, 2, 5	7. Badminton Tournament	7. /
1, 6	8. Doubling on the Cube	8. ///
8	9. M & M Investigation	
4, 6, 7	10. Outdoor Measurement	
4	11. Regular Polyhedra	
2, 5	12. Algebra through Diagrams and Balances	
4	13. Tessellating	

Competency-Based Education Assessment Series

Seventh Grade Mathematics

Type 1 Assessment



Developed by

Adele Cohn

Linda Gojak

Steven Meiring

The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency assessment instrument has a traditional design and is intended to be given in one period of approximately 45 minutes. Twenty-six of the items are short-answer, multiple-choice questions that generally assess one objective.

Items 27 - 30 are questions of a different character. Items 27-29 are worth two points each and call for a somewhat more extended and individual student response that requires a scoring rubric. The following rubric is suggested in assigning 2, 1, or 0 points for the student's work:

2 = correct answer and explanation are given
1 = correct answer without thorough explanation or incorrect answer with an appropriate solution method that can produce the correct answer
0 = other

Question 30 is a more involved item that requires the student to justify his or her solution in two ways and to use a sketch in one of those explanations. Each of the solution explanations is worth 2 points and should be scored according to the above rubric.

Items 27 -30 provide information about students' thinking as contrasted to the preceding items. It is important that assessment determine not only whether or not students have met easily measurable objectives, but also that the assessment itself reflect the full scope of the CBE program. Items requiring students to construct their solutions offer much richer information – for example, as to whether students can:

- *recognize and draw upon patterns* (item 27);
- *discern and differentiate among properties and relationships* (item 28);
- *apply models to real-world phenomena* (item 29);
- *communicate their ideas in writing* (items 28-30); and
- *integrate discretely-learned skills and their thinking to solve more complicated situations* (item 30).

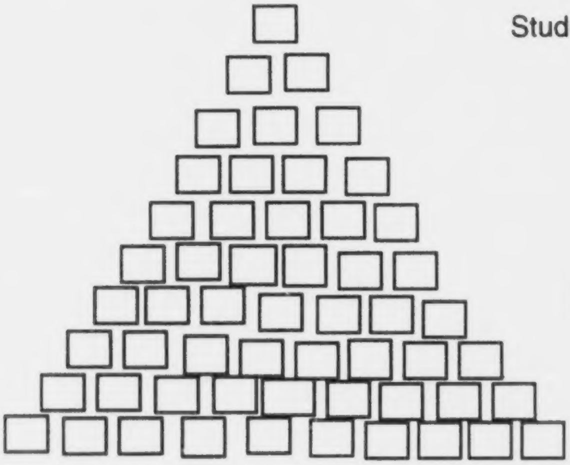
By including some such items on each assessment, we demonstrate a more consistent linkage among instruction, learning, and evaluation.

The discussion that follows provides additional information about and examples of possible student work for items 27-30. Other solution strategies may also be acceptable. On items 27-29 students receive one point for an appropriate solution process and one point for a correct solution.

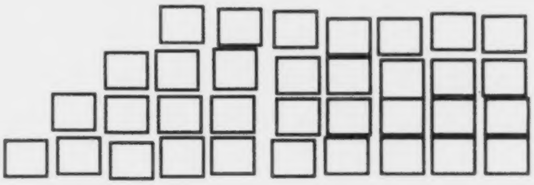
Discussion

27. Solution Strategy 1: Draw a diagram

Two samples of diagrams follow.



Students may not need a complete diagram.



Student work might include a part of the diagram from which they recognize the pattern of adding consecutive integers from 10 to 1.

Solution Strategy 2: Find a pattern

Some students may recognize the pattern without the use of a diagram.

$$10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 = 55$$

28. Correct properties of a square that are also properties of a parallelogram should be stated. Credit should not be given for incorrect information. Information should be given that is appropriate for the question. Examples include:

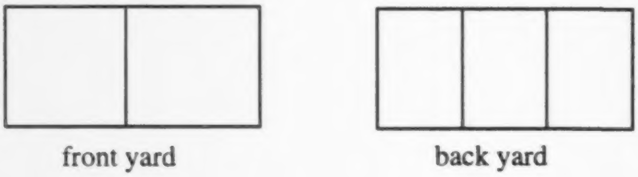
A square has 2 pairs of opposite sides that are parallel as does a parallelogram.

The opposite sides of a square are congruent which is also true of a parallelogram.

Note: The fact that a square has four right angles is not necessarily true for all parallelograms and should not receive credit as an appropriate reason. Any other statements that do not describe all parallelograms should not receive credit.

29. Some students may solve this problem using the rules of integers or they may reason it out using a number line or manipulatives. Credit should be given for any of these strategies.

30. Part A:



When raking the front yard, Tom and Dick each did half of a half or one fourth before Harry came. When they raked the back half of the yard, each of the three boys did a third of a half or one sixth of the total yard. Tom and Dick should get $\frac{1}{4}$ plus $\frac{1}{6}$ which equals $\frac{5}{12}$ of the money. Harry gets $\frac{1}{6}$ of the money.

Tom and Dick get $25 each and Harry gets $10.

Part B:

Another possible solution is to divide the payments into $30 for the front yard and $30 for the back yard. Tom and Dick get $15 each for raking the front yard and $10 each for the back yard. Harry gets $10 for raking the back yard.

Information Sheet

Item	Answer	Objective	Outcome Level*	Critical Objective
1.	A	7-8-2	C	yes
2.	C	7-6-1	K	yes
3.	C	7-3-5	K	yes
4.	D	7-4-3	C	yes
5.	A	7-6-1	K	yes
6.	B	7-6-4	K	yes
7.	D	7-3-1	C	no
8.	B	7-5-5	K	yes
9.	B	7-6-1	PS	yes
10.	D	7-5-2	K	no
11.	B	7-6-4	C	yes
12.	A	7-6-1	C	yes
13.	D	7-3-3	PS	yes
14.	B	7-3-2	PS	no
15.	C	7-3-3	PS	yes
16.	D	7-2-1	C	yes
17.	D	7-8-2	K	yes
18.	C	7-3-2	PS	no
19.	A	7-3-6	C	no
20.	C	7-7-3	K	yes
21.	D	7-5-3	C	yes
22.	D	7-4-6	K	yes
23.	B	7-3-10	PS	no
24.	C	7-8-5	PS	no
25.	A	7-8-9	PS	yes
26.	C	7-6-6	PS	yes
27.	55	7-1-1	PS	no
28.	yes See "Discussion"	7-4-3	C	yes
29.	7° F See "Discussion"	7-3-4	C	yes
30.	$25, $25, $10 See "Discussion"	7-2-2	PS	no

* PS = Problem Solving/Application
C = Concept
K = Knowledge/Skill

Problems 1-26 are worth one point each. Problems 27-29 are worth two each. Problem 30 is worth 4 points.	By Outcome Level: K: 9 C: 10 PS: 11	By Strand: 1: 1 4: 3 7: 1 2: 2 5: 3 8: 4 3: 9 6: 7

Circle the correct answer to the following multiple choice questions.

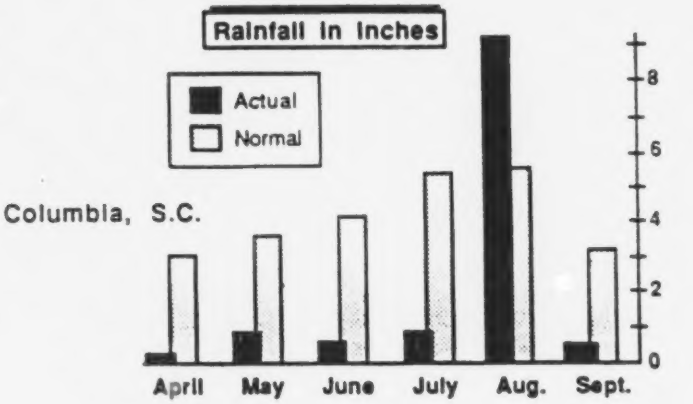
1. How much more rain actually fell in Columbia, SC in July than in September?

A. 0.4 in.

B. 0.7 in.

C. 2.1 in.

D. 3.0 in.



2. If a man can complete a job in 14 days, how much of the job can he do in $\frac{1}{2}$ day?

A. $\frac{1}{7}$

B. $\frac{1}{16}$

C. $\frac{1}{28}$

D. 7

3. Which equivalence is correct?

A. 20% = $\frac{1}{4}$

B. 80% = .08

C. $\frac{2}{5}$ = 40%

D. none of these

4. Which phrase describes a square and only a square?

A. four right angles

B. all sides congruent

C. opposite sides parallel

D. four right angles and all sides congruent

5. To measure the length of a classroom which unit of measure is most appropriate?

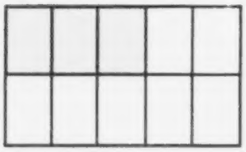
   A. feet
   B. centimeter
   C. kilometer
   D. inch

6. Find the area of a triangle with base 5 cm and height 10 cm.

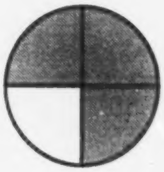
   A. 50 cm^2
   B. 25 cm^2
   C. 15 cm^2
   D. 2 cm^2

7. Which of these figures shows a fractional amount equal to 60%? (The amount is shown by shading, pointer, or circling.)

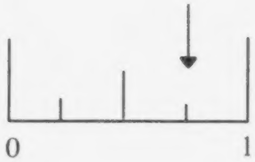
A.



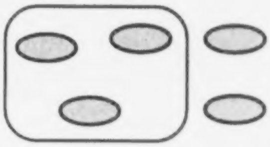
B.



C.



D.



8. Find the value of $2a + 3b$ if $a = 7$ and $b = 9$.

   A. 66
   B. 41
   C. 39
   D. 26

44

9. Mary's checkbook shows she has a balance of $105. She writes checks for $22.50 and $43.75. She also deposits a check for $85. What is her new balance?

Amount of Payment	Amount of Deposit	BALANCE

A. $86.25

B. $123.75

C. $151.25

D. $206.25

10. The diameter of a wheel is 16 in. How far does the wheel travel in 3 complete turns? (Use the formula $C = \pi \bullet d$)

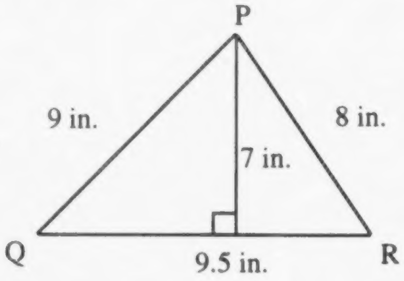
A. 50.24 in.

B. 57.42 in.

C. 59.66 in.

D. 150.72 in.

11. Find the area of triangle PQR.



A. 26.5 in.^{2}

B. 33.25 in.^{2}

C. 33.5 in.^{2}

D. 66.5 in.^{2}

12. Find the perimeter of the triangle PQR in question 11.

A. 26.5 in.

B. 33.25 in.

C. 33.5 in.

D. 66.5 in.

13. A recipe that serves 6 people calls for 4 lbs of chicken. How much chicken is needed to serve 10 people?

A. 2.4 lbs

B. 15 lbs

C. 8 lbs

D. $6\frac{2}{3}$ lbs

14. The Home Shopping Network adds shipping and handling charges to all purchases. The charges are 10% of the cost of the item purchased. You order a set of CD's for $29.50. What will be the total cost?

A. $26.55

B. $32.45

C. $39.50

D. $59.00

15. One inch on a road map represents 35 miles. How many inches on a map represent 175 miles?

A. 14 in.

B. 9.5 in.

C. 5 in.

D. 4.5 in.

16. The difference between twice a number, *x*, and 16 is 30. Which equation states this relationship?

A $30 - 16 = x$

B. $x - 16 = 30$

C. $2x + 16 = 30$

D. $2x - 16 = 30$

17. The graph shows attendance at the school band concerts.

September	&&&&&
December	&&&&&&&&
March	&&&&
May	&&&&&&

Key: each & represents 75 people

How many more people attended the concert in December than in March?

A. 4

B. 150

C. 225

D. 300

18. A coat that regularly sells for $106 is on sale at 20% off. What is the sale price of the coat?

A. $21.20

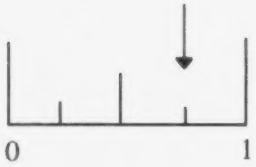
B. $53.00

C. $84.80

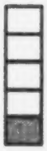
D. $86.00

19. Following are models for showing fractional amounts. Which represents the largest fraction? (The amount is shown by pointer, shading, circling, or numeral.)

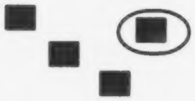
A.



B.



C.



D. $\frac{3}{10}$

20. The population of Ohio in the 1990 census was 10, 847,000. About 1% of the state's population in 1990 was

A. 10,000,000

B. 1,000,000

C. 100,000

D. 10,000

21. A bag of marbles contains 16 red, 12 white, 4 blue, and 10 green. What is the ratio of red and green marbles to the total number of marbles?

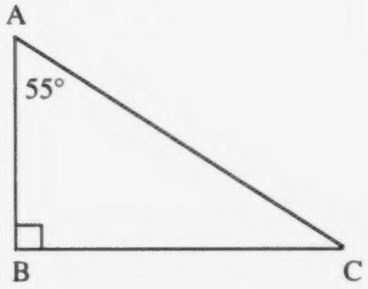
A. 7 : 5

B. 8 : 5

C. 13 : 8

D. 13 : 21

22. ABC is a right triangle. What is the measure of angle ACB?



A. 90°

B. 55°

C. 45°

D. 35°

23. Jim and Connie start a new job on the same day. Jim works every fourth day and Connie works every sixth day. What is the fewest days before they will again be working on the same day?

A. 10 days

B. 12 days

C. 16 days

D. 24 days

24. A snack bar sells five different kinds of sandwiches, four different kinds of juice, and two different kinds of fruit. How many different choices are there for ordering a sandwich and one kind of juice?

A. 10

B. 11

C. 20

D. 40

25. You bowl two games and have scores of 180 and 170. What score in the third game would increase your average score to be 185 for the three games?

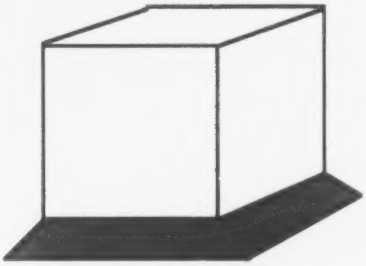
A. 205

B. 200

C. 195

D. 185

26. Park employees placed a cube-shaped sculpture on a base. The sculpture's edges measure 2 feet. What is the total area that must be painted to protect the sculpture's exposed edges from the weather?



A. 12 ft^2
B. 16 ft^2
C. 20 ft^2
D. 24 ft^2

Answer the following questions in the space provided.

27. Margie is building a triangular shaped display of facial tissues in the supermarket. The display is ten rows high. There are 10 boxes of tissues in the bottom row. If there is one less box in each of the rows above, how many boxes does Margie use to build the display? Show your work.

28. Is a square a parallelogram? Explain your answer.

29. The temperature at 7:00 AM was -3 ° Fahrenheit. By noon it had increased by 10 Fahrenheit degrees. What was the temperature at noon? Explain or show how you found the temperature.

30. Mr. Jones hired Tom, Dick and Harry to rake leaves. The house has a back yard and a front yard that are about the same size. The neighbor agrees to pay the three boys $60 for the entire job.

On the day of the job Tom and Dick finished the front yard before Harry arrived. All three boys raked the back yard.

How should the money be split between the boys? Each boy must be paid based on the amount of the yard he raked.

Explain your solution in two ways

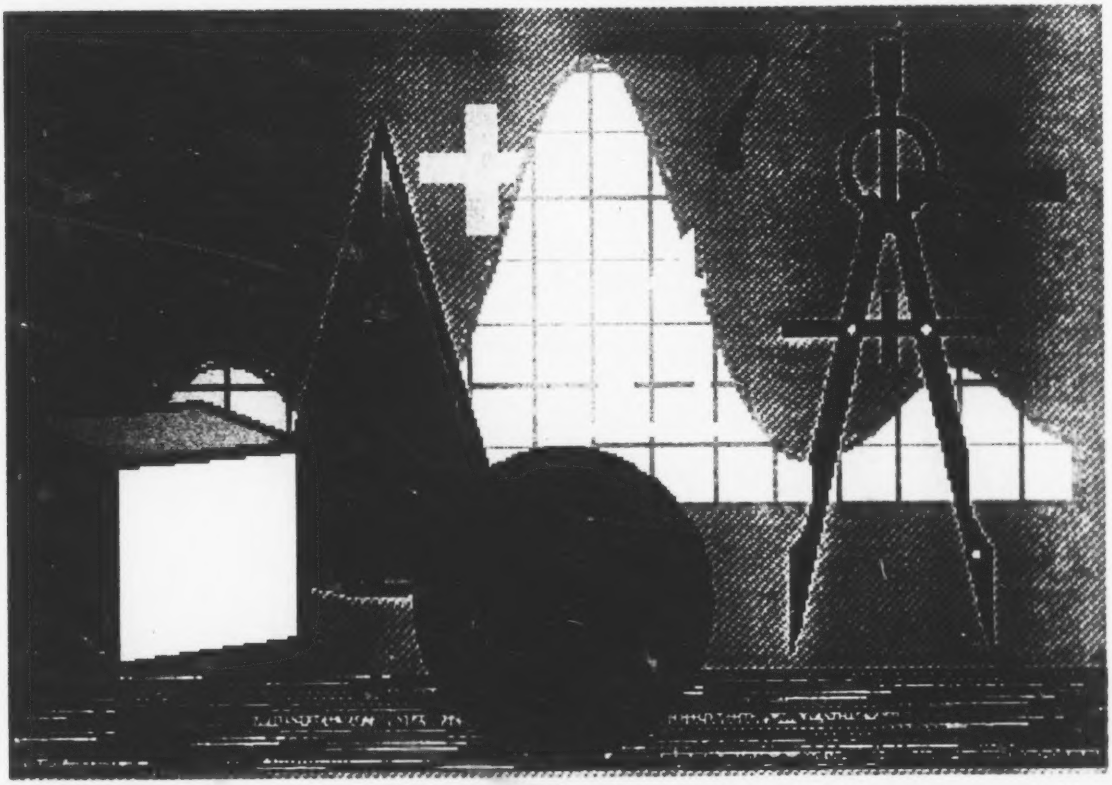
a) Use a diagram to explain your solution.

b) Use another way to explain your solution.

Competency-Based Education Assessment Series

Seventh Grade Mathematics

Type 2 Assessment



Developed by

Adele Cohn

Linda Gojak

Steven Meiring

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The model competency-based education asssessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

Task Force Members

Evelyn Altherr	Mansfield City Schools
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About This Instrument

This model competency assessment is designed to be given in two parts, each requiring one period of approximately 45 minutes for completion. The instrument is a mixture of more traditional design items (multiple-choice), open-form items, and items requiring extended responses. Some items are direct, straight-forward measures of particular objectives. Several items assess combinations of two objectives. A few items require students to integrate several ideas in constructing their responses (15 and 30).

The instrument as a whole assesses a deeper level of understanding than the Type 1, Seventh Grade instrument. Fifteen of the items are written at a problem-solving, outcome level. Items such as (3) require the student to read a pictograph and then to find the average of the data. Items such as (7, 17) do not yield whole number results, thereby requiring student interpretation. Other items such as (10) require careful reading because of the question asked.

Several items (5, 11, 13, 26) provide measures of students' understanding of key concepts. Items such as (8, 20, 23) draw upon students' visualization skills. Item 15 asks students to discern what is wrong with "impossible" geometric figures. Item 29 requires students to describe the rule for a pattern and to predict on the basis of that rule. Item 30 asks students to critically assess the validity of a numerical claim.

Open-form items provide information about students' thinking as contrasted to some of the other items. It is important that assessment determine not only whether or not students have met easily measurable objectives, but also that the assessment itself reflect the full scope of the CBE program. Items requiring students to construct their solutions (9-10, 12-13, 24-25, 26, 29) offer much richer information – for example, as to whether students can:

- *recognize and draw upon patterns*
- *discern and differentiate among properties and relationships*
- *apply models to real-world phenomena*
- *communicate their ideas in writing*
- *integrate discretely-learned skills and their thinking to solve more complicated situations*

By including some such items on each assessment, we demonstrate a more consistent linkage among instruction, learning, and evaluation.

Point values are assigned for ease of scoring items and with some consideration of item difficulty. The following scale is suggested:

2 points: items 1-13, 16-27, 30a, 30b **3 points**: items 14, 15, 28, 29

Additional item-specific scoring suggestions can be found in the discussion that follows.

Discussion

Additional Answers and Suggestions

9. Solution: 21 papers

Comments: Student solution process should show a method of finding the greatest common factor of 252, 168, and 105.

10. Solution: 615,000 square feet

Comments: Some students may prefer to calculate with the numbers given in the problem. Others may use mental computation by finding 40% of the total area and then adding 1% of the total area to that amount. Other mental calculations are also appropriate.

12. Solution: 8 tables

Comments: Students are actually looking for the longest rectangle with a perimeter of 18 which is 8 by 1. Drawing a diagram is an appropriate strategy.

13. Any of these answers are possible:

1 row of 180 | 2 rows of 90 | 3 rows of 60
4 rows of 45 | 5 rows of 36 | 9 rows of 20
10 rows of 18 | 12 rows of 15

and reversals of each of these (e.g., 180 rows of 1, 90 rows of 2 – which appear the same)

Score 0.5 point for each answer including the choice and explanation. Generally, boxes are chosen for pleasing appearance and packing reasons. Any of the answers 9 by 20, 10 by 18, and 12 by 15 satisfy these criteria.

14. Answers are: a) a, c, d, e, f b) b c) a, b, c, d, f

15. For Figure 1: The angle measures sum to 175°. The sum of the angle measures of a triangle is 180°.

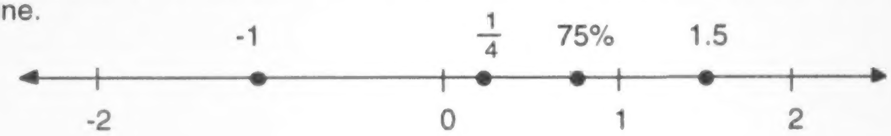
For Figure 2: The diagonal of a square must be longer than a side; OR two sides of the square and the corresponding diagonal form a right triangle whose sides must satisfy $a + b = c$.

For Figure 3: This is an impossible triangle. The sum of the lengths of any two sides of a triangle must be more than the length of the third side.

24. Solution: 48 teachers, 6 administrators

Comments: Students may solve this problem by using ratios or using percents.

26. Score 0.5 point for each number that is approximately located correctly on the number line.



29. Score 2 points for identifying 21 tiles to make the tenth tile figure and one point for the rule. There are many rule patterns that are possible for which the following are examples:

odd whole numbers starting with 3	
position number + next number	ex. 1 + 2 2 + 3 3 + 4 . . . 10 + 11
2 x position number + 1	ex. 2•1 + 1 2•2 +1 2•3 +1 . . . 2•10 +1

30. Score parts a. and b. as two points each. 0, 1, or 2 points can be assigned according to quality of the argument to support or dispute the bus company's claim. The key essentials of the arguments are as follows:

a. The bus company makes this claim upon the basis that many cars have 1 occupant while the bus carries many occupants. The bus could easily carry 45 passengers and therefore claim to replace 45 cars.

b. Buses are seldom full except during rush hours. Therefore, a half-full bus would not carry as many passengers as 45 cars. OR if each of 45 cars had two occupants, this would be 90 passengers which is beyond the bus' capacity. (In later discussion with students, a teacher can note also that the claim does not take into account such factors as operating, maintenance, and original costs; road wear; noise; pollution; nor convenience.)

Information Sheet

Item	Answer	Objective	Outcome Level	Critical Objective
1.	B	7-7-4	K	no
2.	C	7-5-3, 7-6-8	K	no
3.	B	7-8-2, 7-8-9	PS	yes, yes
4.	A	7-3-3	K	yes
5.	D	7-3-5	C	yes
6.	D	7-3-2, 7-7-3	PS	no, yes
7.	C	7-3-3	PS	yes
8.	A	7-6-6	PS	yes
9.	See "Discussion"	7-3-10	PS	no
10.	See "Discussion"	7-3-2, 7-6-4	PS	no, yes
11.	a) 200; b) 50	7-3-1	C	no
12.	See "Discussion"	7-2-4	PS	no
13.	See "Discussion"	7-3-10	C	no
14.	See "Discussion"	7-4-1, 7-4-3	C	no, yes
15.	See "Discussion"	7-4-6	C	yes
16.	A	7-5-5	K	yes
17.	A	6-3-1	PS	yes
18.	D	7-8-2	PS	yes
19.	C	7-3-3	K	yes
20.	B	7-6-8	PS	no
21.	B	7-2-4	PS	no
22.	C	7-6-4, 7-7-5	C	yes, no
23.	C	7-6-5	PS	no
24.	See "Discussion"	7-2-1	K	yes
25.	17 cm	7-6-8	PS	no
26.	See "Discussion"	7-3-5	C	yes
27.	2800 ft	7-3-4, 7-2-4	C	yes, no
28.	a) c ; b) a; c) a, f	7-4-1, 7-4-3	C	no, yes
29.	See "Discussion"	7-1-1, 7-1-4	K, PS	yes, no
30.	See "Discussion"	7-8-6, 7-8-7	PS	no, no

* PS = Problem Solving/Application
C = Concept
K = Knowledge/Skills

Problems 1-13, 16-27, 30a, 30b are worth two points each. Problems 14, 15, 28, 29 are worth 3 points each.	By Outcome Level: K: 7 C: 9 PS: 15	By Strand: 1: 2 4: 5 7: 3 2: 4 5: 2 8: 5 3: 12 6: 7

Part One

Circle the correct answer to the following multiple choice questions.

1. Estimate 3.04 x 5.3

   A. 1.6

   B. 16

   C. 160

   D. 1600

2. The perimeter of a rectangle can be found by the formula $P = 2(l + w)$. Find the perimeter when the length is 13 feet and the width is 11 feet.

   A. 35 feet

   B. 37 feet

   C. 48 feet

   D. 224 feet

3. The graph shows attendance at each of the school band concerts.

September	&&&&&
December	&&&&&&&&&&
March	&&&&
May	&&&&&&

Key: each & represents 75 people

Find the average attendance at the band concerts.

   A. 375

   B. 450

   C. 625

   D. 1800

4. A softball player had 2 hits in 5 times at bat. If she continues to bat at the same rate, how many hits will she get in 20 times at bat?

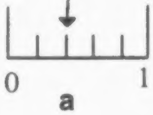
   A. 8

   B. 9

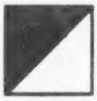
   C. 10

   D. 12

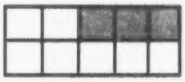
5. The following diagrams represent number amounts (indicated by pointer or shading). Put the amounts in order from least to greatest.



a



b



c

   A. **a, b, c**

   B. **b, a, c**

   C. **c, b, a**

   D. **c, a, b**

6. The Smiths went out for hamburgers and salads. The bill was $18.50. They want to leave a tip of 15%. They also need to add 5% to the cost of the meal for sales tax. What is the total cost of the meal including tax and tip?

   A. $ 3.70

   B. $20.35

   C. $21.28

   D. $22.20

7. If Fred feeds 12 horses for two weeks with 25 bales of hay, how many bales of hay does he need to feed 15 horses for two weeks?

   A. 20

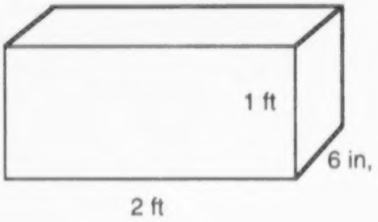
   B. 27

   C. 32

   D. 52

8. How many square feet of contact paper are needed to cover all sides of this package. No overlap is needed. Look at the dimensions carefully.

A. 7
B. 9
C . 12
D. 40



Record your answer in the space provided. Show your work.

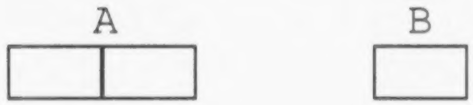
9. Bill, Adele, and Richard deliver newspapers. Bill delivers 252 papers, Adele delivers 168 papers, and Richard delivers 105 papers. The newspapers are packaged in full bundles with the same number of papers in each bundle. What is the greatest number of papers that there could be in a bundle?

Answer

10. The city of Akron plans to build a shopping mall in a rectangular lot measuring 1500 ft by 1000 ft. The city estimates that 59% of this space will be needed for parking. How many square feet will the shopping area cover?

Answer

11. Given



a) A is ________ % of B   b) B is ________ % of A

12. Simone arranged square tables, each seating 4 people, into one long rectangular table so that her 18 dinner guests could eat together. How many tables did she use? Show your work.

Answer

13. You work for a company that designs boxes for other companies. You need to design a box for 180 crayons. One possible box has 6 rows each with 30 crayons. List 3 other ways in which the crayons could be packaged. Which package would you choose? Explain why you would choose that package.

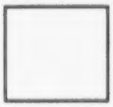
Indicate your choice and reason in the space below.

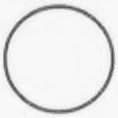
Package Design A

Package Design B

Package Design C

14. Classify the figures below according to common properties.


a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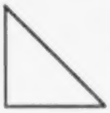
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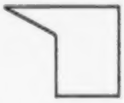
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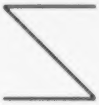
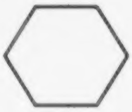

e


f

example: not polygons b, e

a) straight edges ______________________

b) curved edges ______________________

c) simple closed curve ______________________

15. Look at these plane figures some of which are not drawn to scale. Investigate what might be wrong (if anything) with the given information. Briefly write your findings and justify your ideas by geometric principles.

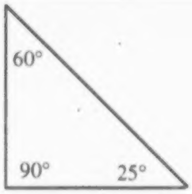


Figure 1

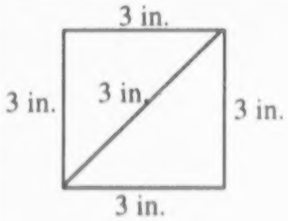


Figure 2

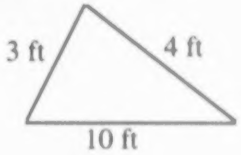


Figure 3

Part Two

Circle the correct answer to the following multiple choice questions.

16. Find the value of $\frac{2a+3b}{a+b}$ if $a = 5$ and $b = 8$.

A. $\frac{34}{13}$

B. $\frac{31}{13}$

C. $\frac{18}{13}$

D. $\frac{63}{13}$

17. Shandra is shipping boxes. It costs $15 to ship each box and there is also a charge of $25 for the entire shipment. Shandra has $135 to spend on shipping. How many boxes can she ship?

A. 7
B. 8
C. 9
D. 10

18. A 40-year-old adult takes the bus to work and back home 5 days a week. How much can be saved by buying a monthly pass over the One Way price? Assume 4 weeks in a month.

A. $ 46.50
B. $ 40.00
C. $ 18.00
D. $ 12.00

Bus Fares

	One Way	Weekly Pass	Monthly Pass
Adult	$ 1.50	$ 12.00	$48.00
Senior Citizen	$ .50	$ 4.00	$10.00
Student	$ .25	$ 2.00	$ 8.00

19. Twenty bald eagles are tagged and released into the wild. The next month nine bald eagles are spotted, two of which are tagged. About how many bald eagles are in the area.

A. 29

B. 31

C. 90

D. 182

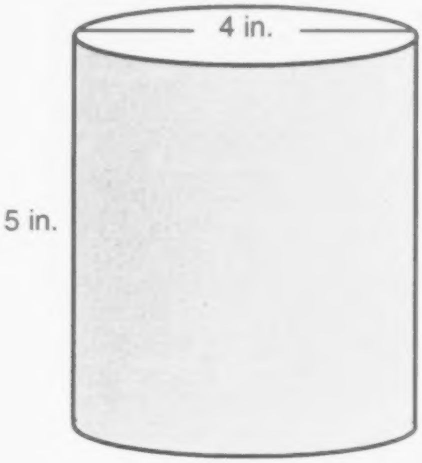
20. If a can of soup has a diameter of 4 inches and a height of 5 inches. What are the dimensions of the label that is wrapped around the can? Assume no overlapping.

A. 5 inches by 4 inches

B. 5 inches by 12.56 inches

C. 4 inches by 15. 7 inches

D. 5 inches by 6.28 inches



21. A telephone pole 47 feet long fell straight across a road. If $2\frac{1}{2}$ ft of the pole were on one side of the road and $14\frac{3}{4}$ ft of the pole were on the other side of the road, how wide was the road? Draw a picture to help you solve this problem.

A. $28\frac{1}{2}$ ft

B. $29\frac{3}{4}$ ft

C. $31\frac{1}{4}$ ft

D. 33 ft

22. The backboard on a basketball hoop is a square with an area of 10 square feet. What is the best estimate of the length of a side?

A. 2.5 ft

B. 2.9 ft

C. 3.2 ft

D. 3.8 ft

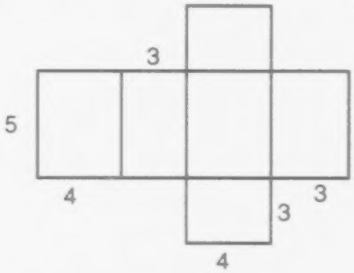
23. Find the volume of the prism (rectangular solid) if the pattern shown were folded. All units are cm.

A. 25 cm^3

B. 50 cm^3

C. 60 cm^3

D. 94 cm^3



Record your answer in the space provided. Show your work.

24. Smallpox Middle School has 1,000 students, 40 teachers, and 5 administrators. If the school grows to 1,200 students and the ratios are maintained, find the number of teachers and administrators that will be needed.

Teachers

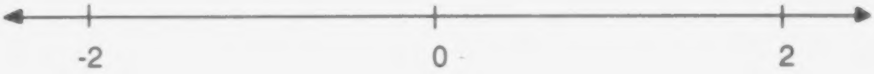
Administrators

25. A square and a quadrilateral have the same perimeter. The quadrilateral has sides with lengths of 18 cm, 23 cm, 12 cm, and 15 cm. Find the length of a side of the square.

Answer

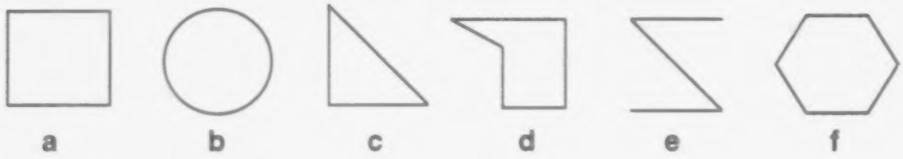
26. Place the following numbers on the number line. Identify the points.

75% -1 $\frac{1}{4}$ 1.5



27. You are visiting Death Valley National Monument. As your car begins to descend into the valley, you see a sign that reads "2600 feet above sea level". When you reach the valley floor you see a sign that reads "200 feet below sea level". Find the change in elevation between the two signs. Draw and label a diagram to solve this problem.

28. Classify the figures below according to common properties.



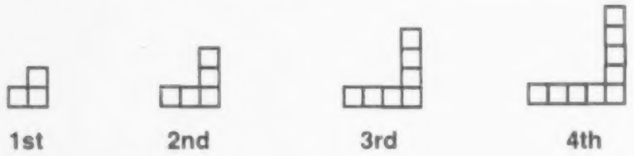
example: not polygons b, e

a) triangles ________________

b) quadrilaterals ________________

c) regular polygons ________________

29. Observe the following pattern of shapes made with tile squares.



a) How many tile squares would it take to make the tenth tile figure?

b) Describe a rule for making the pattern of tile figures.

30. The following sign appeared on a city bus.

when full this
vehicle replaces
45 cars

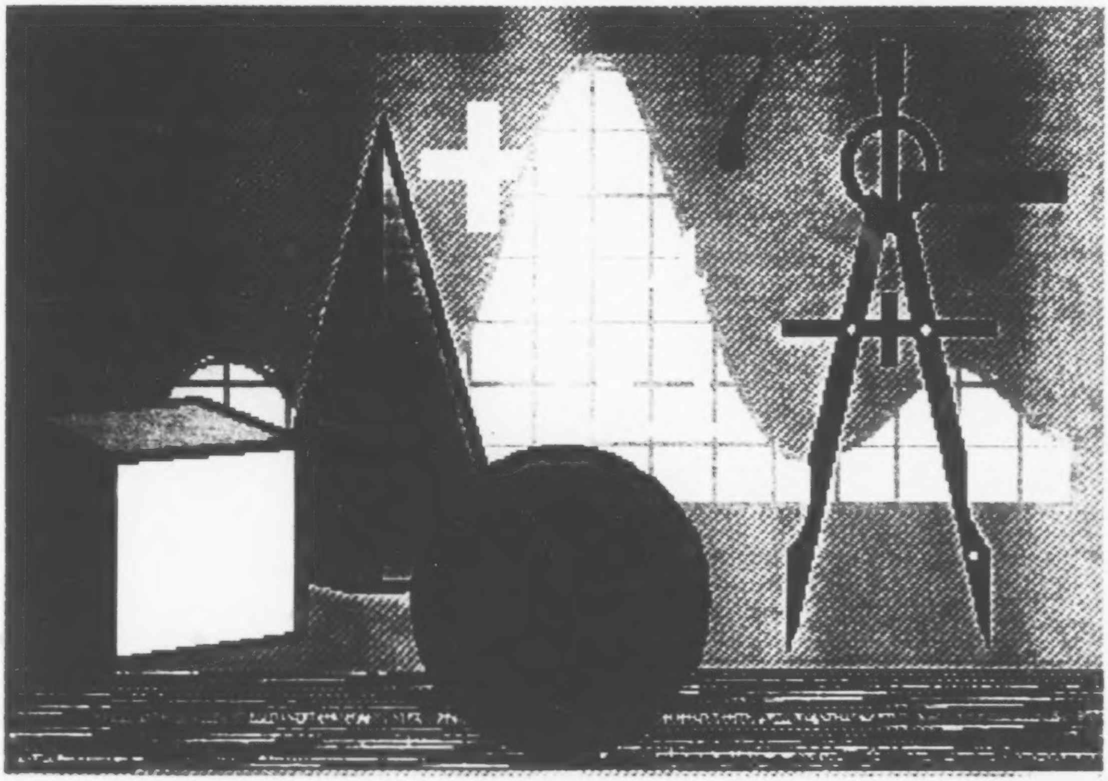
a) Write an argument based upon mathematics ideas to support the bus company's claim.

b) Write an argument based upon mathematics ideas as to why the bus company is NOT justified in making this claim.

Competency-Based Education Assessment Series

Seventh Grade Mathematics

Type 3 Assessment



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About This Instrument

This model competency assessment consists of performance tasks, each requiring a substantial period of time to complete. The tasks involve more authentic-type situations that reflect the ways in which mathematics is applied. Multiple skills, understandings, and thinking are required to complete each task. Rather than assessing single objectives, the tasks address *strands* of the mathematics curriculum. Several objectives may be assessed within the scope of a single task. The manner in which the assessment is carried out may also reflect upon the outcome levels measured relative to those objectives.

For example, Item 6 that follows, *(Directions for Drawing Figures),* addresses objectives 4-1, 4-2, 4-3, 4-4, 6-1, 6-2, 6-3, and 6-8 from the seventh grade strands of the *Model Competency-Based Mathematics Program.* Item 1 *(A Number Investigation),* can be used to assess how a student engages the activity (requiring a process rubric) or to assess outcomes achieved (requiring an outcome rubric). The intent of this instrument is to provide rich, performance tasks whereby a teacher can choose how to assess students and which objectives to focus upon within that activity. Each strand except Estimation and Mental Computation forms the basis of one of the tasks. Objectives from Strand 7 can be assessed in the context of tasks primarily focused toward other strands.

It is the expectation that these performance tasks will be given to students throughout the year at appropriate intervals. Multiple forms of assessment will be employed – for example, student products, teacher observations, interviews, self-assessment, and journal writings. Some tasks may be given individually and some in group settings. Products and records of students' activity may be kept in individual portfolios. At a suitable point toward the end of the year, a thorough review of the work and records within the portfolio can be used to determine how well the student is progressing relative to the competency-based outcomes of the seventh grade.

Characteristic of performance task assessment, the dividing lines between instruction, learning, and assessment become fuzzy. Students will engage many of these tasks within a learning, rather than a testing, environment. They will have the opportunity to converse with others, use appropriate manipulatives, employ calculating tools, and consult resources. The tasks are not time-restrictive for completion, and they permit multiple avenues of approach – making them more equitable and accessible to all students.

For more information about performance assessment, consult the *Performance Assessment* discussion at the end of this instrument and other publications such as the National Council of Teachers of Mathematics resource, *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark, 1991).

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Discussion

Activity 1: Assesses Strands 1 and 2. (See discussion on Performance Assessment pages.)

Solution: a) Powers of two (including $2^0 = 1$ and $2^1 = 2$) cannot be written as a sum of consecutive, positive whole numbers.

b) More than one sum is possible for many of the numbers.

c) Various patterns are possible. Here are some (note the 0, 1,2 are excepted):

1) Any odd number greater than 1 can be written as the sum of two consecutive whole numbers.
2) Numbers that can be written as a sum starting with 1 are: 3, 6, 10, 15, 21, ... For your information this pattern can be written as $\frac{n(n+1)}{2}$ for $n \geq 2$.
3) Primes greater than 2 can only be written as the sum of two consecutive whole numbers.
4) Numbers divisible by 3 can be written as sums of three consecutive whole numbers.
5) Numbers divisible by 5 can be written as sums of five consecutive whole numbers.
6) Numbers that can be written as (2 • odd number), where the odd number is greater than 4, can be written as sums of four consecutive whole numbers.

Methods used to find the answers to part c are as important as the answer.

Comment: This type of investigation activity most appropriately spans portions of several periods and involves students working in groups. For example, during the first period, groups of students could be asked to search for sums for numbers within a certain range (1-15, 16-26, ...) and write their results on large sheets of paper about the room. These collective results can then be searched for patterns in subsequent periods.

Activity 2: Assesses Strands 2 and 3.

Solution: One method of showing the solution is:

Nos. divisible by 2:	2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 34, 36, 38, 40, 42, ...
Nos. divisible by 9:	9, 18, 27, 36, ...
Nos. divisible by 6:	6, 12, 18, 24, 30, 36, 42, ...
Even nos. divisible by 9	18, 36, ...

Comment: A more sophisticated argument might be:
Even numbers can be written as two times another factor, 2 • P.
Divisibility by nine means they also have a factor of nine, 2 • 3 • Q or 2 • 3 • 3 • Q.
Since the number can be written as 2 • 3 • (3 • Q) or 6 • (3 • Q), it is divisible by 6.

Activity 3: Assesses Strand 3.

Solution: The cost of one notebook is $72 divided by 48 = $1.50. 10% of $1.50 = $.15. The notebook sells for $1.65.
Cost of one pen is $12.60 divided by 36 = $.347 or $.35. 10 % of $.35 = $.035 or $.04.
Pens should sell for $.39.
A notebook and pen sell for $1.65 + $.39 = $2.04.

Another solution process is:

To make a 10% profit, we must add 10% of the cost to what each carton (or box) cost us and then divide by the number of items in a carton (box).

Notebooks cost $72 per carton. Adding 10% (or $7.20), we must get $79.20 per carton. Since there are 48 notebooks per carton, we should sell them for $1.65 each.

Pens cost $12.60 per box. Adding 10% (or $1.26), we must get $13.86 per box. Since there are 36 pens per box, we should sell them for $ .385 each. Practically, that means that we could sell them for 2 for $.77 or $.39 each.

Comment: This item is ideal for allowing students to use a calculator. Since 10% is the profit rate, you could also determine whether students are making a suitable mental calculation (Strand 7).

Activity 4: Assesses Strand 3.

Solution: By extending both tables, we find that 13 letters is the deciding point. Store A charges $11.25 and Store B charges $11.30. For T-shirts requiring 13 or fewer letters, choose Store A. For T-shirts requiring 14 or more letters, choose Store B.

Look carefully at students' explanations and accept all logical solutions and reasoning.

Comment: Since patterns and functions are used to create the tables (and extending them), this problem can also be used to assess Strand 1.

Activity 5: Assesses Strand 3.

Solution: Look for ideas shown in the solution below.

The difference in the sales tax rates for the two counties is 1%. One percent of the purchase price of $2000 results in a $20 savings by making the purchase in Lake County.

If it costs 39¢ per mile to travel to (and from) a store in Lake County, your savings will be "eaten up" by whatever extra mileage that the Lake County store is further in distance than the Cuyahoga County store. Dividing $10 by 39¢ per mile (half of $20 going and half coming back), you can still achieve a net savings up to 25.6 miles additional distance.

Students might find the purchase price at the two tax rates and then subtract to find the savings.

Comment: This item is also suitable for allowing students to use a calculator. The 1% difference in sales tax rates (or use of given rates) could enable you to assess whether students can make appropriate mental calculations (Strand 7).

Activity 6: Assesses Strands 4 and 6.

Solution: a) You are going to draw a right triangle PQR with right angle at P. Mark point P on a line. Measuring 4.9 cm along the line to the right of P, mark point Q. Rotate 90° (construct a right angle) at point P from $\overline{PQ}$ in a counterclockwise direction. Along this ray, measure 3.7 cm from P and mark point R. Draw line segment $\overline{RQ}$. PQR is the right triangle.

b) You are going to draw a trapezoid KLMN with right angles at K and L and ***parallel*** sides KN and LN. Mark point K on a line. Measuring 3.3 cm along the line to the right of K, mark point L. At point K, rotate 90° from $\overline{KL}$ in a counterclockwise direction. Along this ray, measure 2.1 cm from K and mark point N. At point L, rotate 90° from $\overline{LK}$ in a clockwise direction. Along this ray, measure 4.3 cm from L and mark point M. Draw line segment $\overline{MN}$. KLMN is the trapezoid.

Note: Check measurements on student pages carefully as reproduction may cause a slight variance from numbers given above. Students may use inches or cenbimeters.

Comment: Student directions will be less efficient than those above and contain some ambiguities (e.g., rotate which way). Be fairly tolerant as long as reasonable choices of two alternatives will result in the correct figure. You may wish to point out, however, that the "other" alternative may result in the mirror image of the intended figure.

Activity 7: Assesses Stand 5.

Solution: a) This is simply a verification activity.

b)

Trick Directions	Explanation
Pick a number	❑
Add 4	❑ ▮
Triple the result	❑ ▮ ❑ ▮ ❑ ▮
Subtract 3	❑❑❑ ▮▮▮
Divide by 3	❑ ▮ ❑ ▮ ❑ ▮
Subtract the original number	▮

Your answer is 3.

c) Answers will vary.

Comment: Students will need assistance to realize that in a division step, only one of the sets remains as the answer (it is circled above). You may also wish to extend this assessment activity by giving students an explanation and asking them to supply the "trick" directions. You may also wish to introduce variables and ask them to write a variable column to show the steps with variables, operation signs, and numerals. The latter two extensions presume that the student has had some prior work with this type of activity prior to assessment.

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Activity 8: Assesses Strand 6.

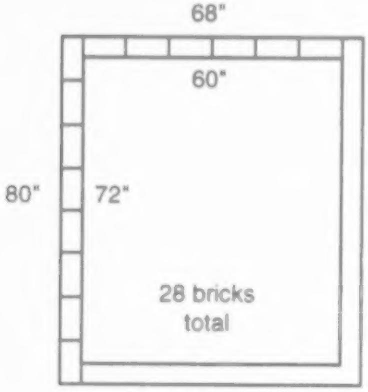
Solution: Answers to a) and b) are in the following table. The minimum perimeter is produced by room dimensions which are closest to be square (8' by 10').

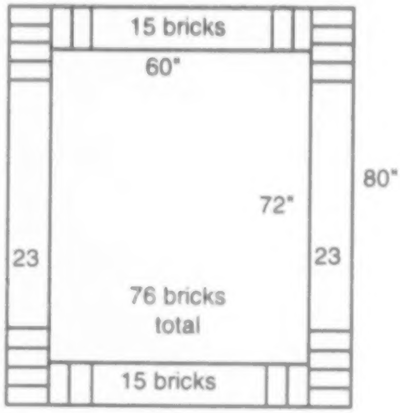
length	80	40	20	16	10
width	1	2	4	5	8
perimeter	162	84	48	21	36

Comment: Look at the methods the students use to develop the table. If a method is not shown, question the students.

Activity 9: Assesses Strand 6.

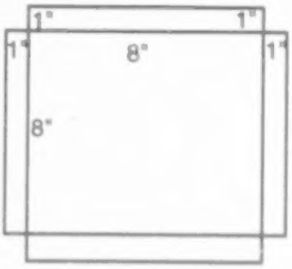
Solution: Two " standard design" borders are possible as shown below, requiring 28 and 76 bricks respectively. Other more creative designs may occur in students' work.





Activity 10: Assesses Strands 4 and 6.

Solution: Two-inch squares cut from each corner result in a maximum volume of 72 cubic inches as shown by the following calculations.

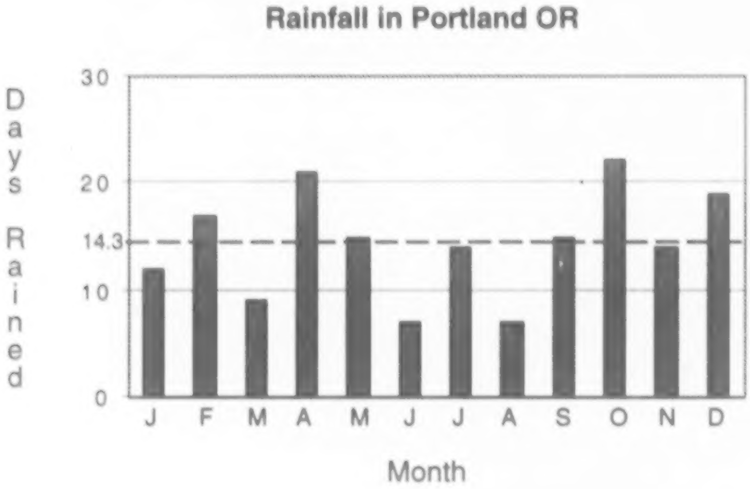


1-in. squares: V = 8 • 1 = 64 in.
2-in. squares: V = 6 • 2 = 72 in.
3-in. squares: V = 4 • 3 = 48 in.
4-in. squares: V = 2 • 4 = 16 in.

Comment: Give students several pieces of 10-in square posterboard, rulers, scissors, and tape to make models as they work to solve this problem. Students may want to verify their solution by using popcorn, sand or other materials to measure the volume of each box.

Activity 11: Assesses Strand 8.

Solution: The graph should look something like the bar-graph below. Notice that the mean-line cuts the bars off so that if all the bar-segments above this line were affixed to short bars below the line, all bars would be the same length – the mean.



The trend of rainfall is for consistent, heavier rains in the fall months through December. Rainfall during the rest of the year is one heavy-rain month followed by a lighter-rain month. The low rainfall months are June and August.

Acttivity 12: Assesses Strand 8.

Solution: This is a fair game, meaning that each player has an equal chance to win. On average, each player should win about 5 times in 10 turns.

Comment: Students can determine this result by simply playing the game many times and keeping track of the data. Alternatively, they could lay out the possiblities for rolls as shown in the table. Each outcome has an equal possibility. Of these, 12 outcomes have A winning 2 points each for a total of 24 points. For the remaining 24 outcomes, B wins one point each, again totaling 24 points. The game is fair.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Activity 13: Assesses Strand 8.

Solution: This is a meaningless statistic. In itself, it is not sufficient reason to buy an alarm system at an inconvenient time.

Memorial Day is the last Monday in May (could occur on 25, 26, 27, 28, 29, 30, 31). Labor Day is the first Monday in September (could occur on 1, 2, 3, 4, 5, 6, 7). Between Memorial Day and Labor Day, there are this many days (averages are used for May and September):

May:	3	days
June:	30	days
July:	31	days
August:	31	days
September:	3	days
	98	days

There are 98 days for a crime to occur out of 365 days for the year. Dividing, we find that this is 26.8% of the year. This is what we would expect if home break-ins occur at the same rate throughout the year.

Comment: It is not expected that students will know when Memorial Day and Labor Day occur (by definition). They may either look up the dates for the current year or consult a dictionary for the definitions. Their differing values will not appreciably affect the results.

Performance Assessments

Overview

"A performance assessment in mathematics involves presenting students with a mathematical task, project, or investigation, then observing, interviewing, and looking at their products to assess what they actually know and can do.

A performance task can:

- allow the examination of the process used as well as the answer or finished project;
- be used with groups as well as individuals;
- document, through observation records or student products, accomplishments not revealed by ordinary tests."

source: *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*

Performance assessments give students an opportunity to display a full range of ability. They emphasize better the nature of mathematics, its processes, and practice. The tasks are interesting and motivational. Assessment does not have to interrupt student learning. Richer and more complete information becomes available for making instructional decisions. A more engaging record of the broad character of the mathematics program is available for parents, administrators, and citizens.

Assessing Performance Tasks

As teacher, you must decided upon which outcomes you wish to focus for a particular task and then decide how to assess those outcomes. For example, let us consider Item 1 *(A Number Investigation)* from this instrument. The goals of the activity are for students to:

- systematically investigate consecutive addends;
- notice patterns about which numbers are sums of consecutive addends;
- formulate written or verbal generalizations about the patterns;
- hypothesize about why the patterns occur and how the sums can be predicted.

You could choose to focus upon how students go about the *process of investigation* and design a corresponding rubric:

1 - Needs significant instruction and direction
2 - Needs some instruction but initiates own investigation
3 - Conducts investigation and finds a few patterns
4 - Meets the expectations of the activity
5 - Exceeds expectations; makes unanticipated discoveries

Alternatively, you could focus upon outcomes the student achieves in either *patterning* or *problem solving:*

Patterning

1 - Shows an attempt at looking for patterns within the specific task
2 - Looks for additional patterns beyond the task and identifies one
3 - Identifies and/or describes two or more relevant patterns
4 - Identifies and makes reasonable generalizations about several patterns

Problem Solving

1 - Unsystematic in investigation
2 - Somewhat systematic; limited discoveries
3 - Systematic approach, but overlooks some evidence or makes an incomplete search
4 - Conducts thorough, systematic investigation with appropriate discoveries
5 - Exceeds level in generalizing and reasoning through discoveries

Another possibility is to focus upon the quality of the response:

Minimal Response. Only some of the essential conditions of the task are met.
Nearly Proficient Response. Most essential conditions are there with some small misconceptions, use of inappropriate concepts or skills, and/or incomplete explanations.
Proficient Response. Full application of knowledge and skills, communication appropriate to task.
Advanced Response. Goes well beyond all expectations along with displays of creativity, elegance, and exceptional reasoning.

Additionallty, a teacher can assess *attitudes* (interaction and participation):

0 = Dependent 1 = Needs Support 2 = Independent

ATTITUDES	Date			Comments
Cooperates				
Shares/Collaborates tries, contributes ideas				
Questions Peers encourages others to participate				
Takes Risks confident in own ability				
Stays on Task perseveres				

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Activity 1

A Number Investigation

Some numbers can be written as the sum of consecutive positive whole numbers

$$9 = 2 + 3 + 4$$

$$30 = 4 + 5 + 6 + 7 + 8$$
$$30 = 6 + 7 + 8 + 9$$

and some cannot:

$$8 \neq 1 + 2 + 3 + 4$$
$$8 \neq 2 + 3 + 4$$
$$8 \neq 3 + 4$$

The examples above show that 9 and 30 can be written as the sum of consecutive positive whole numbers and that 8 cannot be written as the sum of consecutive positive whole numbers.

a) Find all of the whole numbers from 1 - 70 which can be written as the sum of consecutive positive whole numbers. Write the equations you found.

b) The number 30 has at least two equations that fit this pattern. Explore to see if you can find other examples that can be written in more than one way. Write the additional equations you found.

c) Decide which numbers cannot be written in this way and then write a rule about the pattern that describes these numbers.

d) Compare your list from part c with other students. Then state other patterns that you can discover in the lists of these sums.

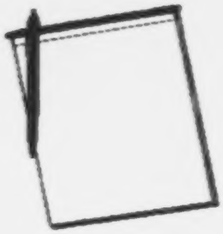
Activity 2

Divisibility

Explain why this statement is true by writing a logical argument. Use examples, as needed.

"Any even number divisible by 9 is also divisible by 6."

Activity 3



Marking Prices

The school store received a shipment of notebooks and pens. There are 10 cartons of notebooks and 25 boxes of pens. Each carton of notebooks costs $72 and contains 48 notebooks. Each box of pens costs $12.60 and contains 36 pens. If the school wants to make a 10% profit, what should be the price of a notebook and of a pen? Show your solution process.

Activity 4

Making the Best Buy

You want to buy T-shirts for your baseball team. The two tables below show the cost of T-shirts of the same quality at two different stores. (The tables continue beyond that shown.) The cost depends on the number of letters on the shirt.

Store A

Number of Letters	0	1	2	3	4	5	6
Cost	$ 8.00	$8.25	$8.50	$8.75	$9.00	$9.25	$9.50

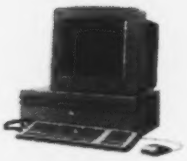
Store B

Number of Letters	0	1	2	3	4	5	6
Cost	$ 10.00	$10.10	$10.20	$10.30	$10.40	$10.50	$10.60

Which store offers the better buy? Explain your reasoning.

Activity 5

Where to Buy?



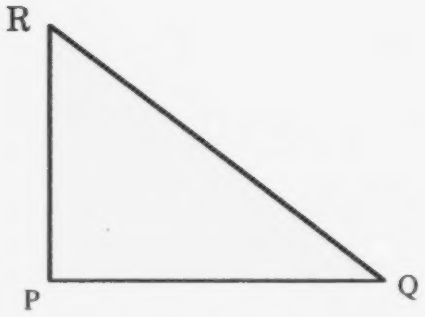
The price of a computer is $2000. You live in Cuyahoga County where the rate of sales tax is 7%. The rate of sales tax in neighboring Lake County is 6%. How much money will you save by buying the computer in Lake County? If it costs you 39 cents a mile to drive your car, how far away could you live from Lake County and still save on the price (including tax) of the computer?

Activity 6

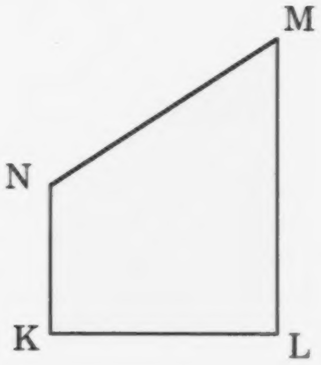
Directions for Drawing Figures

You have drawn the following figures on your paper and would like your friend on the phone to draw them. Write a set of directions so that your friend can draw the figures exactly as shown below. You may need to use a ruler and protractor.

a)



b)



Activity 7

Explaining Number Tricks

Number tricks can be explained by using ❑ to stand for a number (that we are thinking of) and tally marks || when we are adding specific amounts (like two). By following the directions of the trick, using boxes and tally marks, we can see why the trick works.

a) Study the following "explanation" of a number trick. Substitute two numbers of your own choosing to verify that the trick works.

Trick Directions	*Sample Pick*	*Explanation*
Pick a number	12	❑
Double the number	24	❑❑
Add 4	28	❑❑ ||||
Triple your result	84	❑❑ |||| ❑❑ |||| ❑❑ ||||
Divide by 6	14	❑ || ❑ || ❑ || ❑ || ❑ || ❑ ||
Subtract 2	12	❑

Your answer is the number you started with.

b) Provide an explanation for this number trick.

Trick Directions	*Explanation*
Pick a number	
Add 4	
Triple the result	
Subtract 3	
Divide by 3	
Subtract the original number	

Your answer is 3.

c) Devise your own number trick and write an explanation to show that it works.

Activity 8

Room Dimensions

Exactly 80 tiles that are one foot square in size are needed to cover the floor of a rectangular room.

a) What are the possible dimensions of the room in whole feet?

b) Which set of room dimensions has the smallest perimeter?

Activity 9

Brick Border

Hua wants to make a brick border around his garden. Each brick is 4 in. by 10 in. The garden is 5 feet by 6 feet. He does not want to cut any bricks. Draw a picture to show what he should do. How many bricks will he need?

Activity 10

Getting the Largest Volume

You have a piece of cardboard that is 10 inches by 10 inches. You can make a topless box by cutting a square from each corner and folding up the sides. What size square (in inches) cut from each corner would allow you to form a box that has the greatest volume?

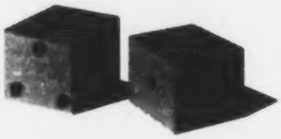
Activity 11

Average Rainfall

The number of days that it rained each month in Portland, Oregon for a complete year was: 12, 17, 9, 21, 15, 7, 14, 7, 15, 22, 14, 19. What was the average number of rain days for a month in Portland? Draw a graph of the number of days of rainfall by month. Indicate the average on your graph. Discuss the trend of rainfall in Portland.

Activity 12

Who Should Win?



The following are rules for a game.

Players take turns rolling two dice.
Player A receives 2 points for a roll of 2, 3, 4, 10, 11, or 12.
Player B receives 1 point for a roll of 5, 6, 7, 8. or 9.
Play continues until each player has had ten turns.

Can you predict who will win? Explain your reasoning.

Activity 13

Radio Claim

The following statement was used on the radio to urge listeners to buy a home alarm system at the beginning of summer.

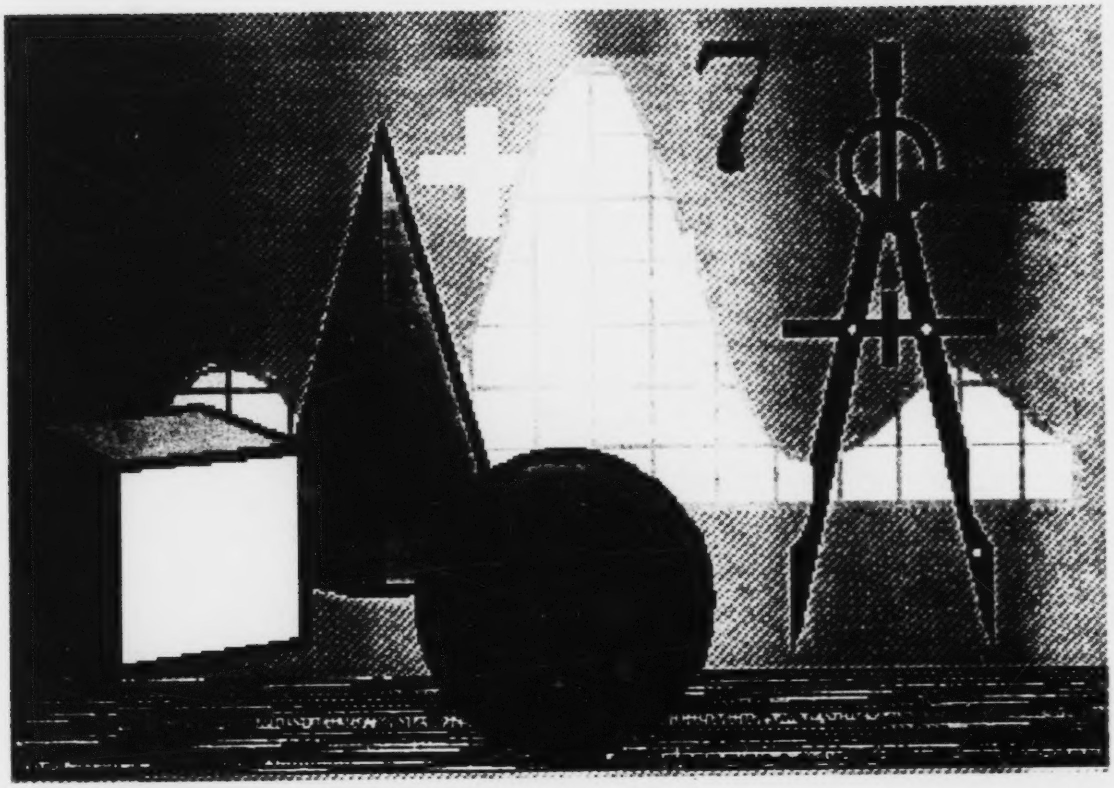
> *"Just over 26% of home break-ins occur between Memorial Day and Labor Day."*

Is this sufficient reason to buy an alarm system now (at the beginning of summer) rather than at a more convenient time? Explain your reasoning.

Competency-Based Education Assessment Series

Eighth Grade Mathematics

Type 1 Assessment



Developed by

Fred Dillon

Margie Raub Hunt

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Steven Meiring

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The model competency-based education assessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency instrument has a traditional design and is intended to be given in one period of approximately 50 minutes. Twenty-five of the items are short-answer, multiple-choice questions that generally assess one objective. Items 26 and 27 are short answer, open-ended questions. Items 28 - 30 are questions of a different character. Item 28 assesses whether students can make a drawing of a descriptive situation and show all listed and relevant information. Item 29 requires students to draw a triangle of specified area and to describe how they know it has the stated area. Item 30 is an open-form item that assesses whether a student can identify, apply, and describe in words a pattern for given dot figures.

The following are optional scoring suggestions for items.

Items 1 - 25: 2 points each
26 - 28* 3 points each
29 - 30 4 points each as determined by the following:

- 2 pts for the required task or answer
- 2, 1, or 0 points for the explanation according to this rubric:

2 = explanation is accurate and complete
1 = explanation is somewhat flawed but complete or accurate
0 = other

* The following rubric is suggested in scoring item 28:

3 = correct drawing with all relevant information shown
2 = runners in correct order, but some information absent
1 = one runner out of correct order and some information absent
0 = other

Items 26 - 30 are valuable in that they provide information about students' thinking as contrasted to the preceding items. It is important that assessment determine not only whether or not students have met easily measurable objectives, but also that the assessment itself reflect the full scope of the CBE program. Items requiring students to construct their solutions offer much richer information – for example, as to whether students can:

- *recognize and draw upon patterns*
- *discern and differentiate among properties and relationships*
- *apply models to real-world phenomena*
- *communicate their ideas in writing*
- *integrate discretely-learned skills and their thinking to solve more complicated situations* .

By including some such items on each assessment, we demonstrate a more consistent linkage among instruction, learning, and evaluation.

Information Sheet

Item	Answer	Objective	Outcome Level	Critical Objective
1.	C	7-3-6	C	no
2.	B	8-5-11	K	yes
3.	D	8-3-1	PS	yes
4.	A	8-6-7	K	yes
5.	D	8-7-2	K	yes
6.	A	8-3-2	K	yes
7.	C	8-5-8	C	no
8.	D	7-5-5	C	yes
9.	B	8-8-5	C	yes
10.	A	8-8-3	PS	yes
11.	B	8-5-4	K	no
12.	D	8-1-2	C	no
13.	A	8-6-3	K	yes
14.	C	8-4-7	C	yes
15.	C	8-4-6	PS	no
16.	B	7-1-4	C	no
17.	D	8-3-10	PS	yes
18.	D	8-8-3	C	yes
19.	B	8-4-8	PS	no
20.	D	8-5-8	K	no
21.	C	8-3-10	K	yes
22.	A	8-3-10	PS	yes
23.	D	8-6-3, 8-5-11	PS	yes, yes
24.	B	8-4-1	PS	yes
25.	A	8-4-9	PS	yes
26.	4	8-2-2	PS	no
27.	YY, BY, BB	8-8-6	C	no
28.	G-50m; M-60m; T-45m; S-55m	8-2-3	PS	yes
29.	triangle half of parallelogram of 12	8-6-2	PS	yes
30.	201; twice position num. + 1	8-1-2, 8-2-3	PS	no, yes

Problems 1-25 are worth two points each. Problems 26 and 28 are worth three each. Problems 29 and 30 are worth four each.	*By Outcome Level:* K: 8 C: 9 PS: 13	*By Strand:* 1: 3 4: 5 7: 1 2: 3 5: 6 8: 4 3: 6 6: 4

Circle the correct answer for questions 1 to 25.

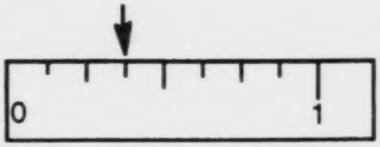
1. Which of the following represents the least amount?

A. 0.3

B. $\frac{2}{5}$

C. 25%

D.



2. Given: $3 \cdot (\square + 5) = 30$

the number in the box should be

A. 2

B. 5

C. 10

D. 95

3. Suppose you want to bake some cakes for a party. You know the flour needed for each of the following cakes:

Pineapple Swirl Cake: $2\frac{1}{3}$ cups flour
Chocolate Velvet Cake: $1\frac{1}{2}$ cups flour

How much flour will be needed to make four Pineapple Swirl Cakes and three Chocolate Velvet Cakes?

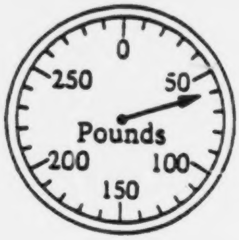
A. $4\frac{1}{5}$ cups

B. $11\frac{1}{5}$ cups

C. $14\frac{5}{6}$ cups

D. $16\frac{5}{6}$ cups

4. What is the weight shown on the scale?



A. 60 pounds

B. 51 pounds

C. 96 pounds

D. 55 pounds

5. A student solved an arithmetic problem with a calculator and the calculator screen showed an answer of 0.52. Which fraction is closest to 0.52?

A. $\frac{1}{50}$

B. $\frac{1}{5}$

C. $\frac{1}{3}$

D. $\frac{1}{2}$

6. On a winter day, the temperature is - 2 C°. As evening progresses, the temperature drops 5° C. Several hours later the temperature rises by 3° C. What is the temperature after these changes?

A. - 4° C

B. - 1° C

C. 0° C

D. 4° C

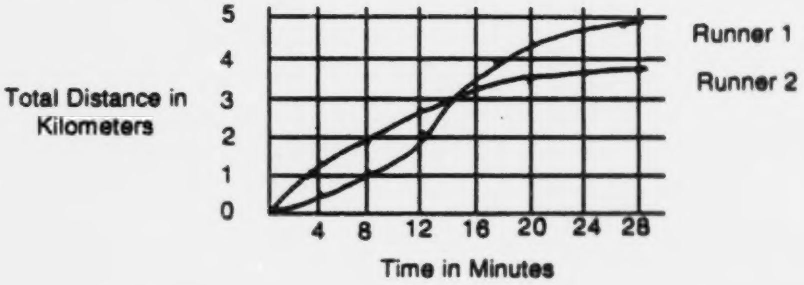
7. The distances covered by two runners during a race are shown in the graph below. How long after the start of the race did one runner pass the other?

A. 3 minutes

B. 12 minutes

C. 14 minutes

D. 28 minutes



8. If k can be replaced by any number, how many different values can the expression $k + 6$ have?

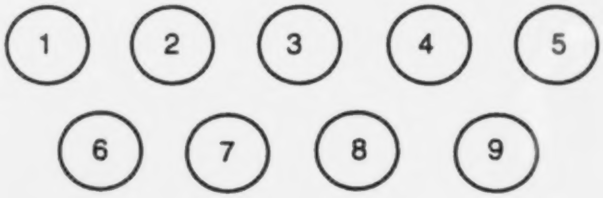
   A. none

   B. one

   C. six

   D. infinitely many

9. The nine chips shown below are placed in a sack and then mixed up. Margie draws one chip from this sack. What is the probability that Margie draws a chip with an even number?



   A.

   B.

   C.

   D.

10. The average weight of 50 prize-winning tomatoes is 2.36 pounds. What is the combined weight, in pounds, of these 50 tomatoes?

    A. 118

    B. 52.36

    C. 11.8

    D. 0.0472

11. What is the least whole number x for which $2x > 11$?

    A. 5

    B. 6

    C. 9

    D. 12

(-1, -1) (1, 3) (2, 5) (3, 7) (4, 9)

12. Given the ordered pairs shown above, what must be done to the first number to obtain the corresponding second number?

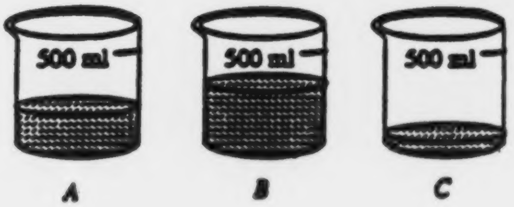
A. Add 2

B. Subtract 3

C. Multiply by 2 and subtract 1

D. Multiply by 2 and add 1

13. The beakers shown below contain different amounts of water. Which beaker has about 200 milliliters of water in it?



A. *A*

B. *B*

C. *C*

14. What is the ratio of the length of a side of an equilateral triangle to its perimeter?

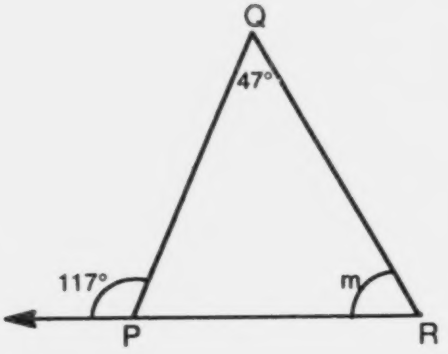
A. 1 : 1

B. 2 : 1

C. 1 : 3

D. 3 : 1

15. Given the information in the figure shown, what is the measure of ∠m in triangle PQR?



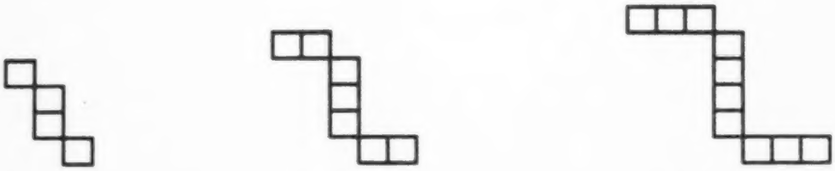
A. 16°

B. 63°

C. 70°

D. 80°

16. The following pattern of figures is made with individual square tiles.



1st figure (4 tiles) 2nd figure 3rd figure

How many tiles would it take to make the sixth figure if this pattern is continued?

A. 16

B. 19

C. 20

D. 22

17. A jacket priced at $80 is on sale for 35% off. What is the sale price?

A. $ 28

B. $ 45

C. $ 49.50

D. $ 52

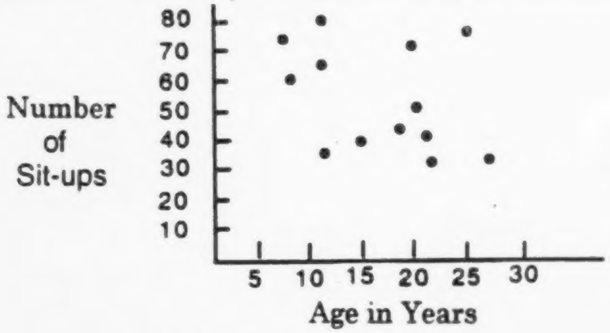
18. In the graph below, each dot shows the number of sit-ups and the corresponding age for one of 13 people. According to this graph, what is the median number of sit-ups for these 13 people?

A. 15

B. 20

C. 45

D. 50



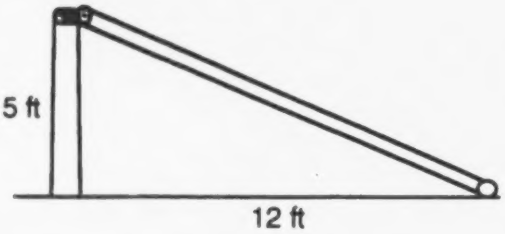
19. A wooden flag pole broke 5 feet from the ground and fell over touching the ground 12 feet away. What was the original height of the flag pole?

A. 30 ft

B. 18 ft

C. 17 ft

D. 16 ft



20. A car averages from 17 to 23 miles per gallon of fuel. What is the maximum distance you could expect the car to travel using 10 gallons of fuel?

A. 40 miles

B. 17 x 23 miles

C. 170 miles

D. 230 miles

21. The Quality Control Department of a company checks with customers and finds that 4,000 of the 200,000 buyers had a defective product. The percent of defective products was:

A. 0.02 %

B. 0.20 %

C. 2 %

D. 20 %

22. A school sports team has 8 seniors, 5 juniors, and 12 sophomores. What percent of this team is not juniors?

A. 80%

B. 52%

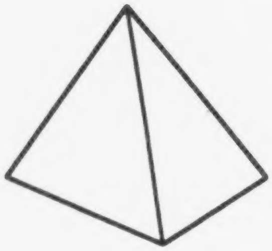
C. 48%

D. 32%

23. The formula for the volume of a square pyramid is

$$V = \frac{1}{3} \cdot h \cdot A$$

where h is the pyramid's height and A is the areaof its base. Find the volume of a pyramid whose height is 5 inches and whose square base is 3 inches on a side.



A. $2\frac{2}{3}$ cubic inches

B. 5 cubic inches

C. 11 cubic inches

D. 15 cubic inches

24. A photographer is making an enlargement of a color portrait. If the original portrait is 8 inches by 11 inches and he wants to make the new width 11 inches, about what should the enlarged length be?

A. 14 inches

B. 15 inches

C. 16 inches

D. 17 inches

25. A rectangular room is 8 ft high, 10 ft wide and 12 ft long. Emile buys wallpaper for the walls. Disregarding doors, windows and pattern matching, what is the minimum area of wall paper he could use to cover all of the walls?

A. 352 square feet

B. 400 square feet

C. 592 square feet

D. 960 square feet

Answer the following questions in the space provided.

26. An assembly worker is making 3-legged and 5-legged stools. She has 67 legs to use in all for the day's work. What is the fewest number of 3-legged stools she can make and still use all the legs up, making 5-legged stools with the rest?

27. Linda was asked to pick two marbles from a bag of yellow marbles and blue marbles. One possible result was one yellow marble first and one blue marble second. She wrote this result in the table below. List all of the other possible results that Linda could get.

y stands for one yellow marble

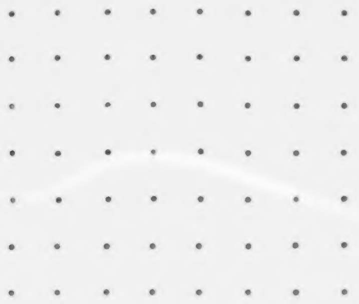
b stands for one blue marble

First Marble	Second Marble
y	b

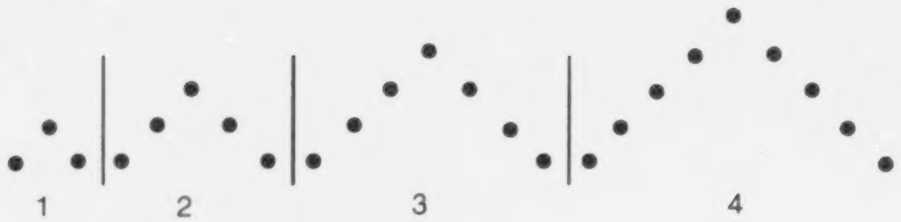
28. Four boys run a race. At the instant Glenn has run 50 meters, Marco is 10 meters ahead of him. Ted is 15 meters behind Marco, and Sam is 5 meters ahead of Glenn.

By drawing a picture, verify that Glenn is in third place at this point. Show all information from the problem in your drawing.

29. Draw a triangle, which is NOT a right triangle, that has a base of 4 units and an area of 6 square units. Tell how you know the area is 6 square units.



30.



If the above pattern of dot-figures is continued:

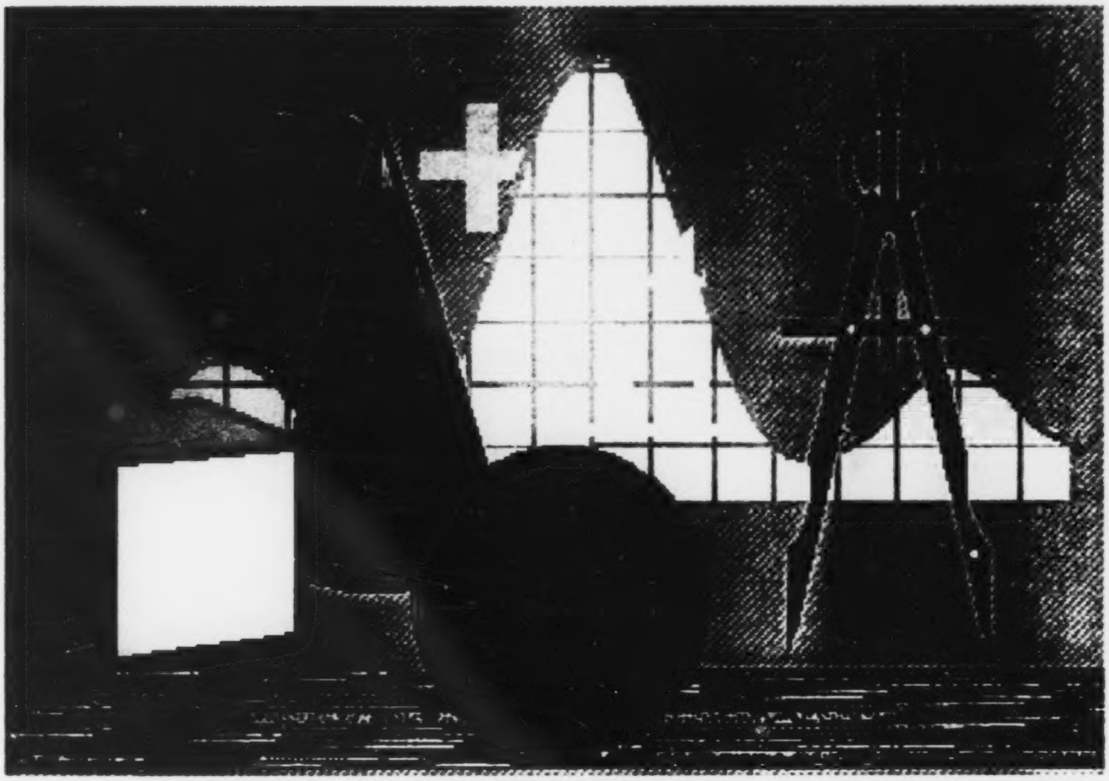
a) How many dots will be in the 100th figure?

b) How do you know? (Explain your reasoning.)

Competency-Based Education Assessment Series

Eighth Grade Mathematics

Type 2 Assessment



Developed by

Fred Dillon

Margie Raub Hunt

William Hunt

Steven Meiring

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The model competency-based education assessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency assessment instrument is designed to be given in two parts, each requiring one period of approximately 50 minutes for completion. The instrument is a mixture of more traditional design items (multiple-choice), open-form items, and items requiring extended responses. Some items are direct, straightforward measures of particular objectives. Several items assess combinations of two objectives. A few items require students to integrate several ideas in constructing their responses.

Items (1, 2, 4, 19, 20, and 21) are essentially review items from earlier grades to assess whether students are maintaining prerequisite skills and concepts from earlier grades that may appear on the Ohio Ninth Grade Proficiency Test in Mathematics. It is the expectation that students will have calculators available to them for all (or most of the assessment). Some items (5, 21, and 24) assess estimation and/or equivalence of numbers. You may choose to have students answer these items before accessing a calculator for the rest of the assessment.

The instrument as a whole assesses a deeper level of understanding than the Type 1, Eighth Grade instrument. Seventeen of the items are written at a problem-solving, outcome level. Items such as (8) and (26) require students to carry out calculations as part of a consumer decision. Item (11) requires the reading of a table for calorie count-ing to make a dietary decision. Item (13) combines the notions of similar figures and perimeters. Items (14, 18, 27, and 36) require logic and problem solving skills for solution.

Several items provide measures of students' understanding of key concepts. Other items draw upon students' visualization skills. Items (3, 23, and 32) entail recognition and application of a pattern. Algebra, geometry, and data analysis are particularly emphasized on this instrument. To the extent possible, questions are couched within authentic contexts.

Open-form items provide information about students' thinking as contrasted to closed form items. It is important that assessment determine not only whether or not students have met easily measurable objectives, but also that the assessment itself reflect the full scope of the CBE program. Items requiring students to construct their solutions offer much richer information – for example, as to whether students can:

- *recognize and draw upon patterns*
- *discern and differentiate among properties and relationships*
- *apply models to real-world phenomena*
- *communicate their ideas in writing*
- *integrate discretely-learned skills and their thinking to solve more complicated situations*

By including some such items on each assessment, we demonstrate a more consistent linkage among instruction, learning, and evaluation.

Discussion

Additional Answers and Suggestions

13. Score 1 point for correct perimeter of larger pentagon, 1 point for a correct proportion for corresponding measures, and 2 points for correct answer.

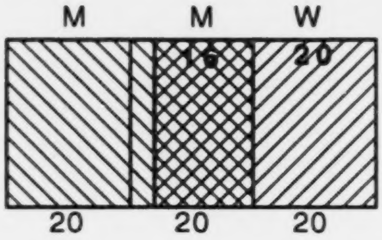
14. Score 1 point for procedure to link scores and 3 points for answers.

15. Subtract 5 from both sides; then divide both sides by 2. The turtle weights 11 units.

16. Score 2 each for the correct drawing and for the correct area.

17. It is not essential that students combine like terms at this grade: b + a + 3 + b + a + 3 is acceptable.

18. a. One possibility is to divide a rectangle (representing the whole amount of 60) according to the fractional amounts and use shading to answer the question.



▧ = 40 men (2/3 of 60)

▨ = 36 w / brown hair (3/5 of 60)

▩ = 16 brown hair men
(36 people - 20 women)

b. A second possibility is to simply subtract the fractions (women from brown haired people) and then multiply the resulting fraction by the reference amount (60 people altogether).

3/5 - 1/3 = 9/15 - 5/15 = 4/15 then 4/15 of 60 = 16 men with brown hair.

Score 2 each for each correct solution.

20. The figures are squared numbers (of the ordinal position) plus two more.

24. It's easiest if you replace one of the given fractions in each sum with a nearly equal fraction to make like denominators:

2/7 + 6/7 = 8/7 3/8 + 2/8 = 5/8 5/6 + 1/6 = 1 4/5 + 2/5 = 6/5

27. Polygons B, C, and D can be referred to A for reference. The perimeter of A is 30 units. Only polygon B has a perimeter less than that of A.

32. The bottom layer can be visualized by pretending that the cubes have been telescoped upward from below. By pushing them back down flat, we realize that all the uppermost cubes showing will push back down into the bottom layer.

33. Combinations are:

Tuna - Milk	Ham - Milk	Beef - Milk
Tuna - Juice	Ham - Juice	Beef - Juice

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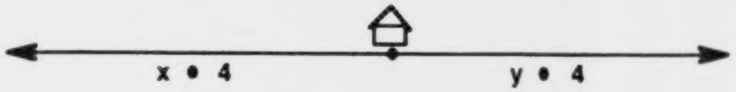
34. Using the formula for *distance* = *rate* × *time*, we can fill in 4 for the time each has driven.



total distance: $d = 4x + 4y$

Score 1 point for a reasonable figure, 1 point for using or stating the distance formula above, and 2 points for the formula.

35. 1% of 2 billion is 20 million; 1/2 % of 2 billion is therefore 10 million. The number of bulbs defective is 20 million ± 1/2% of this amount; that is 20 million ± 10 million. There could be as many as 30 million defective and as few as 10 million defective.

Score 1 point each for the first two answers and 2 points for the range.

36. Conjecture 1 is true and Conjecture 2 is false. Evidence to that effect (certainly not a proof) can be shown with examples involving the 3-4-5 right triangle.

Conjecture 1: $3^2 + 4^2 = 25 = 5^2$
multiply by 2: $6^2 + 8^2 = 100 = 10^2$

Conjecture 2: $3^2 + 4^2 = 25 = 5^2$
add 2: $5^2 + 6^2 = 61 \neq 7^2$
subtract 2: $1^2 + 2^2 = 5 \neq 3^2$

Score 2 points for each for a correct example or counterexample for each conjecture.

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Information

Item	Answer	Objective	Outcome Level	Critical Objective
1.	D	8-3-10	K	yes
2.	B	8-3-1	C	yes
3.	C	7-1-2, 8-2-3	PS	no, yes
4.	B	8-8-3	PS	yes
5.	C	8-7-2, 8-7-6	K	yes, no
6.	D	8-8-5	C	yes
7.	C	8-3-1	C	yes
8.	D	8-3-1, 8-6-2	PS	yes, yes
9.	A	8-4-7, 8-6-2	PS	yes, yes
10.	A	8-4-9	PS	yes
11.	B	7-8-2	PS	yes
12.	A	7-3-3, 8-8-2	PS	yes, no
13.	26 units	7-3-3, 8-4-1	PS	yes, yes
14.	1, 2, 3	8-2-2	PS	no
15.	see discussion	8-5-2	K	no
16.	trapezoid, 14 sq. units	7-8-3, 8-6-2	K, PS	no, yes
17.	2b + 2a + 6 or equivalent	8-4-7, 8-5-8	C	yes, no
18.	see discussion	7-2-2, 8-2-2	PS	no, no
19.	NA, A, A, A	8-7-2	C	yes
20.	NP, P, P, NP	8-8-3	C	yes
21.	C	7-3-5	K	yes
22.	C	8-3-2	K	yes
23.	D	7-1-2	PS	no
24.	C	8-7-2, 8-7-6	PS	yes, no
25.	A	8-8-3	PS	yes
26.	C	8-3-1, 8-6-2	PS, C	yes, yes
27.	B	8-4-6	C	no
28.	B	8-5-11, 8-6-3	PS	yes, yes
29.	D	8-3-2	K	yes
30.	B	8-4-2	C	no
31.	A	8-1-2, 8-5-3	PS	no, no
32.	15	8-4-3	PS	no

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Item	Answer	Objective	Outcome Level	Critical Objective
33.	see discussion	7-8-5	K	no
34.	$d = 4x + 4y$	8-5-8	PS	no
35.	see discussion	8-3-1, 8-3-10	C	yes, yes
36.	see discussion	8-1-4, 8-4-8	PS	no, no

Problems 1-12, 15, 17, 19-32 are worth two points each. Problems 13,14,16,18, 33-36 are worth 4 points each.	*By Outcome Level:* K: 8 C: 10 PS: 20	*By Strand:* 1: 4 4: 8 7: 3 2: 3 5: 5 8: 8 3: 11 6: 5

Part One

Circle the correct answer to items 1 to 12.

1. A newspaper poll states that 40% of the people in Jeremyville like cats. If there are 2.5 million residents in Jeremyville, how many like cats?

   A. 1,000

   B. 10,000

   C. 100,000

   D. 1,000,000

2. A certain reference file contains approximately one billion facts. How many millions is that?

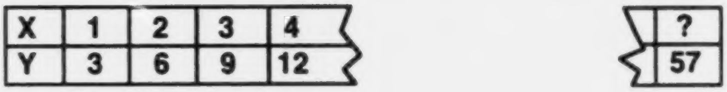
   A. 1,000,000

   B. 1,000

   C. 100

   D. 10

3. The same rule is used to match each number in the X-row of the following table with a number in the Y-row below it. The table became torn and part of it is missing. Using the rule, what number should replace the question mark in the space above the number 57?



X	1	2	3	4
Y	3	6	9	12

?
57

   A. 5

   B. 13

   C. 19

   D. 23

4. William has scored a total of 235 points on the first three math tests taken in his mathematics class. If his mathematics tests are always worth 100 points, how many points must he earn on his next test in order to have an average of 80%?

   A. 80

   B. 85

   C. 90

   D. 95

5. Which of the following is the best estimate of the answer to 396.5 ÷ 15.6 ?

   A. 15

   B. 20

   C. 25

   D. 32

6. In a bag of marbles, $\frac{1}{4}$ are red, $\frac{1}{12}$ are blue, $\frac{1}{6}$ are green, and $\frac{1}{2}$ are yellow. If a marble is taken from the bag without looking, it is more likely to be:

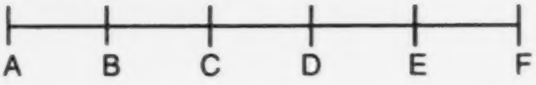
   A. red

   B. blue

   C. green

   D. yellow

7. The line segment AF at the right is marked of into 5 equal parts. If you start at point A and go 77% of the way to point F, between which two letters will you be?



   A. between B and C

   B. between C and D

   C. between D and E

   D. between E and F

8. Plastic edging for flower beds comes in 50-ft rolls and costs $ 6.85 per roll. What is the cost to completely edge two rectangular flower beds which are 40 ft by 15 ft and one circular flower bed which is 16 ft in diameter?

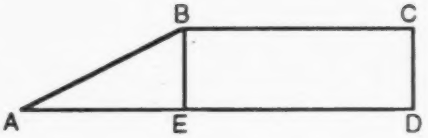
   A. $13.70

   B. $27.40

   C. $34.25

   D. $41.10

9. The area of rectangle BCDE shown to the right is a 60 square inches. If the length of AE is 10 inches and the length of ED is 15 inches, what is the area of trapezoid ABCD, in square inches?



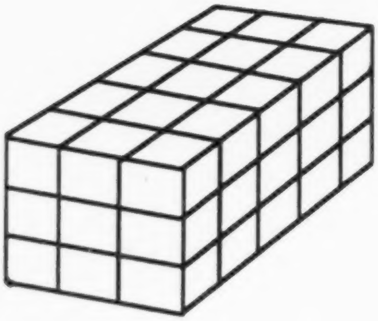
   A. 80

   B. 85

   C. 90

   D. 95

10. A rectangular solid is built by stacking 1 x 1 x 1 cubes as shown in the figure. What is the total number of 1 x 1 squares that form the six faces (surfaces) of the rectangular solid shown to the right?



   A. 78

   B. 45

   C. 42

   D. 39

11. To maintain his weight of 180 pounds, Otis must limit the total number calories he eats to 2400 per day. Today, he has already eaten just a bit more than 1600 calories of food. Which of the meals described below could Otis eat to stay within his 2400-calorie daily limit for today? (Circle the letter of the correct meal.)

CALORIE GUIDE

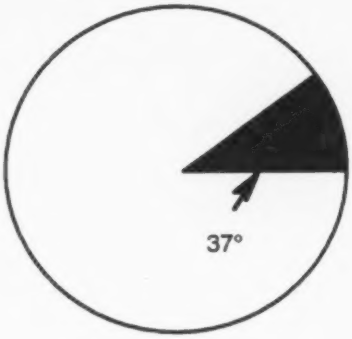
calories per serving

Food	Calories
blueberries	40
bread, French	65
broccoli	20
cake, angel food	135
chicken pot pie	535
chicken, roasted	205
cola drink	70
crackers, rye	30
haddock, fried	140
lemonade	110
macaroni & cheese	430
meat loaf	370
milk, 2%	145

Food	Calories
peaches, canned	100
peas, frozen	60
pie, apple	405
pie, pumpkin	320
potato, baked	90
rice	180
roll, plain	115
spinach	20
stuffing, poultry	195
tea, sugarless	0
tamato juice	25
turkey, roasted	160

Meal A	**Meal B**	**Meal C**	**Meal D**
meat loaf	chicken pot pie	haddock, fried	turkey, roasted
macaroni & cheese	French bread	rice	stuffing
iced tea (sugarless)	tomato juice	crackers, rye	peas
blueberries	cake, angel food	lemonade	milk (2%)
		pie, apple	pie, pumpkin

12. The entire circle shown at the right represents a total of 2,675 radios sold. Of the following, which is the best approximation of the number or radios represented by the shaded sector of the circle?



A. 275

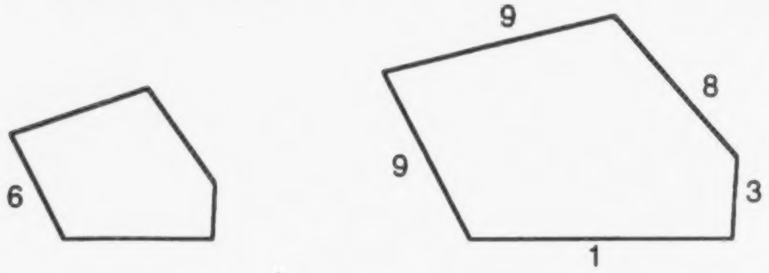
B. 280

C. 285

D. 290

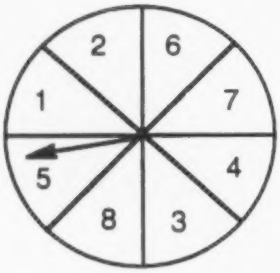
Record your answer in the space provided. Show your work.

13. The following two figures are similar. What is the perimeter of the smaller figure?



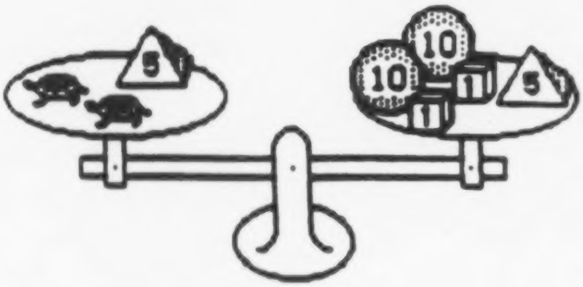
Answer

14. Three people spin the spinner shown once each. Jethro scores 2 more than Quincy. Kwan scores 3 less than Quincy. What are the possible numbers Kwan might have scored?



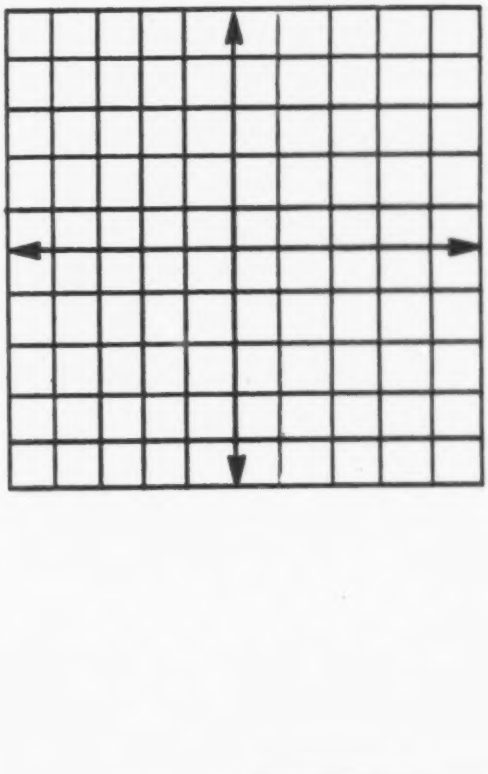
Answer

15. The scale is evenly balanced. Explain how you could find the weight *w* of one turtle.



16. a) Graph the ordered pairs: A (-1, -1), B (4, -1), C (1, 3), D (-1, 3).

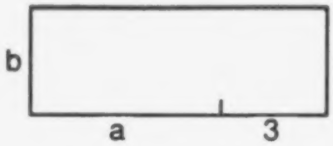
Then join the points in order A, B, C, D, A,



b) What is the area of the figure you made?

Answer

17. What expression can you write for the perimeter of this rectangle?



Answer

18. Of the 60 people in a room, $\frac{2}{3}$ are men and $\frac{3}{5}$ of the people have brown hair. What is the least number of men in the room who could have brown hair?

Solve this problem in two ways. In one way, use a drawing in your solution.

a. Explain your first way.

b. Explain your second way.

Part Two

19. A child's bicycle originally costing $90 is on sale at 10% off. Tell whether each of the following is or is not appropriate for calculating the amount of discount. Indicate by filling in the appropriate circle whether each of the following is possible or not possible.

Appropriate	Not Appropriate	
○	○	$90 \div 0.10$
○	○	90×0.10
○	○	$90 \times \frac{1}{10}$
○	○	$90 \div 10$

20. Raj read from a book on Monday, Tuesday, and Wednesday. He read an average of 10 pages per day. Indicate by filling in the appropriate circle whether each of the following is possible or not possible.

Possible	Not Possible	Monday	Tuesday	Wednesday
○	○	4 pages	4 pages	2 pages
○	○	9 pages	10 pages	11 pages
○	○	5 pages	10 pages	15 pages
○	○	10 pages	15 pages	20 pages

Circle the correct answer to items 21 to 31.

21. Write a decimal that names the same amount as $1\frac{4}{5}$.

A. 1.45

B. 1.54

C. 1.8

D. 14.5

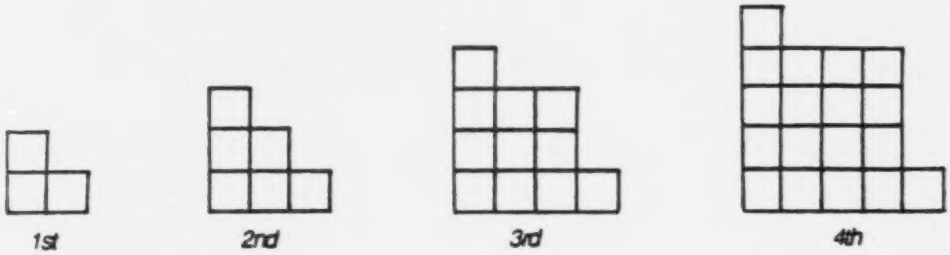
22. The average temperature for February is - 2°C. Which of the following daily temperatures is closest to this average value?

A)- 10° C

B) - 6° C

C) 1° C

D) 3° C



23. If you continue the pattern above, how many blocks will be needed to construct the 20th figure in this sequence?

A. 22

B. 112

C. 202

D. 402

24. Using only the numbers 1, 3, 4, 5, 6, 7 to fill in the single-digit boxes to make fractions, which combination gives a sum closest to 1?

$$\frac{\square}{\square} + \frac{\square}{\square}$$

A. $\frac{1}{3} + \frac{6}{7}$

B. $\frac{3}{7} + \frac{1}{4}$

C. $\frac{5}{6} + \frac{1}{7}$

D. $\frac{4}{5} + \frac{3}{7}$

25. The yearly salaries of the five top executives at the Bigwig Corporation are \$1,000,000, \$250,000, \$130,000, \$90,000, and \$90,000. If we calculate the mean, median, and mode for these salaries and then place these values in order from highest to lowest, the order would be:

    A. mean, median, mode

    B. mode, median, mean

    C. median, mean, mode

    D. mean, mode, median

26. What does it cost to carpet a hallway measuring 15 feet by 4 ft at \$21 per square yard?

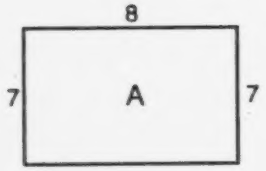
    A. \$40

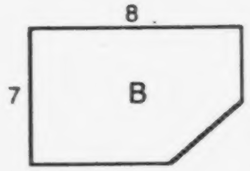
    B. \$84

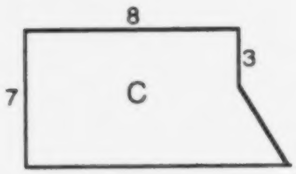
    C. \$140

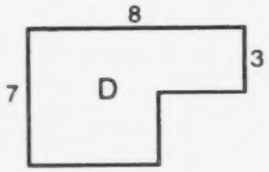
    D. \$1260

27. For each figure below, the lengths of 2 or 3 sides are given, and segments that appear to be perpendicular are perpendicular. Which figure could have a perimeter of 28? (Circle the letter within the correct figure.)



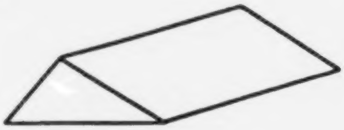






Tent Sale

Floor Size:	7 ft x 6 ft
Peak Height:	4 ft
Weight:	9 lb 3 oz



28. Before B.J. buys the tent shown in the ad above, he wants to make sure that it is as roomy as his old tent. If the average person needs at least 40 cu ft of space, how many people could fit in the tent? (The formula for the volume of a prism is $V = \frac{1}{2} \cdot h \cdot A$ where h is the height of the prism and A is the area of the base.)

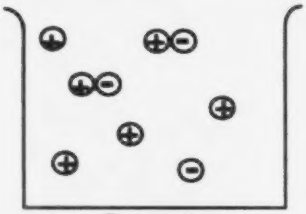
A. 1

B. 2

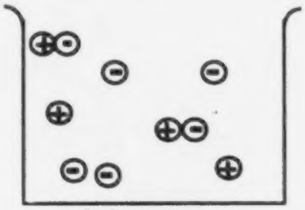
C. 3

D. 4

29. ⊕ means a charge of (+1). ⊖ means a charge of (-1). ⊕⊖ means a neutral charge or a charge of (0). Placing a (+1) charge with a (-1) charge results in a neutral charge ⊕⊖ . The overall charge on a container can be found by combining all possible positive and negative charges and then indicating whatever charge is left over. The charge on the example container is (+3). What is the charge on the laboratory container?



Example Container



Laboratory Container

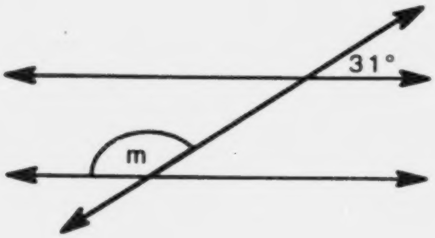
A. (-5)

B. (-2)

C. (+3)

D. (+5)

30. Given the parallel lines in the figure and the given angle, what is the measure of $\angle m$?



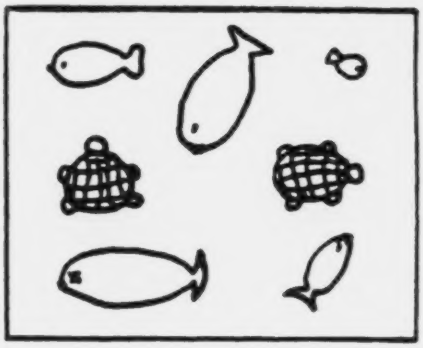
A. 169°

B. 149°

C. 119°

D. 69°

31. The picture shows the captured waterlife from a certain section of a pond. Suppose this sample represents the correct ratio of the number of fish to the number of turtles in the whole pond. Which of the following formulas is correct for finding the number of turtles T given the number of fish F?



A. $T = \frac{2}{5} F$

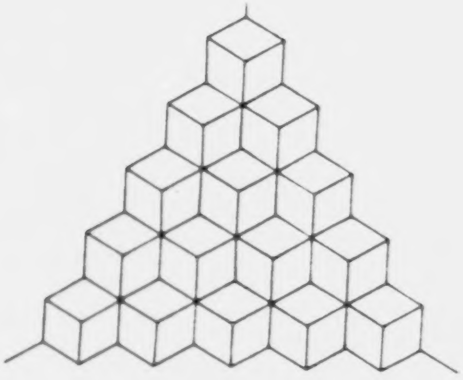
B. $T = 5F - 2$

C. $T = 2F + 5$

D. $T = \frac{5}{2} F$

Record your answer in the space provided. Show your work.

32. The figure shown is the result of stacking individual cubes in a corner. How many cubes are in the bottom layer?



Answer

33. Angelina will choose one sandwich and one drink for lunch. The menu at the right shows the choices. List below all the possible combinations of a sandwich and a drink that Angelina might choose.

MENU

Sandwiches:
Tuna
Ham
Beef

Drinks:
Milk
Juice

34. You and a friend are each driving a car in opposite directions. You both leave your house at the same time. If you are traveling at x miles an hour and your friend is traveling at y miles an hour, make up a formula for how far apart the two cars are after 4 hours. (Hint: draw a picture.)

Answer

35. The Electric Light Company made 2 Billion light bulbs last year. The executives estimated 1% of their output was defective, with a margin of error of $\frac{1}{2}$%.

What is 1% of 2 Billion?

Answer

What is $\frac{1}{2}$ % of 2 Billion?

Answer

What is the range in the number of light bulbs that could be defective?

Answer

36. Provide a convincing argument that the following conjectures are either true or not true.

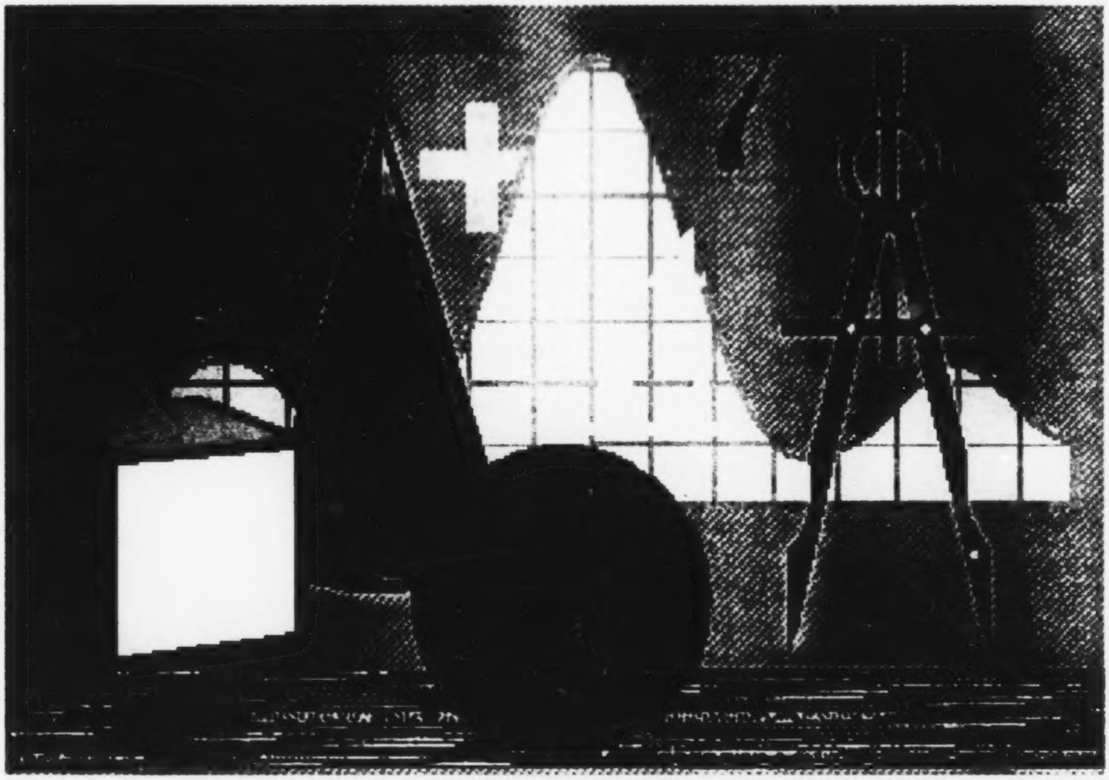
    Conjecture 1: If you multiply the lengths of all three sides of a right triangle by the same number, the new lengths will form another right triangle.

    Conjecture 2: If you add or subtract the same number from each side of a right triangle, the new lengths will form another right triangle.

Competency-Based Education Assessment Series

Eighth Grade Mathematics

Type 3 Assessment



Developed by

Fred Dillon

Margie Raub Hunt

William Hunt

Steven Meiring

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The model competency-based education assessments for mathematics have been developed in cooperation with the Ohio Council of Teachers of Mathematics task force for implementing NCTM Standards.

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About This Instrument

This model competency assessment instrument consists of performance tasks, each requiring a substantial period of time to complete. The tasks involve more authentic-type situations that reflect the ways in which mathematics is applied. Multiple skills, understandings, and thinking are required to complete each task. Rather than assessing single objectives, the tasks address *strands* of the mathematics curriculum. Several objectives may be assessed within the scope of a single task. The manner in which the assessment is carried out may also reflect upon the outcome levels measured relative to those objec-tives.

For example, Activity 8 that follows, *(Misleading Graphs)*, can be used to assess objectives from three strands from the eighth grade of the *Model Competency-Based Mathematics Program*: 1-4, 6-3, 6-6, 8-1, and 8-4. Activity 4*(Calendar Sums)*, can be used to assess how a student engages the task (requiring a process rubric) or to assess outcomes achieved (requiring an outcome rubric). The intent of this instrument is to provide rich, performance tasks whereby a teacher can choose how to assess students and which objectives to focus upon within that activity. Each strand except Estimation and Mental Computation forms the basis of one of the tasks. Objectives from Strand 7 can be assessed in the context of activities primarily focused toward other strands.

It is the expectation that these performance tasks will be given to students throughout the year at appropriate intervals. Multiple forms of assessment will be employed – for example, student products, teacher observations, interviews, self-assessment, and journal writings. Some tasks may be given individually and some in group settings. Products and records of students' activity may be kept in individual portfolios. At a suitable point toward the end of the year, a thorough review of the work and records within the portfolio can be used to determine how well the student is progressing relative to the competency-based outcomes of the seventh grade.

Characteristic of performance task assessment, the dividing lines between instruction, learning, and assessment become fuzzy. Students will engage many of these tasks within a learning, rather than a testing environment. They will have the opportunity to converse with others, use appropriate manipulatives, employ calculating tools, and consult resources. The tasks are not time-restrictive for completion, and they permit multiple avenues of approach – making them more equitable and accessible to all students.

For more information about performance assessment, consult the *Performance Assessment* discussion of this instrument and other publications such as the National Council of Teachers of Mathematics resource, *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions* (Stenmark, 1991).

Performance Assessments

Overview

"A performance assessment in mathematics involves presenting students with a mathematical task, project, or investigation, then observing, interviewing, and looking at their products to assess what they actually know and can do.

A performance task can:

- allow the examination of the process used as well as the answer or finished project;
- be used with groups as well as individuals;
- document, through observation records or student products, accomplishments not revealed by ordinary tests."

source: *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*

Performance assessments give students an opportunity to display a full range of ability. They emphasize better the nature of mathematics, its processes, and practice. The tasks are interesting and motivational. Assessment does not have to interrupt student learning. Richer and more complete information becomes available for making instructional decisions. A more engaging record of the broad character of the mathematics program is available for parents, administrators, and citizens.

Assessing Performance Tasks

As teacher, you must decided upon which outcomes you wish to focus for a particular task and then decide how to assess those outcomes. For example, let us consider Activity 4*(Calendar Sums)* from this instrument. The goals of the activity are for students to:

- systematically investigate column sums on a calendar;
- recognize that the problem is not well-defined and different cases must be considered;
- notice patterns to minimize a column sum;
- generalize the solution for one length of month to others;

You could choose to focus upon how students go about the *process of investigation* and design a corresponding rubric:

1 - Needs significant instruction and direction
2 - Needs some instruction but initiates own investigation
3 - Conducts investigation and finds a few patterns
4 - Meets the expectations of the activity
5 - Exceeds expectations; makes unanticipated discoveries

Alternatively, you could focus upon outcomes the student achieves in either *patterning* or *problem solving:*

Patterning

1 - Shows an attempt at looking for patterns within the specific task
2 - Looks for additional patterns beyond the task and identifies one
3 - Identifies and/or describes two or more relevant patterns
4 - Identifies and makes reasonable generalizations about several patterns

Problem Solving

1 - Unsystematic in investigation; considers only one case
2 - Somewhat systematic; recognizes multiple cases but chooses one
3 - Systematic approach, but overlooks some evidence
4 - Conducts thorough, systematic investigation, considering multiple with cases
5 - Exceeds level in generalizing and reasoning

Another possibility is to focus upon the quality of the response:

Minimal Response. Only some of the essential conditions of the task are met.

Nearly Proficient Response. Most essential conditions are there with some small misconceptions, use of inappropriate concepts or skills, and/or incomplete explanations.

Proficient Response. Full application of knowledge and skills, communication appropriate to task.

Advanced Response. Goes well beyond all expectations along with displays of creativity, elegance, and exceptional reasoning.

Alternatively, a teacher can assess *attitudes* (interaction and participation):

0 = Dependent   1 = Needs Support   2 = Independent

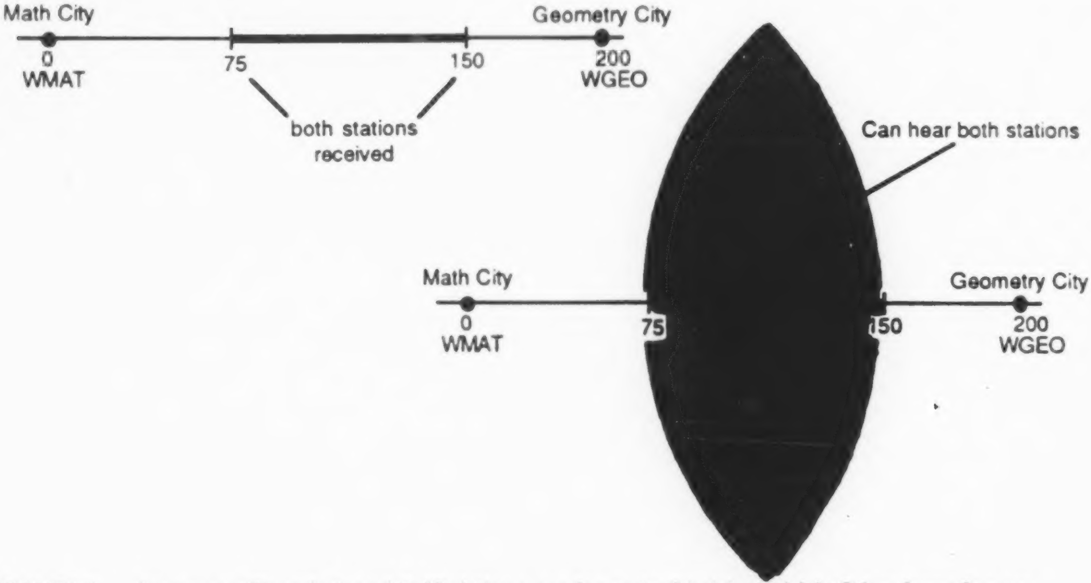
ATTITUDES	Date			Comments
Cooperates				
Shares/Collaborates tries, contributes ideas				
Questions Peers encourages others to participate				
Takes Risks confident in own ability Stays on Task perseveres				

source: *Mathematics Assessment: Myths, Models, Good Questions, and Practical Suggestions*

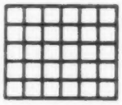
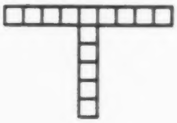
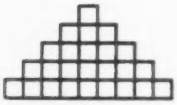
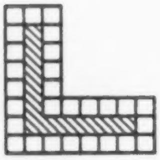
Discussion

ACTIVITY 1. Assesses Strand 4. (See discussion on Performance Assessment pages.)

Solution:


Math City
Geometry City
0
WMAT
75
150
200
WGEO
both stations received
Can hear both stations
Math City
Geometry City
0
WMAT
75
150
200
WGEO

ACTIVITY 2. Assesses Strands 1 and 2. [Solutions are for parts (b), (c), and (e). Other formulas are possible.]

(1)		110	$T = n(n + 1)$
(2)		29	$T = (2n - 1) + n$ OR $T = 3n - 1$
(3)		100	$T = n^2$
(4)		48 11	$T = 4(n + 2)$ $T = 2n + 1$

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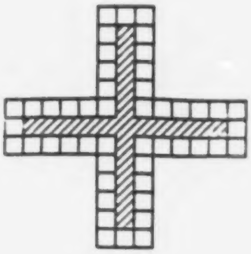
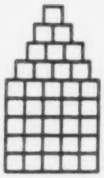
(5)



88 $T = 8 + 8n$ OR $T = 16 + 8(n - 1)$

41 $T = 4n + 1$

(6)



145 $T = n^2 + \frac{1}{2}n(n - 1)$

ACTIVITY 3. Assesses Strands 3 and 8.

Solution: a. \$1.50 - \$ 1.24 = \$ 0.26 b. \$ 3.26 - \$ 2.61 = \$ 0.65

ACTIVITY 4. Assesses Strand 2.

Solution: The minimum sum is achieved by having the last day of the month occur on Sunday, thereby assuring only 4 Mondays in the month and minimizing the sum.

For a 28-day month, the sum is 46.

For a 29-day month, the sum is 50.

For a 30-day month, the sum is 54.

For a 31-day month, the sum is 58.

Comment: Two problem solving features of this task to bring out are these – (a) the problem does not specify the length of the month, so one should *explore all possibilities;* (b) the solution for one month will likely be found by *guess and check* and then must be *generalized* for months of other lengths.

ACTIVITY 5. Assesses Strand 6.

Solution: The adult human lungs have a capacity of about 3.8 liters, although they are always somewhat inflated due to their construction. In quiet breathing, a person inhales about a pint of air. When very active, this amount can increase to about six times that volume.

Comment: Students will need considerable direction for this activity, particularly in calculating the volumes of their spheres or cylinders with hemispeherical ends. They will also need help with the unit, deciliter (100 mL), which is not a common unit. The use of this unit will give students a reasonable scale when making their scatterplots. (The choice of the unit *inches* to measure height was made for the same reason.) Examine the scatterplot for correlations between lung capacity and size, gender, or active lifestyle.

You may wish to carry this activity out as an interdisciplinary lesson with a science teacher. Besides the obvious advantages, an alternative procedure for determining the lung capacity is by means of water displacement. Your science colleague is likely to have the appropriate equipment for this procedure.

ACTIVITY 6. Assesses Strand 6.

Solution: AB = 11.3 cm $m(\angle A) = 75°$ Area = 49 cm^2
AC = 9.0 cm $m(\angle B) = 44°$
BC = 12.4 cm $m(\angle C) = 61°$

Comment: Reproduction machines vary the size of original copy so some discretion should be used in scoring student answers. Once the measurements of the student copy are known, student answers might be counted correct within ± 0.2 cm, ± 2°, and ± 10 cm^2.

Students can find an altitude of triangle ABC by construction or calculation using the sine function. The area can be found by calculation of the areas of the two right triangles with the altitude as one leg. Alternatively, the area of a triangle can be found by this formula, $A = \frac{1}{2} ab \sin \Theta$, where a and b are sides of the triangle and Θ the angle between.

ACTIVITY 7. Assesses Strands 3 and 8.

Solution: a. mean = \$82,542.55 median = \$24,000 mode = \$24,000

b. Select the median/mode to maximize the salary gap between executive salaries and this value for the "average".

c. Select the mean to show how much less hourly salaries are than this" average".

Comment: Actually, the best argument can probably be posed by showing that the percent of the total salaries for the Xylex Corporation that is paid to executives is 70.9%!

ACTIVITY 8. Assesses Strands 1, 6, and 8.

Solution: a. The volume of the Crunchies box is 8 times that of the Oaties box rather than twice its volume, which would be the appropriate comparison to show that the sales are double. This occurs because each dimension has been doubled.

$$\text{If } V_o = l_o w_o h_o \quad \text{then } V_c = (2l_o)(2w_o)(2h_o)$$
$$= 8\, l_o w_o h_o$$
$$= 8\, V_o$$

b. The scale factor is used in each of the three dimensions, length, width, and height. Students need to find a scale factor *k* such that *k* x *k* x *k* = 2. Using guess and check with a calculator, an acceptable scale factor easy to use is *k* = 1.25 (which enables students to multiply each dimension by one and one-fourth).

c. Give the students a net (flat, unfolded box with tabs) for constructing a rectangular box and explain how each dimension should be lengthened by the factor 1.25 in order to produce twice the volume of the box determined by the net.

ACTIVITY 9. Assesses Strand 5.

Solution:

(1) 155 ft
(2) 15 min
(3) no; from 10:00 to 10:07 am
(4) altitude is constant
(5) about 8:05, 8:50 till 9:15, 11:33 am
(6) 450 ft; 55 min
(7) 475 ft and rising
(8) 450 ft and standing still
(9) 10:20 to 11:15 am
(10) 250 - 350 ft
(11) 15 min
(12) 15 min; 49 min (other interpretations possible)
(13) 0 - 125 ft, 350 - 250 ft, 100 - 0 ft
(14) would indicate multiple altitudes at the same time
(15) rather bumpy; descending quite fast, leveled off at the end

ACTIVITY 10. Assesses Strands 2 and 4.

Solution: b. 72.7 ft

Part II (some selected values)

t	$t+1$	$\sqrt{2t+1}$
4	5	3
12	13	5
24	25	7
40	41	9
60	61	11
112	113	15
220	221	21
364	365	27
612	613	35
924	925	43
1300	1301	51
1740	1741	59
2112	2113	65
2520	2521	71
2664	2665	73

ACTIVITY 11. Assesses Strand 4.

Solution:

a. reflection in the x-axis
b. reflection in the y-axis
c. reflection in the origin OR rotate 180° around the origin
d. rotate 90° clockwise
e. replace (x, y) with (-y, x)
f. replace (x, y) with (x+3, y-5)
g. replace (x, y) with (2x, 2y)

ACTIVITY 12. Assesses Strand 3.

Comment: Good sources for students' research are:

The Mathematics Teacher, *Mathematics Teaching in the Middle School*, and *Teaching Children Mathematics*, all available from the National Council of Teachers of Mathematics, 1906 Association Drive, Reston, VA 22091.

Notable Numbers. William T. Stokes. © 1974, P.O. Box 415, Palo Alto, CA 94302.

Fascinating Fibonaccis: Mystery and Magic in Numbers. Trudi Hammel Garland. Dale Seymour Publications, P.O. Box 10888, Palo Alto, CA 94303.

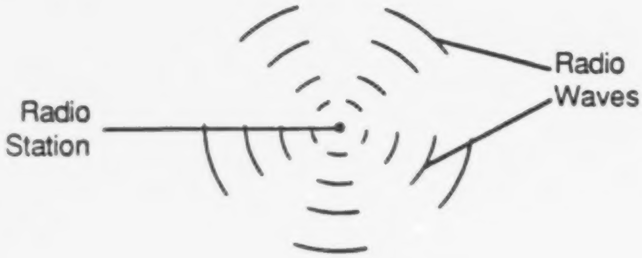
Creative Publications, P.O. Box 10328, Palo Alto, CA 94302

ACTIVITY 1

Radio Transmissions

Radio station WMAT in Math City is 200 miles from radio station WGEO in Geometry City. Highway 8, a straight road, connects the two cities.

WMAT broadcasts can be received up to 150 miles in all directions from the station and WGEO broadcasts can be received up to 125 miles in all directions. Radio waves travel from each station through the air, as represented below.



1. Draw a diagram to scale that shows the following:

   a. Highway 8

   b. The location of the two radio stations.

   c. The part of Highway 8 where both radio stations can be received.

Be sure to label the distances along the highway and the length in miles of the part of the highway where both stations can be received.

2. Construct on your drawing, and then shade, the region on either side of Highway 8 where houses can be located and receive both stations.

ACTIVITY 2

P . A .. T ... T .... E ..... R ...... N .... S

Your teacher will give you a sheet of patterns that have been formed by individual square tiles. You will also be given a number of tiles to use to make some of these figures.

For each SET of figures on the Pattern Worksheet:

a. Use your tiles to make the fourth and fifth figures in each pattern.

b. On square grid paper, draw the fifth figure you have constructed.

c. Predict how many tiles it would take to make the tenth figure in each pattern.

d. In words, describe any patterns that you see in how the patterns "grow".

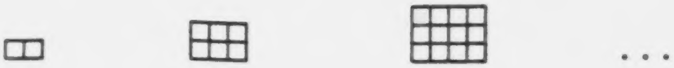
e. Determine a mathematical rule for the pattern. The rule should enable you to determine the number of tiles needed to make a figure if its position (tenth, hundredth) in the pattern is known.

Here are some hints for finding a rule for a pattern. Try the hints in various combinations as they seem appropriate.

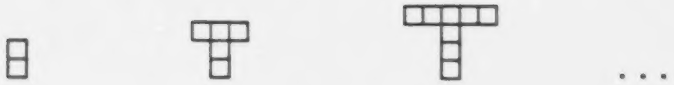
- Build the pattern from scratch. Make the first figure. Then change it into the second. Change that into the third, and so on. What is the operation? (e.g. add 5, multiply by 2)
- Rearrange the tiles after you have made the figure. Make a new figure in which the total number of tiles is easier to determine visually.
- Try to predict the total in terms of the position number of the figure (e.g., three times its position + 1).
- Think of the figure as consisting of more than one part. Maybe the corresponding parts of each figure are growing according to a simpler pattern.

PATTERNS Worksheet

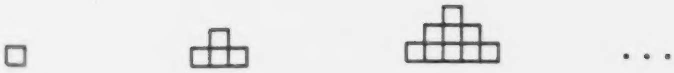
1.



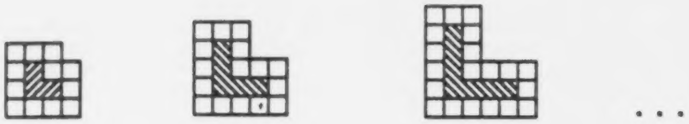
2.



3.

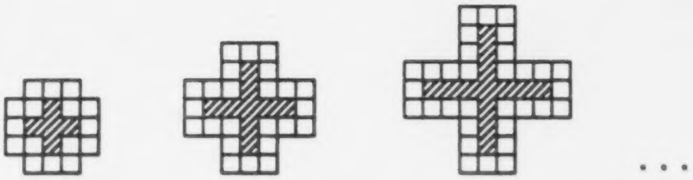


4.

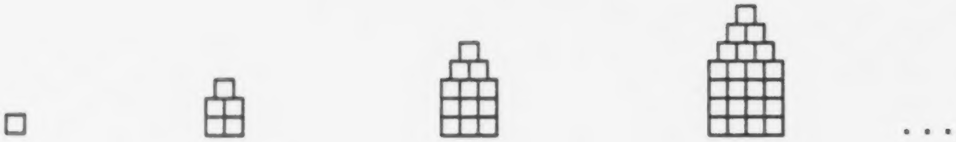


For patterns (4) and (5), carry out the instructions for both the white tiles and then the shaded (black tile) regions of each figure.

5.



6.



ACTIVITY 3

Phone Calls

	Day Rate 8 AM - 5 PM Mon - Fri		**Evening Rate** 5 PM - 11 PM Mon - Fri 8 AM - 11 PM Sat - Sun		**Night Rate** 11 PM - 8 AM ALL DAYS	
From Allenville To	First Minute	Each Additional Minute	First Minute	Each Additional Minute	First Minute	Each Additional Minute
Burneyford	$0.09	$0.03	$0.07	$0.02	$0.05	$0.02
Camptown	$0.28	$0.09	$0.22	$0.07	$0.17	$0.05
Dorning	$0.37	$0.11	$0.30	$0.09	$0.22	$0.07
Edgeton	$0.42	$0.12	$0.34	$0.10	$0.25	$0.07

The table above provides information about the cost of placing phone calls between certain cities at different times during the day.

a. How much more would it cost to place a 10-minute call from Allenville to Edgeton at 3 PM on Friday than at 3 PM on Saturday?

b. What is the difference in cost for a 30-minute call from Allenville to Dorning at 4:45 pm on Monday and 10:45 pm on Monday?

ACTIVITY 4

Calendar Sums

JULY

S	M	T	W	T	F	S
		1	2	3	4	5
6	7	8	9	10	11	12
13	14	15	16	17	18	19
20	21	22	23	24	25	26
27	28	29	30	31		

The dates on a calendar for the Mondays of a month were added. What would be the least possible sum? Consider all possible cases, for any month and any position of dates on the calendar. Show your reasoning.

ACTIVITY 5

Lung Capacity

Two blowhards were arguing over who had the biggest lung capacity. As usual, they were all words and no action. Brainstorm within your work group how they could have put the matter to rest with scientific measurement.

a. Your teacher will distribute two types of balloons to each of you and some measuring tools. One type of balloon will be roughly spherical when blown up. Recall that the formula for the volume of a sphere is $V = 4/3 \cdot \pi r^3$.

b. Taking a deep breath, exhale as much air as possible into the spherical balloon and knot the end. This volume will serve as an approximation of your lung capacity. Decide what measurements to take on this balloon and then determine the volume in terms of deciliters. (1 deciliter = 100 cubic centimeters.)

Measurements: *Volume:*

c. The other balloon that each of you received is cylindrical in shape when inflated, with roughly hemispherical ends (half-spheres). Recall that the formula for the volume of a cylinder is $V = 1/3 \pi r^2 h$.

d. Repeating the directions in (b), exhale your lung capacity into the cylindrical balloon and knot the end. Using a felt tip marker, mark the balloon into three sections: its two hemispherical ends and the cylinder in between. Decide what measurements to take on this balloon and then determine the volume in deciliters.

Measurements: *Volume:*

e. How do the two calculated volumes compare? To what do you account any differences? Decide on the value you will use as your lung capacity and report it on a piece of paper to your teacher along with your height in inches and gender.

f. Your teacher will put individual data on a table with the gender noted. Create a scatterplot from this data to determine whether there is any relationship between height and gender vs lung capacity. Use ● for males and ✖ for females in plotting your values. Do you think there is a relationship? Why or why not?

ACTIVITY 6

Triangular Area

For this activity you will be using a centimeter ruler and a protractor. Measure all lengths to the closest tenth of a centimeter (or closest millimeter) and angles to the nearest degree. You can draw any other segments or angles you feel will be necessary to find the area of the triangle. Label all answers correctly.

AB =

m (∠A) =

AC =

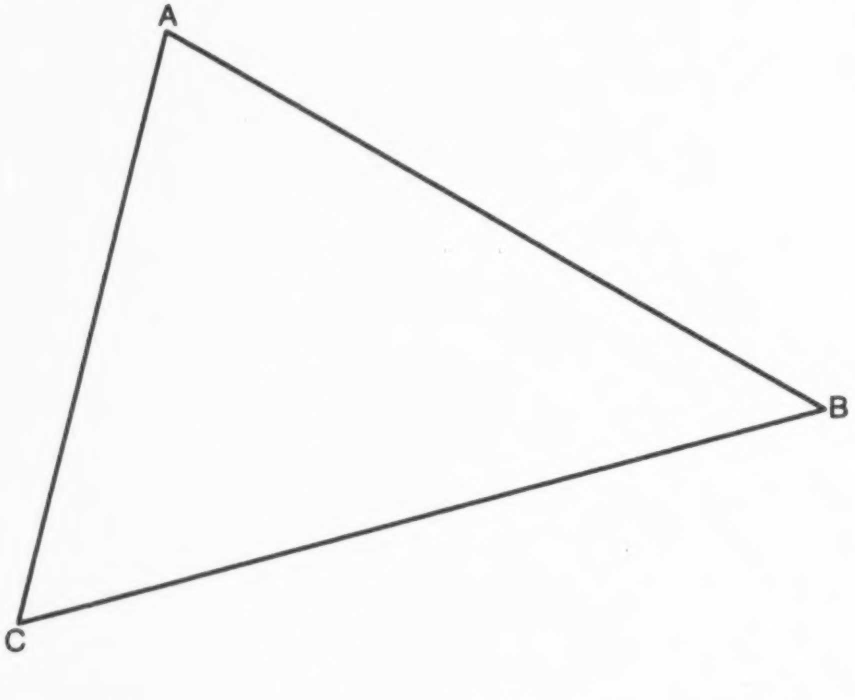
m (∠B) =

BC =

m (∠C) =

Measures of any other parts you drew:

Determine the area of ΔABC and justify your answer (that is, how did you find the area).



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ACTIVITY 7

Salary Negotiations

The Xylex Corporation listed the salaries of its employees as follows:

Xylex Salaries

CEO	$ 1.8 million
President	$ 500,000
Executive Vice President	$ 200,000
Vice President-Sales	$ 125,000
Vice President-Advertising	$ 125,000
Plant Manager (2)	$ 55,000
Foreman (6)	$ 35,000
Sales Representatives (6)	$ 32,000
Assembly Workers (20)	$ 24,000
General Office (5)	$ 18,500
Custodian (3)	$ 15,000

(numbers in parentheses are number of employees at that position)

a. Find the mean salary, median salary, and mode salary.

b. If you wanted to argue that executives (vice-president and above) were overpaid, which of these measures of the "average" would you use? Why?

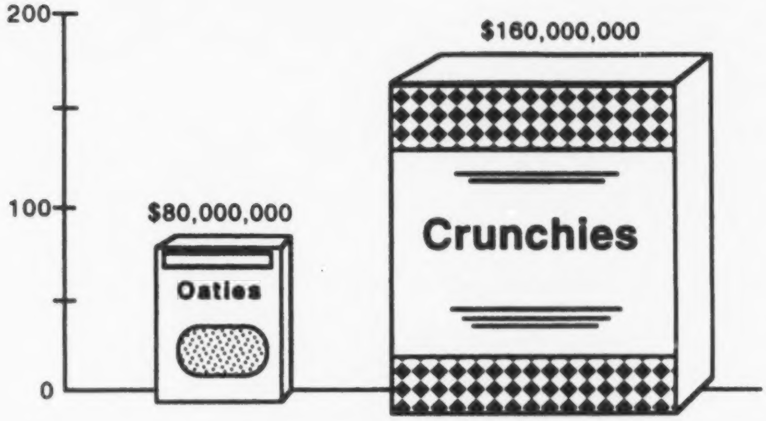
c. If you wanted to argue that hourly employees (nonexecutives) need more pay, which of these measures would you use? Why?

ACTIVITY 8

Misleading Graphs

The picture graph shown below is misleading.

ANNUAL CEREAL SALES



a. Explain why the graph is not a fair representation of the information.

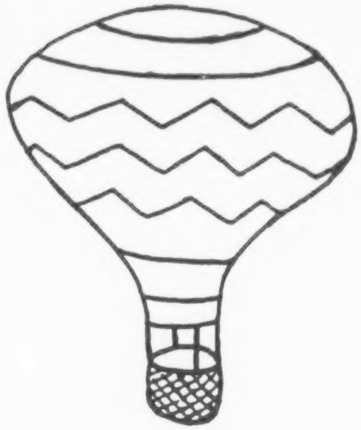
b. What scale factor should you use to make the volume of the Crunchies box twice that of the Oaties box? (That is, by what constant factor should each of the measurements of the Oaties box be multiplied to result in the Crunchies box appearing to be twice the volume of the Oaties box?)

c. On a sheet of graph paper, draw the graph as it should be.

d. Bring a rectangular box from home. Using poster board supplied by your teacher, create a pattern for a box with double the volume of the box from home. Make the box. Using packing material or some other material supplied by your teacher, verify that your created box has a volume twice that of the box from home.

ACTIVITY 9

Hot Air Ballooning



Story: At 7:30 am, three balloonists lift off the ground in the gondola of a hot air balloon. They ascend to an altitude of 125 feet where they pause to check the ballon's controls. They then rise to an altitude of 350 feet. A circling airplane concerns them, so they descend to 250 feet and wait until it leaves. They then lift to their target altitude of 600 feet, later coming down to 450 feet where they take a number of photographs. Afterward they descend, touching the ground again at 11:45 am.

Your teacher will give you a copy from the altimeter graphing device that recorded the balloon's altitude with the passage of time. Use this graphical record to answer the following questions.

Part I

Questions:

1. At what altitude was the balloon at 8:00 am?
2. How long did the balloon remain at 350 ft?
3. Did the balloon remain at 600 ft very long? How do you know?
4. What does it mean when the graph is flat?
5. When was the balloon at 250 ft?
6. At what altitude did the balloon remain the longest? How long was that?

7. At what height was the balloon at 9:45 am and what was it doing?

8. At what height was the balloon at 10:30 am and what was it doing?

9. During what time interval was the balloon at 450 ft?

10. The balloon was at some altitudes four different times. What were they?

11. How long did it take the balloonists to check their controls when they stopped first?

12. How long did it take them to go from 325 ft to 475 ft? (two answers)

13. When was the balloon rising the fastest? descending the fastest?

14. Why doesn't the graph ever go straight up?

15. Was the balloon landing safe? Why do you think so?

Part II

Make up your own story of some interesting acitivity in which some aspect of that activity can be described by a graph. Then make the graph that goes along with the acitivity. Make sure the graph has all the important information to describe the activity in the story. Then make up at least ten questions about the activity in your story that can be answered from your graph.

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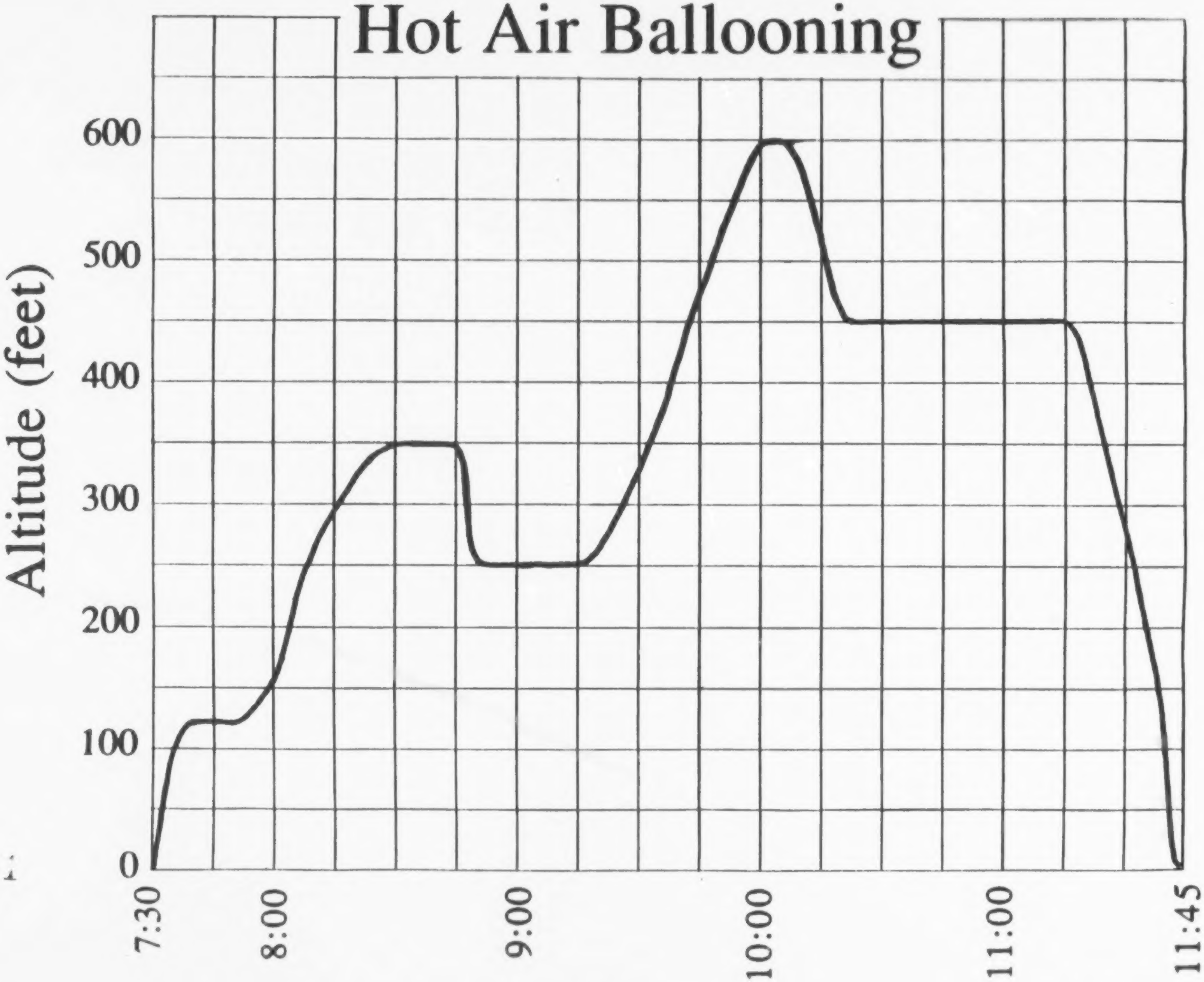
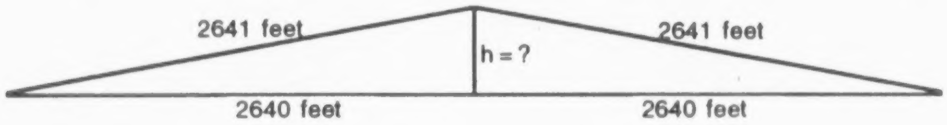

Hot Air Ballooning
Altitude (feet)
600
500
400
300
200
100
0
7:30
8:00
9:00
10:00
11:00
11:45

ACTIVITY 10

Why Expansion Joints?

Suppose a steel rail for a railroad track were made in a single length of one mile (5280 ft). Suppose the rail is securely anchored on each end. During the day, as the temperature increases, the steel expands and the rail lengthens by 2 feet. If the rail rises off the ground and forms two right triangles with a common altitude **h**, how high off the ground would the rail reach at this maximum point?



Part I

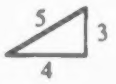
a. First make a guess as to how much **h** might be.

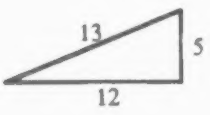
b. Then calculate h using the Pythagorean Theorem. (A calculator is helpful.)

c. Does this result seem plausible?

Part II

Pythagorean triplets are sets of whole numbers a, b, c that satisfy the Pythagorean Theorem $a^2 + b^2 = c^2$. To determine whether the result in Part I is reasonable, we will look at the pattern of Pythagorean triplets where the base of the triangle is one less than the hypoteneuse (3-4-5 and 5-12-13 triangles are examples). The altitudes of these two trianglesare 3 and 5 respectively.





To find such triangles easily, suppose the base is t. Then the hypoteneuse will be t + 1 and the altitude will be $\sqrt{2t + 1}$.

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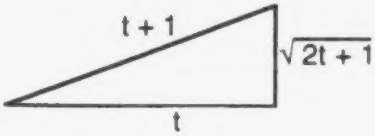
IF the expression $2t + 1$ is a perfect square, then t, $t + 1$, and $\sqrt{2t + 1}$ give us Pythagorean Triplet.



Using a calculator or a spreadsheet, complete a table of Pythagorean Triplets up to the situation in Part I to determine whether that result *is* reasonable.

t	$t + 1$	$\sqrt{2t + 1}$
4	5	3
12	13	5

(Hint: the altitudes in this table will be consecutive odd whole numbers.)

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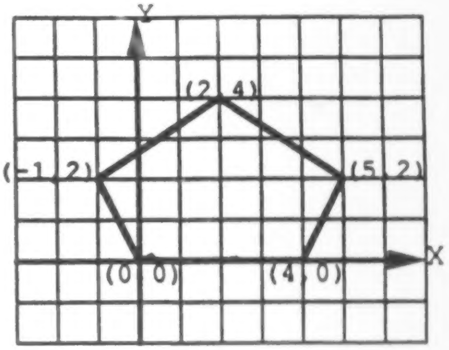
ACTIVITY 11

Reflections, Rotations, Slides, and Scaling

Once created and described mathematically, simple or complicated figures can be easily changed (location, position, size) by giving simple instructions to a device – such as a computer. For example, consider that the pentagon on the grid below is described by the coordinates at its vertices.



By giving instructions for these coordinates, we can slide a figure over, show its mirror reflection, rotate it, and change its size. We describe how to make these changes by telling how each of its coordinate pairs should be replaced.

Replace (x,y) with (x,-y) means:

> plot a new point for each given point by using the *same* x-coordinate and the *opposite* y-coordinate. For example, the point P(3,-2) would be replaced by the new point P'(3,2).

Part I

Working with a piece of graph paper and the coordinates of the pentagon above,

- make the following indicated changes in each coordinate pair
- draw the new figure
- describe in words the type of change that occurred in the figure

a. Replace (x,y) with (x,-y) .

b. Replace (x,y) with (-x,y) .

c. Replace (x,y) with (-x,-y) .

d. Replace (x,y) with (y,-x) . [Switches the coordinates, but uses the opposite of the x-coordinate].

Part II

e. Describe a change that will rotate the figure by 90° in a counter-clockwise direction.

f. Describe a change that will slide the figure over by 3 units to the right and 5 units down.

g. Describe a change that will enlarge the figure by a scale factor of 2.

ACTIVITY 12

Fascinating Numbers

Numbers have fascinated people across the ages. Some have considered specific numbers to have special properties (lucky, magical, appealing). Others have decided to name numbers with particular characteristics. For example, the Greeks considered the even counting numbers to be *male* and the odd counting numbers to be *female.*

Your project is to do research in the media center (or other information source as directed by your teacher) and to write a paper on one of the following topics.

Amicable numbers

Palindromic numbers

Fibonacci numbers

Narcissistic numbers

The Golden Ratio

Perfect Squares

Clock Arithmetic (5-hr clock)

Factorial numbers

Abundant, perfect, deficient numbers

Polygonal numbers (triangular, square, pentagonal, ...)

Goldbach's Conjecture

Divisibility Rules (2 through 11)

History of symbols for numbers

Your paper should:

- be at least 3 pages in length, neat, legible, and finished in appearance
- should include definitions, if appropriate
- provide examples and/or drawings
- list any special properties of these particular numbers
- list any discoveries that you might make about the numbers
- identify any source(s) used
- include any relevant history about the particular numbers